Buffer-Aided Physical-Layer Network Coding Techniques for Cooperative Networks

Jiaqi Gu [1], Rodrigo C. de Lamare [2], [1] and Mario Huemer [3]
[1] Communications Research Group, Department of Electronics, University of York, United Kingdom
[2] CETUC, PUC-Rio, Rio de Janeiro, Brazil
[3] Institute of Signal Processing, Johannes Kepler University Linz, Austria
Emails: jg849@york.ac.uk, rodrigo.delamare@york.ac.uk, Mario.Huemer@jku.at

Abstract—**In this work, we present buffer-aided physical-layer network coding (PLNC) schemes and develop novel PLNC algorithms for cooperative direct-sequence code-division multiple access systems. In the proposed buffer-aided PLNC schemes, we employ a relay pair selection algorithm to select the relay pair and the packets in the buffer entries with the best performance and the associated link combinations used for data transmission. We then develop PLNC algorithms based on the design of linear network coding matrices using maximum likelihood and minimum mean-square error criteria to generate the network coded symbols that are sent to the destination. Simulation results show that the proposed techniques significantly outperform prior art in sum-rates and bit error rates.**

I. INTRODUCTION

Interference suppression techniques for wireless communications have been widely investigated in the last decades. Unlike traditional strategies that treat interference as a nuisance to be avoided or mitigated, physical-layer network coding (PLNC) techniques take advantage of the superposition of radio signals and exploit the interference to improve throughput performance [1]. PLNC techniques have generated a number of fertile theoretical and application-oriented studies in the last few years, and are expected to be successfully implemented in future wireless applications [2], [3], [4], [5], [6], [7], [8].

PLNC has key advantages in wireless multi-hop networks such as higher sum rates and enhanced bit error rate (BER) performance as compared to standard cooperative techniques. In a scenario where multiple relay nodes are employed in the network to transmit data from sources to the destination [9], PLNC techniques allow a node to exploit signals that are received simultaneously, rather than treating them as interference [9]. Additionally, instead of decoding each incoming data stream separately, a node detects and forwards a function of the incoming data streams [10]. In this context, there are several network coding approaches, namely, XOR mapping schemes and linear network coding designs [1], [11], [2], [12], [3], [7].

In cooperative relaying systems, strategies that employ relays have been investigated in [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26]. In order to further increase the quality and reliability of cooperative schemes, buffer strategies have been introduced and used to equip relay nodes in cooperative relaying scenarios [27], [28], [29], [30], [31]. In this setting, relay selection algorithms must be employed to obtain the combination of links in the buffer entries that optimize a desired criterion and may result in performance improvements.

In this work, we present buffer-aided PLNC schemes and develop relay selection and PLNC algorithms for cooperative direct-sequence code-division multiple access (DS-CDMA) systems. In the proposed buffer-aided PLNC schemes, a relay pair selection algorithm is devised to obtain the relay pair and the packets in the buffer entries with the highest SINR and the associated link combinations used for data transmission. We then develop PLNC techniques based on optimal code designs, which employ linear network coding matrices computed according to the maximum likelihood (ML) and the minimum mean-square error (MMSE) criteria. The proposed ML and MMSE linear network code designs are used to generate the network coded symbols (NCS) that are sent to the destination. Simulations show that the proposed techniques significantly outperform prior art.

This paper is organized as follows: Section I introduces the cooperative DS-CDMA system model. Section II presents the proposed ML and MMSE linear network code designs, whereas Section III describes the proposed buffer-aided PLNC transmission scheme. Section IV presents the simulations and Section V gives the conclusions.

This work is funded by the ESII consortium in the UK under Task 26 for low-cost wireless ad hoc and sensor networks.

II. COOPERATIVE DS-CDMA SYSTEM MODEL

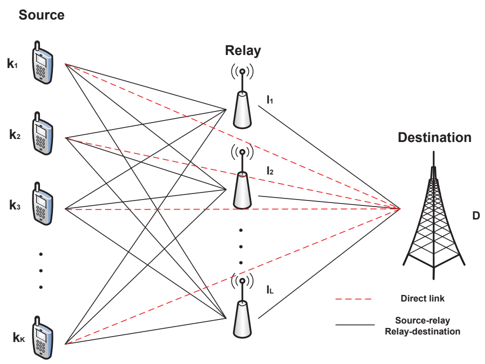

Fig. 1. Uplink of a cooperative DS-CDMA system.

Let us consider the uplink of a cooperative synchronous DS-CDMA system with two transmission phases, K users, L relays equipped with finite-size buffers store up to J packets and use N chips per symbol to transmit data over flat fading channels, as shown in Fig. 1. The network is equipped with a cooperative protocol at each relay and the transmit data are organized in packets of P symbols. The received signals are filtered by a matched filter, sampled at chip rate to obtain sufficient statistics and organized in $N \times 1$ vectors $\mathbf{y}_{sr}$, $\mathbf{y}_{sd}$ and $\mathbf{y}_{rd}$, which represent the signals received from the sources to the relays, the sources to the destination and the relay to the destination, respectively. Every set of m users and m relays are assigned to a group, forming pairs of users and relays.

In the first phase, the signals received at the destination and the l-th relay can be described as

$$\mathbf{y}_{sd} = \sum_{k=1}^{K} a_{sd}^{k} \mathbf{s}_k h_{sd,k} b_k + \mathbf{n}_{sd}, \quad (1)$$

$$\mathbf{y}_{sr_l} = \sum_{k=1}^{K} a_{sr_l}^{k} \mathbf{s}_k h_{sr_l,k} b_k + \mathbf{n}_{sr_l}, \quad (2)$$

where b_k are the transmitted modulated symbols. a_{sd}^k and $a_{sr_l}^k$ are the k-th user's amplitude from the source to the destination and from the source to relay l, respectively. The vector $\mathbf{s}_k = [s_k(1), s_k(2), ...s_k(N)]^T$ is the signature sequence for user k. $h_{sd,k}$ and $h_{sr_l,k}$ contain the complex channel coefficients from user k to the destination and from user k to the l-th relay, respectively. $\mathbf{n}_{sd}$ and $\mathbf{n}_{sr_l}$ are the noise vectors with samples of zero mean complex Gaussian noise with variance σ^2.

