Iterative Detection and Decoding for Large-Scale Multiple-Antenna Systems with 1-Bit ADCs

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***Abstract*—We present a novel iterative detection and decoding (IDD) scheme for uplink (UL) large-scale multiuser multiple-antenna systems. In order to reduce the receiver's power consumption and computational complexity, 1-bit analog to digital converters (ADCs) are used in the front-end. The performance loss due to the 1-bit quantization can be mitigated by using large-scale antenna arrays. We propose a linear low-resolution-aware minimum mean square error (LRA-MMSE) detector for soft multiuser interference mitigation. Moreover, short block length low-density parity-check (LDPC) codes are considered for avoiding high latency. In the channel decoder, a quasi-uniform quantizer with scaling factors is devised to lower the error floor of LDPC codes. Simulations show good performance of the system in terms of bit error rate (BER) as compared to prior work.**

***Index Terms*—Large-scale multiple-antenna systems, 1-Bit quantization, IDD schemes, MMSE detectors, quantized LDPC decoders.**

I. Introduction

Large-scale multiple-antenna systems have been identified as a promising technology for the next generation communication systems [1]. In fact, large spatial degrees of freedom (DoFs) can increase the spectral and energy efficiency. However, as the antennas scale up, the receiver design will become more complex and the power consumption will be higher. For overcoming these issues, one solution is to use low-resolution ADCs at the receiver. As one extreme case, 1-bit quantization can drastically simplify the receiver design. Prior work on multiple-antenna systems with low-resolution ADCs includes the study in [2], where the authors reported a novel linear MMSE precoder design for multiple-antenna systems using 1-bit DAC/ADC both at the transmitter and the receiver. An analytical approach to calculate a lower bound on capacity for a wideband system with multiple-antenna and one-bit ADCs, which employs low-complexity channel estimation and symbol detection, has been described in [3].

In recent years, LDPC codes have been used in many industry standards including DVB-S2 and IEEE 802.11n (Wi-Fi). They have also been adopted for the next generation communication system, since they approach the Shannon capacity and have low complexity. Compared to LDPC codes with large block length, short block LDPC codes result in much lower latency. As one branch of LDPC codes, regular LDPC codes, they have high error floor phenomenon, which is commonly attributed to the existence of certain error-prone structures in the corresponding Tanner graph. This is partially because of trapping sets [4] and absorbing sets [5]. The authors in [6] have proposed a new LDPC decoder with low error floors and low computational complexity. This approach quasi-uniformly quantizes the passing messages into different ranges of reliability. It extends the saturation level to prevent the messages from being trapped and can be helpful for 1-bit quantized data. In receiver designs with channel codes, it is often useful to employ iterative detection and decoding (IDD) schemes. The key mechanism of the IDD process is the soft information exchange between the detector and the channel decoder, which leads to successive performance improvement [7]. The soft information exchanged often has the form of log likelihood ratio (LLR) of a certain bit. In [8] an IDD algorithm with LDPC codes for multiple-antenna systems under block-fading channels is proposed. Results show that by properly manipulating the LLR output of decoder the system performance can be largely improved.

In this work, we develop an IDD scheme for 1-bit quantized systems and derive a linear low-resolution-aware MMSE (LRA-MMSE) receive filter suitable for 1-bit ADCs and soft interference mitigation. Moreover, we also develop an adaptive decoding approach that combines a quasi-uniform quantization of the passing messages with adjustable scaling factors, which can avoid trapping sets and refine the exchange of LLRs between the detector and the decoder.

The rest of this paper is organized as follows: Section II shows the system model and presents some statistical properties about 1-bit quantization. Section III describes the derivation of the proposed LRA-MMSE detector and decoding technique. Section IV discusses the simulation results and section V concludes the work.

Notation: Bold capital letters indicate matrices while vectors are in bold lowercase. $\mathbf{I}_n$ denotes $n \times n$ identity matrix while $\mathbf{0}_n$ is $n \times 1$ zeros vector. Additionally, diag($\mathbf{A}$) is a diagonal matrix only containing the diagonal elements of $\mathbf{A}$.

