

# Low-Complexity Set-Membership Channel Estimation for Cooperative Wireless Sensor Networks

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**Abstract**—In this paper, we consider a general cooperative wireless sensor network (WSN) with multiple hops and the problem of channel estimation. Two matrix-based set-membership (SM) algorithms are developed for the estimation of complex matrix channel parameters. The main goal is to significantly reduce the computational complexity, compared with existing channel estimators, and extend the lifetime of the WSN by reducing its power consumption. The first proposed algorithm is the SM normalized least mean squares (SM-NLMS) algorithm. The second is the SM recursive least squares (RLS) algorithm called BEACON. Then, we present and incorporate an error bound function into the two channel estimation methods, which can automatically adjust the error bound with the update of the channel estimates. Steady-state analysis in the output mean-square error (MSE) is presented, and closed-form formulas for the excess MSE and the probability of update in each recursion are provided. Computer simulations show good performance of our proposed algorithms in terms of convergence speed, steady-state mean square error, and bit error rate (BER) and demonstrate reduced complexity and robustness against time-varying environments and different signal-to-noise ratio (SNR) values.

**Index Terms**—Channel estimation, cooperation, data selection, set membership (SM), time-varying bounds (TVBs), wireless sensor networks (WSNs).

## I. INTRODUCTION

RECENTLY, there has been increasing research interest in wireless sensor networks (WSNs) because their unique features allow a wide range of applications in the areas of the military, the environment, health, and home [1]. They are usually composed of a large number of densely deployed sensing devices, which can transmit their data to the desired user through multihop relays [2]. Low complexity and high energy efficiency are the most important design characteristics of communication protocols [3] and physical-layer techniques employed for WSNs. The performance and capacity of WSNs can be significantly enhanced through exploitation of spatial diversity with cooperation between the nodes [2]. In a cooperative WSN, nodes relay signals to each other to propagate redundant copies of the same signals to the destination nodes. Among the

existing relaying schemes, the amplify-and-forward (AF) and the decode-and-forward are the most popular approaches [4]. Due to limitations in sensor node power, computational capacity, and memory [1], some power-constrained relay strategies [5], [6] and power allocation methods [7] have been proposed for WSNs to obtain the best possible signal-to-noise ratio (SNR) or best possible quality of service at the destinations. Most of these ideas are based on the assumption of perfect synchronization and available channel state information (CSI) at each node [1]. Therefore, more accurate estimates of the CSI will bring about better performance in WSNs.

The normalized least mean squares (NLMS) estimation method is appropriate for WSNs due to its simplicity. However, the main problem of the NLMS is that the tradeoff between convergence speed and steady-state performance is achieved through the introduction of a step size [8]. It is not possible to achieve the best solution on these two aspects using a conventional NLMS estimation method. Channel estimation with the NLMS algorithm can be improved by introducing the set-membership filtering (SMF) framework [9], which modifies the objective function of the NLMS algorithm. It specifies an error bound on the magnitude of the estimation error, which can make the step size adaptive. Therefore, the SM-NLMS channel estimation method can achieve good convergence and tracking performance for each update. An SM-NLMS channel estimation algorithm for cooperative WSNs is proposed in [10]. Compared with the NLMS channel estimation method, the recursive least squares (RLS) channel estimator can provide better performance in terms of convergence speed and steady state [8]. However, it is not suitable for WSNs due to its high computational complexity [8]. To overcome this shortcoming, the SMF framework can also be introduced to devise a computationally efficient version of the conventional RLS channel estimation method, which is called BEACON channel estimation [11]. It can be considered as a constrained optimization problem where the objective function is the least squares (LS) cost function and the constraint is a bound on the magnitude of the estimation error. As a result, an adaptive forgetting factor can be derived to achieve the optimal performance for each update. Most importantly, the set-membership (SM) algorithms possess a feature that allows updating for only a small fraction of time, which is expressed as the update rate (UR). Therefore, the UR of the two SM channel estimation algorithms decreases due to the data-selective update, which can significantly reduce the computational complexity and extend the lifetime of the WSN by reducing its power consumption.

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The biggest issue for the SM channel estimation is the appropriate selection of the error bound, because it has a critical effect on the estimation performance. For SM-NLMS channel estimation, the extreme settings of the bound, i.e., overbounding (the error bound being too large) and underbounding (the error bound being too small), will result in performance degradation [12], [13]. In practice, the bound depends on environmental parameters such as the SNR. It is very difficult to accurately determine the optimal error bound because there is usually insufficient knowledge about the underlying system. For the BEACON channel estimation, the value of the error bound can be varied to trade off achievable performance against computational complexity [11]. A higher error bound would result in lower UR but worse performance. For WSNs, the aim is to quickly achieve an acceptable CSI with low power consumption. Therefore, the bound for BEACON channel estimation should be adjusted to ensure good estimation performance, lower computational complexity, and low UR. In addition, the required error bound may be time variant due to changing environmental conditions.