After the detected symbols are obtained, a PLNC scheme is employed at each relay. When the bit-wise XOR operation (the modulo-2 sum in the binary field) is considered, we assume that the mapping from user symbol c_k to a modulated symbol b_k is denoted as $\mathcal{M}_{\mathcal{B}}$. This mapping is then expressed as

$$b_k = \mathcal{M}_B = 1 - 2c_k. \quad (3)$$

We then perform the above mapping from $\hat{b}_{r_l d,k}$, which represents the decoded symbol for user k at the output of relay l using a relay protocol, to its corresponding user symbol as given by

$$\hat{b}_{r_l d,k} \rightarrow c_l^k. \quad (4)$$

Similarly, when linear network coding is adopted at the l-th relay, the corresponding mapping $\mathcal{L}_{\mathcal{B}}$ is described by

$$b_l = \mathcal{L}_B = \sum_{k=1}^{K} g_{kl} \hat{b}_{r_l d,k}. \quad (5)$$

In the following, we briefly review existing PLNC schemes that employ bit-wise XOR or linear network codes at each relay. When the signals are sent to the relays, we obtain the detected symbol as given by

$$\hat{b}_{r_{\Omega(l)}d,\Upsilon(k)} = Q\Big((\mathbf{w}_{s_{\Upsilon(k)} r_{\Omega(l)}})^H \mathbf{y}_{sr_l}\Big), \quad (6)$$

where $Q(\cdot)$ is the decision device, Υ is a $1 \times m$ user set and Ω is the corresponding $1 \times m$ relay set since we combine every m users and m relays into a sub transmission group. The symbol $\hat{b}_{r_{\Omega(l)}d,\Upsilon(k)}$ is the detected symbol for user $\Upsilon(k)$ at relay $\Omega(l)$, $\mathbf{w}_{s_{\Upsilon(k)} r_{\Omega(l)}}$ refers to the linear receive filter for user $\Upsilon(k)$ at relay $\Omega(l)$. Then, the detected symbol is mapped through (3) to obtain the detected user symbol given by

$$c_{\Omega(l)}^{\Upsilon(k)} = (1 - \hat{b}_{r_{\Omega(l)}d,\Upsilon(k)})/2. \quad (7)$$

If the bit-wise XOR operation is used at relay $\Omega(l)$ then we apply

$$c_{\Omega(l)} = c_{\Omega(l)}^{\Upsilon(1)} \oplus c_{\Omega(l)}^{\Upsilon(2)} \oplus ... \oplus c_{\Omega(l)}^{\Upsilon(m)}, \quad (8)$$

where $c_{\Omega(l)}$ denotes the NCS at relay $\Omega(l)$ and $\oplus$ is the bit-wise XOR operation. After that and prior to transmitting the encoded data to the destination the following mapping is required:

$$b_{\Omega(l)} = 1 - 2c_{\Omega(l)}. \quad (9)$$

Consequently, in the second phase the encoded symbols are stored and prepared to be sent to the destination. The NCS is then transmitted from relay set Ω to the destination as described by

$$\mathbf{y}_{r_\Omega d} = \sum_{l=1}^{m} \mathbf{h}_{r_{\Omega(l)}d} b_{\Omega(l)} + \mathbf{n}_{rd}, \quad (10)$$

where $\mathbf{h}_{r_{\Omega(l)}d} = a_{r_{\Omega(l)}d}^{\Upsilon} \mathbf{s}_\Upsilon h_{r_{\Omega(l)}d}$ denotes the $N \times 1$ channel vector from relay $\Omega(l)$ to the destination, $a_{r_{\Omega(l)}d}^{\Upsilon}$ is the amplitude for the combined source symbols (user set Υ) from the $\Omega(l)$-th relay to the destination, $h_{r_{\Omega(l)}d}$ is the complex channel fading coefficient from the $\Omega(l)$-th relay to the destination, $\mathbf{s}_\Upsilon$ is the $N \times 1$ spreading sequence assigned to the NCS $b_{\Omega(l)}$ and $\mathbf{n}_{rd}$ is the $N \times 1$ zero mean complex Gaussian noise with variance σ^2.

At the destination, the detected symbol for user $\Upsilon(k)$ from the direct transmissions is obtained as

$$\hat{b}_{sd}^{\Upsilon(k)} = Q\Big((\mathbf{w}_{sd}^{\Upsilon(k)})^H \mathbf{y}_{sd}\Big), \quad (11)$$

where $\mathbf{w}_{sd}^{\Upsilon(k)}$ is the detector for user $\Upsilon(k)$ used at the destination. After that, according to (3), the corresponding user symbol $\hat{c}_{sd}^{\Upsilon(k)}$ is then generated as given by

$$\hat{c}_{sd}^{\Upsilon(k)} = (1 - \hat{b}_{sd}^{\Upsilon(k)})/2. \quad (12)$$

Similarly, the detected symbol for relay $\Omega(l)$ from the second phase transmission is given by

$$\hat{b}_{\Omega(l)} = Q\Big((\mathbf{w}_{r_{\Omega(l)}d})^H \mathbf{y}_{r_\Omega d}\Big), \quad (13)$$

where $\mathbf{w}_{r_{\Omega(l)}d}$ is the receive filter for the $\Omega(l)$-th relay-destination link. Consequently, its corresponding user symbol is obtained through (3) as given by

$$\hat{c}_{\Omega(l)} = (1 - \hat{b}_{\Omega(l)})/2. \quad (14)$$

Thus, the final detected symbol $\hat{c}^{\Upsilon(k)}$ when the XOR operation is adopted at the target relays is given by

$$\hat{c}^{\Upsilon(k)} = \hat{c}_{\Omega(l)} \oplus \hat{c}_{sd}^{\Upsilon(1)} \oplus ...\hat{c}_{sd}^{\Upsilon(k-1)} \oplus \hat{c}_{sd}^{\Upsilon(k+1)} ... \oplus \hat{c}_{sd}^{\Upsilon(m)}. \quad (15)$$

Consequently, the final detected symbol for user $\Upsilon(k)$ is obtained according to the following mapping

$$\hat{b}_{\Upsilon(k)} = 1 - 2\hat{c}^{\Upsilon(k)} \quad (16)$$

If a linear network coding technique is adopted at the relays, every relay group will be allocated a unique $m \times m$ linear network coding matrix $\mathbf{G}$ whose structure is given by

$$\mathbf{G} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1m} \\ g_{21} & g_{22} & \cdots & g_{2m} \\ & & \vdots & \\ g_{m1} & g_{m2} & \cdots & g_{mm} \end{bmatrix}. \quad (17)$$

Thus, the following linear combination can be obtained as

$$b_{\Omega(l)} = \sum_{k=1}^{m} g_{kl}\, \hat{b}_{r_{\Omega(l)}d,\Upsilon(k)}, \quad (18)$$

where $\hat{b}_{r_{\Omega(l)}d,\Upsilon(k)}$ is the detected symbol for user $\Upsilon(k)$ at relay $\Omega(l)$. Therefore, the following transmission is operated according to (10) when a suitable time interval comes. Equivalently, (10) can be rewritten as

$$\mathbf{y}_{r_\Omega d} = \mathbf{H}_{r_\Omega d}\mathbf{G}\mathbf{b}_\Omega + \mathbf{n}_{r_\Omega d} \tag{19}$$

where $\mathbf{H}_{r_\Omega d}$ is an $N \times m$ channel matrix, each $N \times 1$ column $\mathbf{h}_{r_{\Omega(l)}d}$ in this matrix represents the channel vector from the $\Omega(l)$-th relay to the destination, $\mathbf{b}_\Omega$ is a $m \times 1$ vector containing the detected symbols for all users from set Υ at any relay that belongs to Ω. The vector $\mathbf{n}_{r_\Omega d}$ is the $N \times 1$ noise vector.