II. System Model and Statistical Properties of 1-bit Quantization

A single-cell multi-user large-scale multiple-antenna uplink scenario is considered, which is depicted in Fig. 1. There are K single-antenna users and the receiver is equipped with M antennas, where $M \gg K$. The information symbols b_k are firstly encoded by each user's channel encoder and modulated to x_k according to the modulation scheme. The transmit symbols x_k have zero-mean and the same energy $E[|x_k|^2] = \sigma_x^2$. The modulated symbols are then transmitted over block-fading channels. The vector $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}_M, \sigma_n^2 \mathbf{I}_M)$ contains complex

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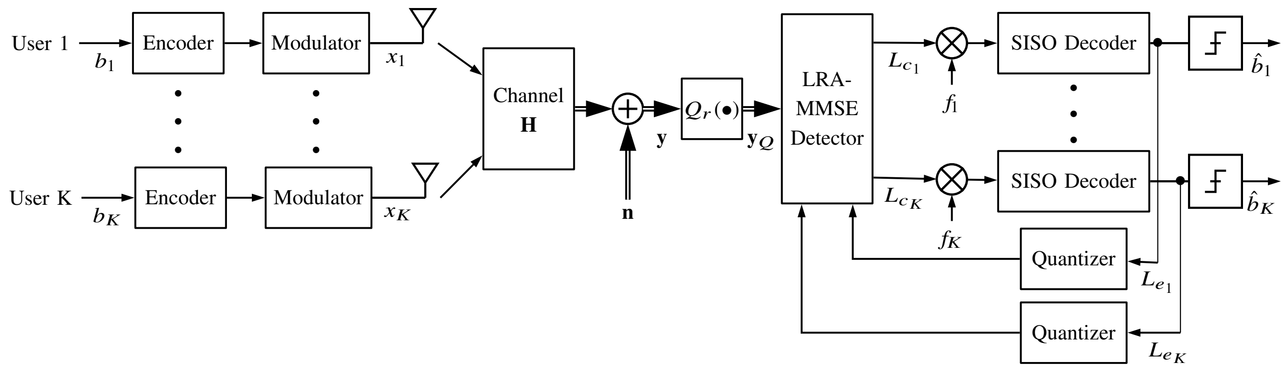

Fig. 1: System model of a multi-user multiple-antenna system

Gaussian noise samples with zero mean and noise variance σ_n^2. The received unquantized signal is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} = \sum_{k=1}^{K} \mathbf{h}_k x_k + \mathbf{n}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{M\times K}$ is the channel matrix. $Q_r(\bullet)$ represents the 1-bit quantization. The real and imaginary parts of the unquantized signal $\mathbf{y}$ are element-wisely quantized to $\{\pm 1\}$ based on a threshold. The resulting quantized signal $\mathbf{y}_Q$ is

$$\mathbf{y}_Q = Q_r(\Re\{\mathbf{y}\}) + jQ_r(\Im\{\mathbf{y}\}), \tag{2}$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ get the real and imaginary part.

Since quantization strongly changes the properties of signals, we show here some statistical properties of quantization for a Gaussian input signal. In [9] the autocorrelation function of the output of a hard limiter has been derived while the input signal is real Gaussian distributed. In [2] the authors have introduced the statistical properties of 1-bit quantization for complex Gaussian input signals. The cross-correlation between the unquantized complex-valued signal $\mathbf{s}$ with covariance matrix $\mathbf{C}_\mathbf{s}$ and its 1-bit quantized signal $\mathbf{s}_Q$ is described by

$$\mathbf{C}_{\mathbf{s}_Q\mathbf{s}} = \sqrt{\frac{4}{\pi}}\mathbf{K}\mathbf{C}_\mathbf{s}, \text{where } \mathbf{K} = \text{diag}(\mathbf{C}_\mathbf{s})^{-\frac{1}{2}}. \tag{3}$$

Furthermore, the covariance matrix of the 1-bit quantized signal $\mathbf{s}_Q$ is given by

$$\mathbf{C}_{\mathbf{s}_Q} = \frac{4}{\pi}(\sin^{-1}(\mathbf{K}\Re\{\mathbf{C}_\mathbf{s}\}\mathbf{K}) + j\sin^{-1}(\mathbf{K}\Im\{\mathbf{C}_\mathbf{s}\}\mathbf{K})). \tag{4}$$

III. Proposed Iterative Detection and Decoding

A. Proposed LRA-MMSE Detector

Inspired by prior work on IDD schemes [7] [8], we propose a LRA-MMSE detector, which employs a modified linear MMSE receive filter and performs soft parallel interference cancellation. The soft estimate of the kth transmitted symbol is firstly calculated based on the extrinsic LLR L_{e_k} provided by the channel decoder from a previous stage:

$$\tilde{x}_k = \sum_{x\in\mathcal{A}} xp(x_k = x) = \sum_{x\in\mathcal{A}} x(\prod_{l=1}^{M_c}[1+\exp(-x^l L_{e_k}^l)]^{-1}), \tag{5}$$

where $\mathcal{A}$ is the complex constellation set with 2^{M_c} possible points. The symbol x^l corresponds to the value $(+1,-1)$ of the lth bit of symbol x. Denote $\tilde{\mathbf{x}} = [\tilde{x}_1, ..., \tilde{x}_K]^T$ and

$$\tilde{\mathbf{x}}_k = \tilde{\mathbf{x}} - \tilde{x}_k\mathbf{e}_k, \tag{6}$$

where $\mathbf{e}_k$ is a column vector with all zeros, except that the kth element is equal to 1. For each user k, the interference from the other $K-1$ users is canceled according to

$$\mathbf{y}_{Q_k} = \mathbf{y}_Q - \sum_{j=1, j\neq k}^{K} \tilde{x}_j\mathbf{h}_j = \mathbf{y}_Q - \mathbf{H}\tilde{\mathbf{x}}_k. \tag{7}$$

The linear LRA-MMSE filter is then applied to $\mathbf{y}_{Q_k}$, to obtain

$$\hat{x}_k = \mathbf{w}_k^H\mathbf{y}_{Q_k}, \tag{8}$$

where $\mathbf{w}_k$ is chosen to minimize the mean square error (MSE) between the transmitted symbol x_k and the filter output, i.e.

$$\mathbf{w}_k = \arg\min_{\mathbf{w'}_k} E[||x_k - \mathbf{w'}_k^H\mathbf{y}_{Q_k}||^2]. \tag{9}$$

The solution of the LRA-MMSE receive filter is given by

$$\mathbf{w}_k = \mathbf{C}_{y_{Q_k}}^{-1}\mathbf{c}_{x_k\mathbf{y}_{Q_k}} = (E[\mathbf{y}_{Q_k}\mathbf{y}_{Q_k}^H])^{-1}E[x_k\mathbf{y}_{Q_k}], \tag{10}$$

where the covariance matrix is

$$\mathbf{C}_{y_{Q_k}} = \mathbf{C}_{\mathbf{y}_Q} - (\mathbf{C}_{\mathbf{y}_Q\tilde{\mathbf{x}}_k}\mathbf{H}^H)^H - \mathbf{C}_{\mathbf{y}_Q\tilde{\mathbf{x}}_k}\mathbf{H}^H + \mathbf{H}\mathbf{C}_{\tilde{\mathbf{x}}_k}\mathbf{H}^H \tag{11}$$

and the cross-correlation vector is

$$\mathbf{c}_{x_k\mathbf{y}_{Q_k}} = \sigma_x^2\sqrt{\frac{4}{\pi}}\mathbf{K}\mathbf{h}_k. \tag{12}$$

The covariance matrix of the quantized data vector $\mathbf{y}_Q$ is described by

$$\mathbf{C}_{\mathbf{y}_Q} = \frac{4}{\pi}(\sin^{-1}(\mathbf{K}\Re\{\mathbf{C}_\mathbf{y}\}\mathbf{K}) + j\sin^{-1}(\mathbf{K}\Im\{\mathbf{C}_\mathbf{y}\}\mathbf{K})), \tag{13}$$

with the approximation $\sin^{-1}(x) \simeq x$, for $x \neq 1$, the linear expression of (13) can be written as

$$\mathbf{C}_{\mathbf{y}_Q} \simeq \frac{4}{\pi}(\mathbf{K}\mathbf{C}_\mathbf{y}\mathbf{K} + (\frac{\pi}{2} - 1)\mathbf{I}_K). \tag{14}$$

The cross-correlation vector between $\mathbf{y}_Q$ and $\tilde{\mathbf{x}}_k$ is

$$\mathbf{C}_{\mathbf{y}_Q\tilde{\mathbf{x}}_k} = \sqrt{\frac{4}{\pi}}\mathbf{K}\mathbf{H}\mathbf{C}_{\tilde{\mathbf{x}}_k}. \tag{15}$$