Similarly, at the destination side, the detected symbol for users from set Υ after using the linear network coding operation at each of the relays is given by

$$\hat{b}_{\Omega(l)} = Q\Big(\mathbf{G}^{-1}(\mathbf{w}_{r_\Omega d})^H \mathbf{y}_{r_\Omega d}\Big). \tag{20}$$

Traditionally, the decoding process for PLNC is expressed by

$$\begin{aligned} \hat{b}_{\Omega(1)} &= g_{\Upsilon(1)\Omega(1)}\hat{b}_{\Upsilon(1)} + g_{\Upsilon(2)\Omega(1)}\hat{b}_{\Upsilon(2)} + ... + g_{\Upsilon(m)\Omega(1)}\hat{b}_{\Upsilon(m)} \\ &\vdots \\ \hat{b}_{\Omega(m)} &= g_{\Upsilon(1)\Omega(m)}\hat{b}_{\Upsilon(1)} + g_{\Upsilon(2)\Omega(m)}\hat{b}_{\Upsilon(2)} + ... + g_{\Upsilon(m)\Omega(m)}\hat{b}_{\Upsilon(m)} \end{aligned} \tag{21}$$

The m unknown detected results $\hat{b}_{\Upsilon(k)}, k = 1, 2, ...m$ for each user in user set Υ can easily be sorted out with the above m equations. However, in this scenario, since we introduce the direct transmissions, there is another approach that can also be applied at the decoding process. Specifically, $\hat{b}_{\Upsilon(k)}$ is given by

$$\hat{b}_{\Upsilon(k)} = \frac{\hat{b}_{\Omega(l)} - \big(g_{\Upsilon(1)\Omega(l)}\hat{b}_{sd}^{\Upsilon(1)} + \ldots + g_{\Upsilon(m)\Omega(l)}\hat{b}_{sd}^{\Upsilon(m)}\big)}{g_{\Upsilon(k)\Omega(l)}}. \tag{22}$$

With direct transmissions, we can reduce the bit error rate. In particular, there is a possibility that the detection for $\hat{b}_{\Omega(l)}, l \in \Omega$ is incorrect, which could directly lead to a problem when solving the system of equations in (21), causing incorrect decisions. However, this can be addressed when applying (22) as we can select another detected symbol $\hat{b}_{\Omega(s)}, s \neq l$ as the minuend in (22) instead of the possibly incorrect detected symbol $\hat{b}_{\Omega(l)}$.

III. Proposed Cooperative Linear Network Coding Schemes

In this section, we present novel linear network code designs for $\mathbf{G}$. The idea is inspired by the work in [32] but applied in a different context. In the cooperative PLNC transmission scenario, the simplest way to compute $\mathbf{G}$ is to generate it randomly [33], however, this approach often does not result in the optimum performance as it does not satisfy an optimality criterion. Therefore, in order to obtain performance advantages, we devise methods based on the ML and MMSE criteria.

A. Maximum Likelihood Design

When a linear network coding scheme is adopted at the relay, we introduce the $m \times m$ linear network code matrix $\mathbf{G}$, which is used to generate the NCS. An ML design of $\mathbf{G}$ that requires the evaluation of all possible (2^{m^2}) binary matrices is given by

$$\mathbf{G}^{\mathrm{ML}} = \arg\min_{\mathbf{G}} \left\| \begin{bmatrix} b_{\Omega(1)} \\ \vdots \\ b_{\Omega(m)} \end{bmatrix} - \mathbf{G}^{-1} \begin{bmatrix} \mathbf{w}^H_{r_{\Omega(1)}d}\mathbf{y}_{r_{\Omega(1)}d} \\ \vdots \\ \mathbf{w}^H_{r_{\Omega(m)}d}\mathbf{y}_{r_{\Omega(m)}d} \end{bmatrix} \right\|^2, \tag{23}$$

where $\mathbf{w}_{r_l d}$ is the linear detector used at relay l, $\mathbf{y}_{r_l d}$ is the signal transmitted from relay l. In summary, the principle of this approach is to obtain an $m \times m$ code matrix $\mathbf{G}^{\mathrm{ML}}$ to match $\mathbf{G}$ (or $\mathbf{G}^{\mathrm{ML}}$). Thus, the minimum distance between the transmitted symbols and detected symbols can be obtained. Note that this approach corresponds to a combinatorial problem where we must test all 2^{m^2} possible candidates to obtain $\mathbf{G}^{\mathrm{ML}}$ associated with the best performance. The computational cost is one of the disadvantages of this approach but the design of $\mathbf{G}^{\mathrm{ML}}$ can be carried out online. At the destination side, we employ the same matrix $\mathbf{G}^{\mathrm{ML}}$ that is applied at the relays in order to obtain better detection performance.

B. MMSE design

The ML approach in (23) does not consider the influence of noise. This fact motivates us to seek another efficient decoding method that can consider both interference and noise. In order to further exploit the minimum distance and improve the transmission performance, we introduce a code matrix $\tilde{\mathbf{G}}$ at the destination to match the generated binary matrix $\mathbf{G}$ and perform the linear network coding operation. The proposed MMSE linear network coding matrix $\mathbf{G}^{\mathrm{MMSE}}$ is obtained as follows:

$$\mathbf{G}^{\mathrm{MMSE}} = \arg\min_{\tilde{\mathbf{G}}} E\left[\left\| \begin{bmatrix} b_{\Omega(1)} \\ \vdots \\ b_{\Omega(m)} \end{bmatrix} - \tilde{\mathbf{G}} \begin{bmatrix} \mathbf{w}^H_{r_{\Omega(1)}d}\mathbf{y}_{r_{\Omega(1)}d} \\ \vdots \\ \mathbf{w}^H_{r_{\Omega(m)}d}\mathbf{y}_{r_{\Omega(m)}d} \end{bmatrix} \right\|^2 \right], \tag{24}$$

where ideally we have $b_{\Omega(1)} = b_{\Omega(2)} = ... = b_{\Omega(m)}, k \in [1, m]$. This design problem can be recast as the following:

$$\mathbf{G}^{\mathrm{MMSE}} = \arg\min_{\tilde{\mathbf{G}}} E\Big[\Big\|\mathbf{a} - \tilde{\mathbf{G}}\mathbf{b}\Big\|^2\Big], \tag{25}$$