In the above equations, we have $\mathbf{K} = \text{diag}(\mathbf{C_y})^{-\frac{1}{2}}$ and

$$\mathbf{C_y} = E[(\mathbf{Hx} + \mathbf{n})(\mathbf{Hx} + \mathbf{n})^H] = \sigma_x^2 \mathbf{HH}^H + \sigma_n^2. \quad (16)$$

In order to calculate $p(\hat{x}_k|x)$, we use Cramer's central limit theorem [10]: the LRA-MMSE filter output can be approximated by a complex Gaussian distribution due to the large number of independent variables. Conditioned on the transmitted symbol x, the mean and the variance of the estimated symbol $\hat{x}_k$ are given by

$$\mu_k = E[\hat{x}_k x] = \sigma_x^2 \sqrt{\frac{4}{\pi}} \mathbf{w}_k^H \mathbf{K} \mathbf{h}_k \quad (17)$$

$$\eta_k^2 = \text{var}[\hat{x}_k] = E[||\hat{x}_k||^2] - \mu_k^2 = \mu_k - \mu_k^2. \quad (18)$$

Therefore, the likelihood function is approximated by

$$P(\hat{x}_k|x) \simeq \frac{1}{\pi \eta_k^2} \exp(-\frac{1}{\eta_k^2} |\hat{x}_k - \mu_k x|^2), \quad (19)$$

where $\mathbf{h}_k x = Q_r(\mathbf{h}_k x + \mathbf{H}\tilde{\mathbf{x}}_k) - \mathbf{H}\tilde{\mathbf{x}}_k$. Then the LLR computed by the LRA-MMSE detector for the lth bit ($l \in \{1, ..., M_c\}$) of the symbol $\hat{x}_k$ is given by

$$\begin{aligned} L_{c_k}^l &= \log \frac{P(b_k^l = +1|\hat{x}_k)}{P(b_k^l = -1|\hat{x}_k)} - \log \frac{P(b_k^l = +1)}{P(b_k^l = -1)} \\ &= \log \frac{\sum_{x \in \mathcal{A}_l^{+1}} P(\hat{x}_k|x)P(x)}{\sum_{x \in \mathcal{A}_l^{-1}} P(\hat{x}_k|x)P(x)} - L_{e_k}^l \end{aligned} \quad (20)$$

where $\mathcal{A}_l^{+1}$ is the set of hypotheses x for which the lth bit is +1 and $\mathcal{A}_l^{-1}$ is similarly defined.

B. Proposed Soft Information Processing and Decoding

The soft information provided by the LRA-MMSE detector is then fed into a channel decoder that adaptively scales the input LLRs and quasi-uniformly quantizes the messages.

1) Iterative Decoder: the decoding method is based on the message passing, which iteratively computes the distributions of variables in graph-based models. In the system we have used the box-plus sum product algorithm (SPA) [11], which is an approximation of SPA decoding. One drawback of SPA is the hyperbolic tangent function, which has numerical saturation problems when computed with finite precision. To avoid such problems, thresholds on the magnitudes of messages must be applied. In the box-plus SPA, the message sent from check node (CN) j to variable node (VN) i is

$$L_{j \to i} = \boxplus_{i' \in N(j) \backslash i} L_{i' \to j}, \quad (21)$$

where $\boxplus$ is the pairwise "box-plus" operator defined as

$$\begin{aligned} x \boxplus y &= \log \left(\frac{1 + e^{x+y}}{e^x + e^y} \right) \\ &= \text{sign}(x)\text{sign}(y) \min(|x|, |y|) \\ &\quad + \log(1 + e^{-|x+y|}) - \log(1 + e^{-|x-y|}). \end{aligned} \quad (22)$$

The message from VN i to CN j is then calculated as

$$L_{i \to j} = L_i + \sum_{j' \in N(i) \backslash j} L_{j' \to i}, \quad (23)$$

where L_i is the LLR at VN i. The quantity $j' \in N(i) \backslash j$ represents all CNs connected to VN i except CN j.