where the quantities in the argument are $\mathbf{a} = [b_{\Omega(1)} \ \ldots \ b_{\Omega(m)}]^T$ and $\mathbf{b} = [\mathbf{w}^H_{r_{\Omega(1)}d}\mathbf{y}_{r_{\Omega(1)}d} \ \ldots \ \mathbf{w}^H_{r_{\Omega(m)}d}\mathbf{y}_{r_{\Omega(m)}d}]^T$. By taking the gradient of the cost function with respect to $\tilde{\mathbf{G}}$ and equating the terms to zero, we obtain the MMSE linear network code matrix given by

$$\mathbf{G}^{\mathrm{MMSE}} = \mathbf{P}_{ab}\mathbf{R}_b^{-1}, \tag{26}$$

where the statistical quantities in the MMSE code matrix are the cross-correlation matrix $\mathbf{P}_{ab} = E[\mathbf{a}\mathbf{b}^H]$ and the covariance matrix $\mathbf{R}_b = E[\mathbf{b}\mathbf{b}^H]$, and the expected value is over the symbols and the noise. The computational cost is $\mathcal{O}(N^3)$ and the details can be found at [34]. The elements of $\mathbf{P}_{ab}$ are given by

$$[\mathbf{P}_{ab}]_{k,j} = (\sum_{i=1}^{m} g_{ik}g_{ik}\sigma_i^2)\mathbf{h}^H_{\mathbf{r}_{\Omega(j)d}}\mathbf{w}_{\mathbf{r}_{\Omega(j)d}}, \text{ for } k, j = 1, \ldots, m, \tag{27}$$

whereas the main diagonal entries of $\mathbf{R}_b$ are described by

$$[\mathbf{R}_b]_{j,j} = \mathbf{w}^H_{\mathbf{r}_{\Omega(j)d}}(\mathbf{h}_{\mathbf{r}_{\Omega(j)d}}\mathbf{h}^H_{\mathbf{r}_{\Omega(j)d}} + \sigma^2\mathbf{I})\mathbf{w}_{\mathbf{r}_{\Omega(j)d}}, \text{ for} j = 1, \ldots, m. \tag{28}$$

IV. Proposed Buffer-Aided PLNC Scheme

In this section, we consider groups of $m = 2$ users and $m = 2$ relays and develop a buffer-aided PLNC scheme, where each relay is equipped with a buffer. This allows the received data to be stored and then the relay can wait until the link pair associated with the best performance is selected. Consequently, encoded data are stored at the buffer entries and then forwarded to the destination when the appropriate time interval comes. Moreover, the destination is also equipped with a buffer so that the detected symbols from the direct transmissions can be stored. After that, the detected symbols are obtained through PLNC decoding and mapping operations at the appropriate time instants.

Transmission between a selected user-relay pair ($m = 2$)
At the relays, the received signal is processed by linear network coding and forwarded to the destination, which employs a decoding matrix and a linear receiver. Specifically, the proposed algorithm starts with a selection procedure using all possible link combinations associated with all relay pairs from both source-relay and relay-destination phases. Since any linear network coding technique can be adopted, every ($m = 2$) relays are combined into a group and paired with ($m = 2$) users. Note that arbitrary numbers of users and relays can also be considered but for the sake of simplicity, we only consider $m = 2$ relays and users. In this case, the signal transmitted from other users are seen as the interference component. All possible links are considered and their SINR are then calculated:

$$\mathrm{SINR}_{sr\Omega} = \frac{\sum_{k=1}^{K}\sum_{l=1}^{m} \mathbf{w}^H_{s_k r_{\Omega(l)}} \rho_{s_k r_{\Omega(l)}} \mathbf{w}_{s_k r_{\Omega(l)}}}{\sum_{k=1}^{K}\sum_{\substack{j=1\\ j\notin\Omega}}^{L} \mathbf{w}^H_{s_k r_j}\rho_{s_k r_j}\mathbf{w}_{s_k r_j} + \sum_{l=1}^{m}\sigma^2 \mathbf{w}^H_{s_k r_{\Omega(l)}}\mathbf{w}_{s_k r_{\Omega(l)}}}, \tag{29}$$

$$\mathrm{SINR}_{r\Omega d} = \frac{\sum_{k=1}^{K}\sum_{l=1}^{m} (\mathbf{w}^k_{r_{\Omega(l)}d})^H \rho^k_{r_{\Omega(l)}d}\mathbf{w}^k_{r_{\Omega(l)}d}}{\sum_{k=1}^{K}\sum_{\substack{j=1\\ j\notin\Omega}}^{L} (\mathbf{w}^k_{r_j d})^H\rho^k_{r_j d}\mathbf{w}^k_{r_j d} + \sum_{l=1}^{m}\sigma^2 (\mathbf{w}^k_{r_{\Omega(l)}d})^H\mathbf{w}^k_{r_{\Omega(l)}d}}, \tag{30}$$

where $\rho_{s_k r_{\Omega(l)}} = \mathbf{h}^H_{s_k r_{\Omega(l)}}\mathbf{h}_{s_k r_{\Omega(l)}}$ is the correlation coefficient of the desired user k between the source and relay $\Omega(l)$, $\rho^k_{r_{\Omega(l)}d} = (\mathbf{h}^k_{r_{\Omega(l)}d})^H\mathbf{h}^k_{r_{\Omega(l)}d}$ is the correlation coefficient for user k from relay $\Omega(l)$ to the destination. $\mathbf{h}_{s_k r_{\Omega(l)}} = a_{s_k r_{\Omega(l)}}\mathbf{s}_k h_{s_k r_{\Omega(l)}}$ is the channel vector from user k to relay $\Omega(l)$. In (29), $\mathrm{SINR}_{sr\Omega}$ denotes the SINR for the combined paths from all users to relay set Ω, $\mathbf{w}_{s_k r_{\Omega(l)}}$ is the detector used at the relay $\Omega(l)$. When the RAKE receiver is adopted at the corresponding relay, $\mathbf{w}_{s_k r_{\Omega(l)}}$ is expressed as

$$\mathbf{w}_{s_k r_{\Omega(l)}} = \mathbf{h}_{s_k r_{\Omega(l)}}. \tag{31}$$

Similarly, if the linear MMSE receiver [35] is employed at the relays, $\mathbf{w}_{s_k r_{\Omega(l)}}$ is equal to

$$\mathbf{w}_{s_k r_{\Omega(l)}} = \left(\sum_{k=1}^{K}\mathbf{h}_{s_k r_{\Omega(l)}}\mathbf{h}^H_{s_k r_{\Omega(l)}} + \sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_{s_k r_{\Omega(l)}}, \tag{32}$$