2) Quasi-uniform Quantizer: this quantizer is used both in the decoder and the extrinsic message quantizer to refine or compensate for the effect of 1-bit quantization on the LLRs. The algorithm is based on the quasi-uniform quantization in [6], which represents a compromise between conflicting objectives of retaining fine precision, allowing large dynamic range and implementation complexity. It is a combination of non-uniform and uniform quantization and realized as follows:

$$Q_\Delta^*(L_{c_k}) = \begin{cases} d^{N+1} N\Delta & \text{if } d^{N+1} N\Delta \le L_{c_k} \\ d^r N\Delta & \text{if } d^r N\Delta \le L_{c_k} < d^{r+1} N\Delta, \\ & \text{for } N \ge r \ge 1 \\ Q_\Delta(L_{c_k}) & \text{if } -dN\Delta < L_{c_k} < dN\Delta \\ -d^r N\Delta & \text{if } -d^{r+1} N\Delta \le L_{c_k} < -d^r N\Delta, \\ & \text{for } 1 \le r \le N \\ -d^{N+1} N\Delta & \text{if } L_{c_k} \le -d^{N+1} N\Delta \end{cases} \quad (24)$$

with

$$Q_\Delta(L_{c_k}) = \begin{cases} N\Delta & \text{if } N\Delta - \frac{\Delta}{2} \le L_{c_k} \\ m\Delta & \text{if } m\Delta - \frac{\Delta}{2} \le L_{c_k} < m\Delta + \frac{\Delta}{2}, \\ & \text{for } N > m > 0 \\ 0 & \text{if } -\frac{\Delta}{2} < L_{c_k} < \frac{\Delta}{2} \\ -m\Delta & \text{if } m\Delta - \frac{\Delta}{2} < L_{c_k} \le m\Delta + \frac{\Delta}{2}, \\ & \text{for } -N < m < 0 \\ -N\Delta & \text{if } L_{c_k} \le -N\Delta + \frac{\Delta}{2} \end{cases} \quad (25)$$

where d is the growth rate parameter, Δ is the step size, N is the total bits for representing each range and L_{c_k} is the passing message at the kth decoder.

3) Adaptive Scaling Factors: for improving the decoding performance we have deployed two scaling factors, which are obtained offline and online respectively.

- Offline Scaling Factor: this factor is utilized to correct the LLR values used in iterative decoding based on the LLR distribution [12]. In the training phase, data packets are sent to the receiver for obtaining sufficient LLR statistics. For a given SNR, the following steps are carried out:
  1) Calculate the probabilities of $P(L_{c_k}|b_k)$ conditioned on transmitted bits b_k through histograms.
  2) Obtain $f(L_{c_k}) = \log \frac{P(L_{c_k}|b_k=1)}{P(L_{c_k}|b_k=0)}$.
  3) Approximate $f(L_{c_k}) = \alpha_k L_{c_k}$.

  This factor α_k is only applied in the first iteration at the kth decoder input during the data transmitting phase. Moreover, the scaled mean absolute value $\alpha_k \overline{L}_{c_k}$ for each user is stored for calculating the online scaling factor.
- Online Scaling Factor: the factor f_k is calculated at the kth decoder input in the second iteration and applied for all the iterations except the first iteration. It aims to correct the LLR errors caused by quantizer. The scaled LLR should be approximated to the scaled LLR in 3). We propose a linear scaling factor that is calculated as

$$f_k = \alpha_k \overline{L}_{c_k} / \overline{L}_{c_k}^{\text{2nd iteration}}, \quad (26)$$

  where $\overline{L}_{c_k}^{\text{2nd iteration}}$ is the mean absolute value of LLRs for the kth user in the second iteration.

IV. Numerical Results

This section compares the BER performance under different iterative decoding techniques. We consider a short length regular LDPC code with block length $n = 256$ and rate 0.5. In the simulation there are $K = 12$ users and $M = 32$ receive antennas. The modulation scheme is QPSK and the parameters of the quasi-uniform quantizer are $\Delta = 0.25$, $d = 1.3$ and $N = 6$. The channel is assumed to experience block fading and is modeled by independent complex Gaussian random variables with zero mean and unit variance. The channel matrix is estimated through the Bussgang-based channel estimator [13]. During training, all K users simultaneously transmit pilot sequences of $\tau = K$ (for simplicity) symbols to the receiver. To match the matrix form to the vector form of (2), the vectorized received signal is described by

$$\mathbf{y}_{Q_p} = Q_r(\mathbf{y}_p) = Q_r(\tilde{\mathbf{X}}_p\mathbf{h} + \mathbf{n}_p), \tag{27}$$

where $\tilde{\mathbf{X}}_p = (\mathbf{X}_p^T \otimes \mathbf{I}_M)$ and $\mathbf{h} = \text{vec}(\mathbf{H})$. $\mathbf{X}_p \in \mathbb{C}^{K\times\tau}$ is the pilot matrix. $\mathbf{h} \in \mathbb{C}^{KM\times 1}$ is the vectorized channel matrix $\mathbf{H}$. Then the estimated channel vector is given by

$$\hat{\mathbf{h}} = \mathbf{C}_{\mathbf{h}}(\mathbf{A}_p\tilde{\mathbf{X}}_p)^H \mathbf{C}_{\mathbf{y}_{Q_p}}^{-1} \mathbf{y}_{Q_p}, \tag{28}$$

where $\mathbf{A}_p = \sqrt{\frac{4}{\pi}}\text{diag}(\mathbf{C}_{\mathbf{y}_p})^{-\frac{1}{2}}$. $\mathbf{C}_{\mathbf{h}}$ is the covariance matrix of $\mathbf{h}$. $\mathbf{C}_{\mathbf{y}_{Q_p}}$ and $\mathbf{C}_{\mathbf{y}_p}$ are calculated according to (13) and (16), respectively.

The BER performances of the traditional MMSE and the proposed LRA-MMSE detector are depicted in Fig.2, where LRA-MMSE obtains a large gain. Moreover, the BER performance of IDD schemes with and without quasi-uniform quantization and scaling factors is depicted in Fig.3, which shows the system has a significant performance gain after 2 iterations. The system reaches saturation at the third iteration. This results also show that the quantizer and the scaling factors offer extra performance gains.

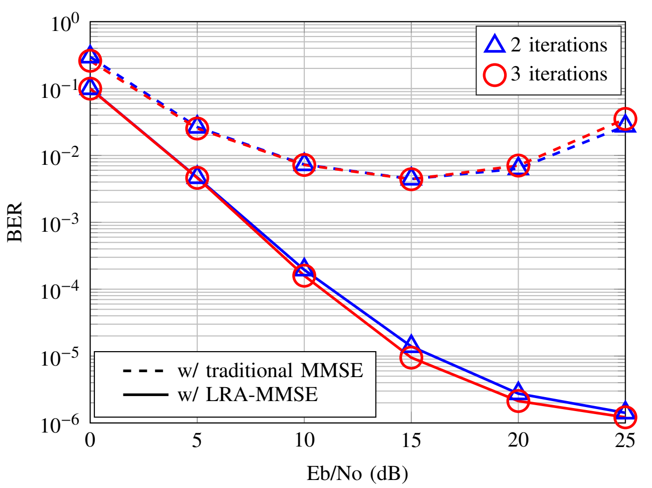

Fig. 2: BER performance comparison between the traditional MMSE and the proposed LRA-MMSE detector

V. Conclusion

In this work, we have developed an IDD scheme for 1-bit quantized systems and proposed a LRA-MMSE detector for 1-bit systems. The simulation results have shown a great performance gain after several iterations. Moreover, we have devised an adaptive channel decoder using a quantizer together with scaling factors for further performance improvement.

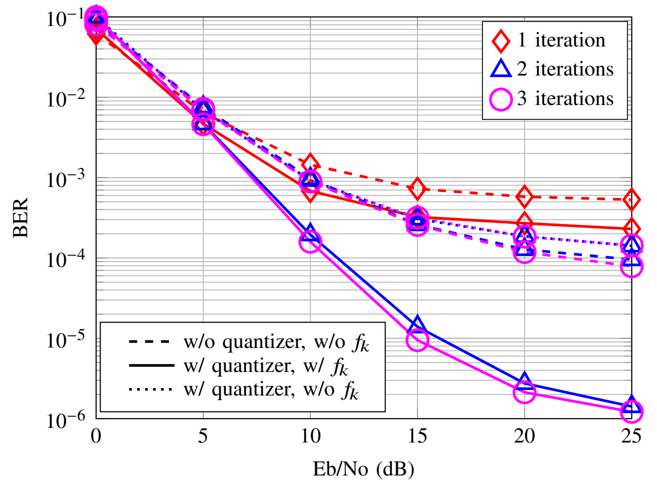

Fig. 3: BER performance of IDD schemes using the proposed LRA-MMSE detector with and without quantizer and scaling factors

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