$\mathbf{h}_{s_k r_{\Omega(l)}} = a_{s_k r_{\Omega(l)}}\mathbf{s}_k h_{s_k r_{\Omega(l)}}$ is the effective signature vector from user k to the relay $\Omega(l)$. In (30), $\mathrm{SINR}_{r\Omega d}$ represents the SINR for the combined paths from relay set Ω to the destination. The receive filter $\mathbf{w}^k_{r_{\Omega(l)}d}$ is employed by the detector used at the destination. When the RAKE receiver is adopted at the destination, $\mathbf{w}^k_{r_{\Omega(l)}d}$ is expressed as

$$\mathbf{w}^k_{r_{\Omega(l)}d} = \mathbf{h}^k_{r_{\Omega(l)}d}. \tag{33}$$

In an analog way, if the linear MMSE receiver is employed at the relays, $\mathbf{w}^k_{r_{\Omega(l)}d}$ is equal to

$$\mathbf{w}^k_{r_{\Omega(l)}d} = \left(\sum_{k=1}^{K}\mathbf{h}^k_{r_{\Omega(l)}d}(\mathbf{h}^k_{r_{\Omega(l)}d})^H + \sigma^2\mathbf{I}\right)^{-1}\mathbf{h}^k_{r_{\Omega(l)}d}. \tag{34}$$

Both RAKE and MMSE receivers are considered here due to their reasonably low complexity and it should be mentioned that other detectors [36], [37], [38], [39], [40], [41], [42], [43] including the ML and MAP [44], [45] detectors can also be used. Moreover, precoding techniques [46], [47], [48], [49], [50], [51], [52], [53], [54], [55] could also be considered for the downlink.

After the computation of receive filters and SINR values, we sort all user signals according to decreasing SINR values and choose the relay pair with the highest SINR as given by

$$\mathrm{SINR_{i,j}} = \arg\max_{\Omega\in[1,2,\ldots,L]}\{\mathrm{SINR_{sr\Omega}}, \mathrm{SINR_{r\Omega d}}\}, \tag{35}$$

where $\mathrm{SINR_{i,j}}$ denotes the highest SINR associated with the relay i and relay j. After the relay pair ($m = 2$) with the highest SINR is selected, the signal is transmitted through the corresponding channels. In this case, two different situations of the buffer mode need to be considered as follows.

Transmission mode:
If the link combinations associated with the selected relay set belongs to the relay-destination phase, the buffers are turned to the transmission mode. A buffer space check needs to be conducted first to ensure the corresponding buffer entries are not empty, namely:

$$\Phi_i^{\text{buffer}} \neq \varnothing,\ i\in[1,2,...,L], \tag{36}$$

and

$$\Phi_j^{\text{buffer}} \neq \varnothing,\ j\in[1,2,...,L], \tag{37}$$

where Φ_i^{buffer} and Φ_j^{buffer} denote the buffers equipped at relay i and relay j. If the corresponding buffer entries are not empty, we transmit the NCS according to (10). On the other hand, if the buffer condition does not satisfy the transmission requirements, namely, the buffer entries are empty, the selected relay pair cannot help to forward the NCS.

In this case, we drop the current relay pair i and j and choose another relay pair with the second highest SINR as given by

$$\mathrm{SINR^{pre}_{i,j}} = \mathrm{SINR_{i,j}} \tag{38}$$

$$\mathrm{SINR_{u,v}} \in \max\{\mathrm{SINR_{sr\Omega}}, \mathrm{SINR_{r\Omega d}}\} \setminus \mathrm{SINR^{pre}_{i,j}}, \tag{39}$$

where $\mathrm{SINR_{u,v}}$ denotes the second highest SINR associated with the updated relay pair u and v. $\{\mathrm{SINR_{sr\Omega}}, \mathrm{SINR_{r\Omega d}}\} \setminus \mathrm{SINR^{pre}_{i,j}}$ represents a complementary set, where we drop the $\mathrm{SINR^{pre}_{i,j}}$ from the set of SINR links $\{\mathrm{SINR_{sr\Omega}}, \mathrm{SINR_{r\Omega d}}\}$. Consequently, the above process repeats until the buffer condition achieves the transmission requirement.

Reception mode:

If the link combination associated with the highest relay pair SINR belongs to the source-relay phase, the buffers are switched to reception mode. Similarly, the buffer is checked to ensure that the corresponding buffer entries are not full. We then have

$$\Phi_i^{\text{buffer}} \neq \text{U},\ i \in [1, 2, ..., L], \tag{40}$$

and

$$\Phi_j^{\text{buffer}} \neq \text{U},\ j \in [1, 2, ..., L], \tag{41}$$

where U represents a full buffer set. If the buffers are not full, then the sources send the data to the selected relay pair i and j according to (2). Otherwise, the algorithm reselects a new relay pair as in (38) and (39). The re-selection process stops when the buffer entries are not full. The size J of buffers plays an important role in the system performance. When we increase the buffer size, better channels can be selected as a relatively larger candidate pool is generated. Therefore, extra degrees of freedom in the system are also available.

V. Sum-Rate Analysis

The sum rate is an important parameter to evaluate the system performance and it is usually measured according to the source, channel relay and destination parameters of a given system. Thus, in the following, we analyze the sum rate of the proposed buffer-aided cooperative PNC schemes and algorithms, and derive expressions for them according to the receive filters.

In particular, when XOR is applied at the selected relays, the sum rate for the corresponding system is given by

$$\begin{aligned} R \ &\leq \frac{1}{N+N(J+1)} \min \{ I_{DF}^{sr},\ I_{DF}^{sd,rd} \} \\ &= \frac{1}{N+N(J+1)} \min \{\ E[\log \det(\sigma^2 \mathbf{I}_{2N}\ +\ \mathbf{H}_{sr}\mathbf{Q}\mathbf{H}_{sr}^H)], \\ &\quad E\Big[\log\,(1 + \sum_{k=1}^{K} \text{SINR}_{sd}^k + \sum_{l=1}^{m} \text{SINR}_{r_{\Omega(l)}d})\Big] \\ &= \frac{1}{N+N(J+1)} \min \{\ E[\log \det(\sigma^2 \mathbf{I}_{2N}\ +\ \mathbf{H}_{sr}\mathbf{Q}\mathbf{H}_{sr}^H)], \\ &\quad E\Big[\log\,(1 + \sum_{k=1}^{K} \frac{(\mathbf{w}_{sd,k})^H \rho_{sd,k} \mathbf{w}_{sd,k}}{\sum\limits_{\substack{p=1 \\ p\neq k}}^{K} (\mathbf{w}_{sd,k})^H \rho_{sd,p} \mathbf{w}_{sd,k} + \sigma^2 (\mathbf{w}_{sd,k})^H \mathbf{w}_{sd,k}} \\ &\quad + \sum_{l=1}^{m} \frac{\mathbf{w}_{r_{\Omega(l)}d}^H \rho_{r_{\Omega(l)}d} \mathbf{w}_{r_{\Omega(l)}d}}{\sigma^2 \mathbf{w}_{r_{\Omega(l)}d}^H \mathbf{w}_{r_{\Omega(l)}d}})\Big]\}, \end{aligned} \tag{42}$$

where the last summation term $\sum_{l=1}^{m} \text{SINR}_{r_{\Omega(l)}d}$ denotes the overall SINR for all NCS received from the selected relay pair Ω by the destination. When linear network coding is applied at the destination, the NCS is the linear combination of the detected symbols at the output of relay $\Omega(l)$. Consequently, the SINR for the corresponding NCS from the $\Omega(l)$-th relay to the destination is given by

$$\text{SINR}_{r_{\Omega(l)}d} = \frac{\sum\limits_{k=1}^{K} \mathbf{w}_{r_{\Omega(l)}d}^H \rho_{r_{\Omega(l)}d} g_{k\Omega(l)}^2 \mathbf{w}_{r_{\Omega(l)}d}}{\sigma^2 \mathbf{w}_{r_{\Omega(l)}d}^H \mathbf{w}_{r_{\Omega(l)}d}} \tag{43}$$

The sum rate for the system when linear network coding is employed at the relays is expressed by

$$\begin{aligned} R &\leq \frac{1}{N+N(J+1)} \min \{ I_{DF}^{sr},\ I_{DF}^{sd,rd} \} \\ &= \frac{1}{N+N(J+1)} \min \{\ E[\log \det(\sigma^2 \mathbf{I}_{2N}\ +\ \mathbf{H}_{sr}\mathbf{Q}\mathbf{H}_{sr}^H)], \\ &\quad E\Big[\log\,(1 + \sum_{k=1}^{K} \text{SINR}_{sd}^k + \sum_{l=1}^{m} \text{SINR}_{r_{\Omega(l)}d})\Big] \\ &= \frac{1}{N+N(J+1)} \min \{\ E[\log \det(\sigma^2 \mathbf{I}_{2N}\ +\ \mathbf{H}_{sr}\mathbf{Q}\mathbf{H}_{sr}^H)], \\ &\quad E\Big[\log\,(1 + \sum_{k=1}^{K} \frac{(\mathbf{w}_{sd,k})^H \rho_{sd,k} \mathbf{w}_{sd,k}}{\sum\limits_{\substack{p=1 \\ p\neq k}}^{K} (\mathbf{w}_{sd,k})^H \rho_{sd,p} \mathbf{w}_{sd,k} + \sigma^2 (\mathbf{w}_{sd,k})^H \mathbf{w}_{sd,k}} \\ &\quad + \sum_{l=1}^{m} \frac{\sum\limits_{k=1}^{K} \mathbf{w}_{r_{\Omega(l)}d}^H \rho_{r_{\Omega(l)}d} g_{k\Omega(l)}^2 \mathbf{w}_{r_{\Omega(l)}d}}{\sigma^2 \mathbf{w}_{r_{\Omega(l)}d}^H \mathbf{w}_{r_{\Omega(l)}d}})\Big]\}, \end{aligned} \tag{44}$$

where the last summation term is the overall SINR for all NCS transmitted by the selected relay pair Ω to the destination.

VI. Simulations

In this section, a simulation study of the proposed buffer-aided PLNC scheme and linear network coding techniques is carried out. The DS-CDMA network uses randomly generated spreading codes of length $N = 16$. The corresponding channel coefficients are modeled as complex Gaussian random variables. We assume perfectly known channels at the relays and receivers and remark that results with channel estimation have the same performance hierarchy. We consider $K = 6$ users, $L = 6$ relays, equal power allocation and packets with 1000 BPSK symbols in the transmissions. For PLNC techniques, we first compare the XOR against linear network coding schemes using different designs of the matrix $\mathbf{G}$ with and without buffers ($J = 4$) in Fig. 2. The results show that the use of buffers can provide a significant gain in performance and that linear network coding techniques outperform XOR-based approaches.

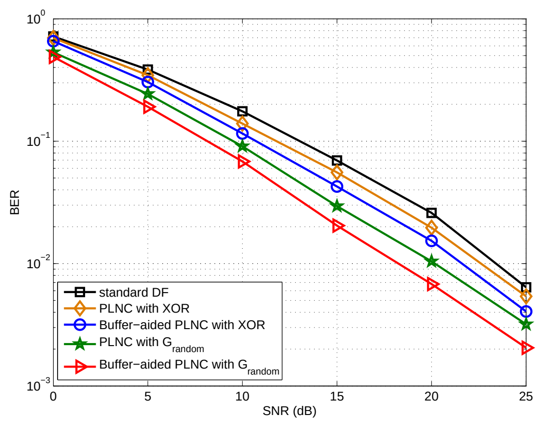

Fig. 2. Performance of linear network coding techniques with and without buffers using BPSK modulation.

The BER performance of buffer-aided ($J = 4$) PLNC schemes with the proposed and existing linear network coding techniques is shown in Fig. 3. The results show that the introduction of $\mathbf{G}^{\text{MMSE}}$ provides the best performance, followed by

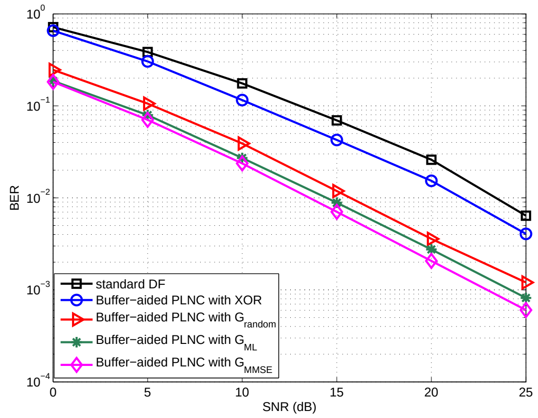

Fig. 3. Performance of linear network coding techniques with and without buffers using BPSK modulation.

the introduction of $\mathbf{G}^{\mathrm{ML}}$ and $\mathbf{G}^{\mathrm{random}}$. In particular, the adoption of $\mathbf{G}^{\mathrm{MMSE}}$ results in a gain of up to 1.5dB in SNR over $\mathbf{G}^{\mathrm{ML}}$ and up to 3 dB in SNR as compared to $\mathbf{G}^{\mathrm{random}}$ for the same BER performance. The results also demonstrate that with the use of buffers the overall system performance significantly improves, achieving a gain of up to 5 dB in terms of SNR for the same BER performance as compared to the system without buffers even though the diversity order is the same.

Finally, we compare the sum-rate performance versus SNR of the proposed techniques as shown by Fig. 4. The simulation results demonstrate that the PLNC design with $\mathbf{G}^{\mathrm{MMSE}}$ provides the highest sum rate under the same level of input SNR, followed by the PLNC with $\mathbf{G}^{\mathrm{ML}}$, the PLNC with $\mathbf{G}^{\mathrm{Random}}$ and the cooperative network coding design with XOR mapping applied.

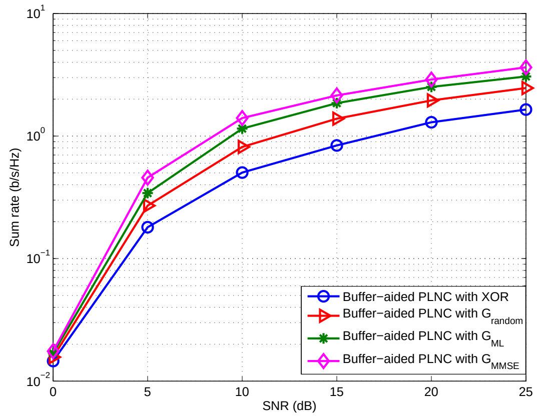

Fig. 4. Sum rate versus SNR for different network coding techniques.

VII. Conclusions

We have presented a buffer-aided PLNC scheme and devised novel ML and MMSE linear network coding algorithms for cooperative DS-CDMA systems with different relay pair selection techniques. Simulation results show that the BER and sum-rate performances of the proposed scheme and algorithms are significant better than those of previously reported techniques.

References

[1] S. Zhang, S. C. Liew, and P. K. Lam, "Hot topic: Physical-layer network coding," *in Proc. Annual Int. Conf. on Mobile Computing and Networking(MobiCom).*, LA, USA, Sep. 2006.

[2] M. Sanna and E. Izquierd, "A survey of linear network coding and network error correction code constructions and algorithms," *International Journal of Digital Multimedia Broadcasting*, vol. 2011, 2011.

[3] J. He and S. C. Liew, "Building blocks of physical-layer network coding," *IEEE Transactions on Wireless Communications*, vol. 14, no. 5, pp. 2711–2728, May 2015.

[4] A. Uchoa, C. T. Healy, R. C. de Lamare, and R. D. Souza, "Design of ldpc codes based on progressive edge growth techniques for block fading channels," *IEEE Communications Letters*, vol. 15, no. 11, pp. 1221–1223, 2011.

[5] L. Yang, T. Yang, Y. Xie, J. Yuan, and J. An, "Linear physical-layer network coding and information combining for the k-user fading multiple-access relay network," *IEEE Transactions on Wireless Communications*, vol. 15, no. 8, pp. 5637–5650, Aug 2016.

[6] M. Nokleby and B. Aazhang, "Cooperative compute-and-forward," *IEEE Transactions on Wireless Communications*, vol. 15, no. 1, pp. 14–27, Jan 2016.

[7] H. Zhang, L. Zheng, and L. Cai, "Design and analysis of heterogeneous physical layer network coding," *IEEE Transactions on Wireless Communications*, vol. 15, no. 4, pp. 2484–2497, April 2016.

[8] C. T. Healy and R. C. de Lamare, "Design of ldpc codes based on multipath emd strategies for progressive edge growth," *IEEE Transactions on Communications*, vol. 64, no. 8, pp. 3208–3219, 2016.

[9] A. Burr and D. Fang, "Linear physical layer network coding for multihop wireless networks," in *22nd European Signal Processing Conference (EUSIPCO)*, September 2014, pp. 1153–1157.

[10] ——, "Linear physical-layer network coding for 5G radio access networks," in *5G for Ubiquitous Connectivity (5GU), 2014 1st International Conference on*, November 2014, pp. 116–121.

[11] R. Ahlswede, N. Cai, S. Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Trans. Inf. Theory.*, vol. 46, no. 4, p. 12041216, 2000.

[12] S. Y. R. Li, Q. T. Sun, and Z. Shao, "Linear network coding: Theory and algorithms," *Proceedings of the IEEE*, vol. 99, no. 3, pp. 372–387, March 2011.

[13] Y. Jing and H. Jafarkhani, "Single and multiple relay selection schemes and their achievable diversity orders," *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1084–1098, Mar 2009.

[14] T. Wang, R. C. de Lamare, and P. D. Mitchell, "Low-complexity set-membership channel estimation for cooperative wireless sensor networks," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 6, pp. 2594–2607, 2011.

[15] P. Clarke and R. C. de Lamare, "Joint transmit diversity optimization and relay selection for multi-relay cooperative mimo systems using discrete stochastic algorithms," *IEEE Communications Letters*, vol. 15, no. 10, pp. 1035–1037, 2011.

[16] R. C. D. Lamare, "Joint iterative power allocation and linear interference suppression algorithms for cooperative ds-cdma networks," *IET communications*, vol. 6, no. 13, pp. 1930–1942, 2012.

[17] P. Clarke and R. C. de Lamare, "Transmit diversity and relay selection algorithms for multi-relay cooperative MIMO systems," *IEEE Trans. Veh. Technol*, vol. 61, no. 3, pp. 1084–1098, Mar 2012.

[18] T. Wang, R. C. de Lamare, and A. Schmeink, "Joint linear receiver design and power allocation using alternating optimization algorithms for wireless sensor networks," *IEEE Transactions on Vehicular Technology*, no. 99, pp. 1–1, 2012.

[19] T. Peng, R. de Lamare, and A. Schmeink, "Adaptive distributed space-time coding based on adjustable code matrices for cooperative mimo relaying systems," *IEEE Transactions on Communications*, vol. 61, no. 7, pp. 2692–2703, 2013.

[20] S. Talwar, Y. Jing, and S. Shahbazpanahi, "Joint relay selection and power allocation for two-way relay networks," *IEEE Signal Process. Lett*, vol. 18, no. 2, pp. 91–94, Feb 2011.

[21] T. Wang, R. C. De Lamare, and A. Schmeink, "Alternating optimization algorithms for power adjustment and receive filter design in multihop wireless sensor networks," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 1, pp. 173–184, 2015.

[22] J. Gu and R. de Lamare, "Joint sic and multi-relay selection algorithms for cooperative DS-CDMA systems," in *Proceedings of the 22nd European Signal Processing Conference (EUSIPCO)*, Sept 2014, pp. 556–560.

[23] S. Xu, R. C. de Lamare, and H. V. Poor, "Distributed compressed estimation based on compressive sensing," *IEEE Signal Processing Letters*, vol. 22, no. 9, pp. 1311–1315, 2015.

[24] ——, "Adaptive link selection algorithms for distributed estimation," *EURASIP Journal on Advances in Signal Processing*, vol. 2015, no. 1, p. 86, 2015.

[25] T. Peng, R. de Lamare, and A. Schmeink, "Adaptive distributed space-time coding based on adjustable code matrices for cooperative MIMO relay-

ing systems," *IEEE Transactions on Communications*, vol. 61, no. 7, pp. 2692–2703, July 2013.

[26] J. Gu and R. C. de Lamare, "Joint interference cancellation and relay selection algorithms based on greedy techniques for cooperative ds-cdma systems," *EURASIP Journal on Wireless Communications and Networking*, 2016.

[27] N. Zlatanov, A. Ikhlef, T. Islam, and R. Schober, "Buffer-aided cooperative communications: opportunities and challenges," *IEEE Communications Magazine*, vol. 52, no. 4, pp. 146–153, May 2014.

[28] N. Zlatanov, R. Schober, and P. Popovski, "Throughput and diversity gain of buffer-aided relaying," *IEEE Globecom*, pp. 1–6, Dec 2011.

[29] I. Krikidis, T. Charalambous, and J. Thompson, "Buffer-aided relay selection for cooperative diversity systems without delay constraints," *IEEE Trans. Wireless Commun.*, vol. 11, no. 5, pp. 1592–1967, May 2012.

[30] A. Ikhlef, D. S. Michalopoulos, and R. Schober, "Max-max relay selection for relays with buffers," *IEEE Trans. Wireless Commun.*, vol. 11, no. 5, pp. 1124–1135, Jan 2012.

[31] T. Peng and R. C. de Lamare, "Adaptive buffer-aided distributed space-time coding for cooperative wireless networks," *IEEE Transactions on Communications*, vol. 64, no. 5, pp. 1888–1900, May 2016.

[32] M. Huemer, C. Hofbauer, and J. B. Huber, "Non-systematic complex number rs coded ofdm by unique word prefix," *IEEE Trans. Signal Processing*, vol. 60, no. 1, pp. 285–299, Jan. 2012.

[33] T. Ho, M. Medard, R. Koetter, D. R. Karger, M. Effros, J. Shi, and B. Leong, "A random linear network coding approach to multicast," *IEEE Trans. Inf. Theory*, vol. 52, no. 10, pp. 4413–4430, 2006.

[34] J. Gu, R. C. de Lamare, and M. Huemer, "Buffer-aided physical-layer network coding with optimal linear code designs for cooperative networks," *IEEE Transactions on Communications*, 2017.

[35] R. de Lamare and R. Sampaio-Neto, "Adaptive reduced-rank processing based on joint and iterative interpolation, decimation, and filtering," *IEEE Trans. Signal Processing*, vol. 57, no. 7, pp. 2503–2514, July 2009.

[36] R. C. de Lamare and R. Sampaio-Neto, "Blind adaptive code-constrained constant modulus algorithms for cdma interference suppression in multipath channels," *IEEE Communications Letters*, vol. 9, no. 4, pp. 334–336, 2005.

[37] R. de Lamare and R. Sampaio-Neto, "Minimum mean-squared error iterative successive parallel arbitrated decision feedback detectors for ds-cdma systems," *IEEE Transactions on Communications*, vol. 56, no. 5, pp. 778–789, May 2008.

[38] P. Li, R. C. de Lamare, and R. Fa, "Multiple feedback successive interference cancellation detection for multiuser mimo systems," *IEEE Transactions on Wireless Communications*, vol. 10, no. 8, pp. 2434–2439, 2011.

[39] P. Li and R. C. de Lamare, "Adaptive decision-feedback detection with constellation constraints for mimo systems," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 2, pp. 853–859, 2012.

[40] R. C. de Lamare, "Adaptive and iterative multi-branch mmse decision feedback detection algorithms for multi-antenna systems," *IEEE Transactions on Wireless Communications*, vol. 12, no. 10, pp. 5294–5308, 2013.

[41] Y. Cai, R. C. de Lamare, B. Champagne, B. Qin, and M. Zhao, "Adaptive reduced-rank receive processing based on minimum symbol-error-rate criterion for large-scale multiple-antenna systems," *IEEE Transactions on Communications*, vol. 63, no. 11, pp. 4185–4201, 2015.

[42] A. Uchoa, C. Healy, and R. C. de Lamare, "Iterative detection and decoding algorithms for mimo systems in block-fading channels using ldpc codes," *IEEE Transactions on Vehicular Technology*, 2016.

[43] J. Gu and R. C. de Lamare, "Joint interference cancellation and relay selection algorithms based on greedy techniques for cooperative ds-cdma systems," *EURASIP Journal on Wireless Communications and Networking*, vol. 2016, no. 1, p. 59, 2016.

[44] T. Hesketh, R. C. de Lamare, and S. Wales, "Joint maximum likelihood detection and link selection for cooperative mimo relay systems," *IET Communications*, vol. 8, no. 14, pp. 2489–2499, 2014.

[45] P. Li and R. C. de Lamare, "Distributed iterative detection with reduced message passing for networked mimo cellular systems," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 6, pp. 2947–2954, 2014.

[46] Y. Cai, R. C. de Lamare, and R. Fa, "Switched interleaving techniques with limited feedback for interference mitigation in ds-cdma systems," *IEEE Transactions on Communications*, vol. 59, no. 7, pp. 1946–1956, 2011.

[47] W. Zhang, H. Ren, C. Pan, M. Chen, R. C. de Lamare, B. Du, and J. Dai, "Large-scale antenna systems with ul/dl hardware mismatch: Achievable rates analysis and calibration," *IEEE Transactions on Communications*, vol. 63, no. 4, pp. 1216–1229, 2015.

[48] R. C. de Lamare, "Massive mimo systems: Signal processing challenges and future trends," *Radio Science Bulletin*, 2013.

[49] K. Zu and R. C. de Lamare, "Low-complexity lattice reduction-aided regularized block diagonalization for mu-mimo systems," *IEEE Communications Letters*, vol. 16, no. 6, pp. 925–928, 2012.

[50] K. Zu, R. C. de Lamare, and M. Haardt, "Generalized design of low-complexity block diagonalization type precoding algorithms for multiuser mimo systems," *IEEE Transactions on Communications*, no. 99, pp. 1–11, 2013.

[51] ——, "Multi-branch tomlinson-harashima precoding design for mu-mimo systems: Theory and algorithms," *IEEE Transactions on Communications*, vol. 62, no. 3, pp. 939–951, 2014.

[52] L. Zhang, Y. Cai, R. C. de Lamare, and M. Zhao, "Robust multi-branch tomlinson-harashima precoding design in amplify-and-forward mimo relay systems," *IEEE Transactions on Communications*, vol. 62, no. 10, pp. 3476–3490, 2014.

[53] H. Ruan and R. C. de Lamare, "Robust adaptive beamforming using a low-complexity shrinkage-based mismatch estimation algorithm," *IEEE Signal Processing Letters*, vol. 21, no. 1, pp. 60–64, 2014.

[54] Y. Cai, R. C. de Lamare, L.-L. Yang, and M. Zhao, "Robust mmse precoding based on switched relaying and side information for multiuser mimo relay systems," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 12, pp. 5677–5687, 2015.

[55] H. Ruan and R. C. de Lamare, "Robust adaptive beamforming based on low-rank and cross-correlation techniques," *IEEE Transactions on Signal Processing*, vol. 64, no. 15, pp. 3919–3932, 2016.