In this paper, we develop two matrix-based SM algorithms for channel estimation in cooperative WSNs using the AF cooperation protocol. The major novelty in these algorithms presented here is that they are matrix-based SM channel estimation algorithms, as opposed to vector-based SM techniques for filtering applications [14]–[16]. Therefore, we specify a bound on the norm of the estimation error vector, instead of the magnitude of the scalar estimation error. Then, a novel error bound function is introduced to automatically change the error bound to obtain optimal performance with the proposed SM channel estimation. Furthermore, we propose analytical expressions of the steady-state output excess mean-square error (MSE) of the two SM channel estimation methods. Further novelty in this analysis is that we employ the chi-square distribution to describe the probability of the update for estimating the channel matrix, as opposed to the Gaussian distribution for estimating the filter vector [17]–[19]. A key contribution of this paper is the consideration of techniques to reduce the complexity of the channel estimation for WSNs.

This paper is organized as follows: Section II describes the general cooperative WSN system model and its constrained form. Section III introduces two conventional channel estimation methods for reference. Section IV proposes two channel estimation methods using the SMF framework and presents an error bound function, which automatically tunes the error bound. Section V contains the analysis of the steady-state output excess MSE and the computational complexity. Section VI presents and discusses the simulation results, whereas Section VII provides some concluding remarks.

## II. COOPERATIVE WIRELESS SENSOR NETWORK SYSTEM MODEL

Consider a general  $m$ -hop WSN with multiple parallel relay nodes for each hop, as shown in Fig. 1. The WSN consists of  $N_s$  sources,  $N_d$  destinations, and  $N_r$  relays, which are separated into  $m - 1$  groups:  $N_{r(1)}, N_{r(2)}, \dots, N_{r(m-1)}$ . All these nodes are assumed to be within communication range. We will concentrate on a time-division scheme with perfect syn-

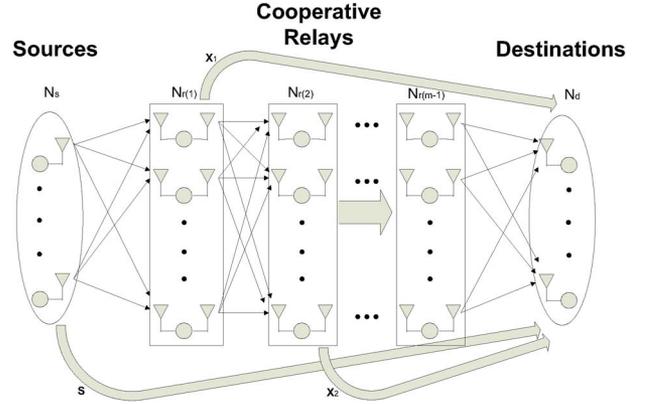


Fig. 1.  $m$ -hop cooperative WSN with  $N_s$  sources,  $N_d$  destinations, and  $N_r$  relays.

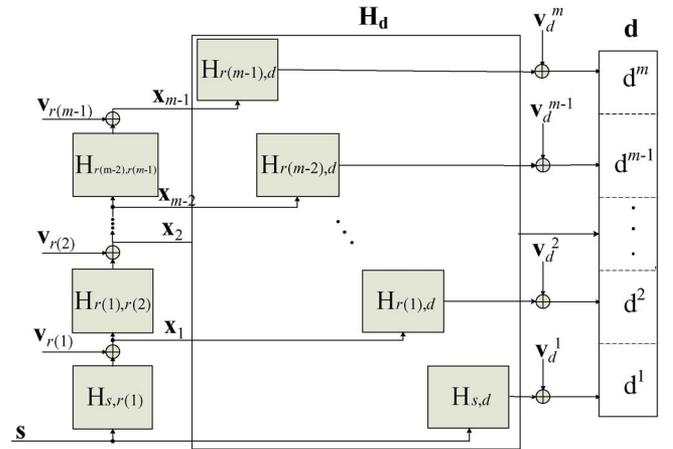


Fig. 2. Block diagram of the cooperative WSN system with transmission constraints.

chronization, for which all signals are transmitted and received in separate time slots. The sources first broadcast the  $N_s \times 1$  signal vector  $\mathbf{s}$  to the destinations and all groups of relays. We consider an AF cooperation protocol in this paper. Each group of relays receives the signal from the sources and the previous groups of relays and amplifies and rebroadcasts them to the next groups of relays and the destinations. In practice, we need to consider the constraints on the transmission policy. For example, each transmitting node would transmit during only one phase. In our WSN system, we assume that each group of relays directly transmits the signal to the nearest group of relays and the destinations. We can use a block diagram to indicate the cooperative WSN system with these transmission constraints, as shown in Fig. 2.

Let  $\mathbf{H}_{s,r(i)}$  denote the  $N_{r(i)} \times N_s$  channel matrix between the sources and the  $i$ th group of relays,  $\mathbf{H}_{r(i),d}$  denote the  $N_d \times N_{r(i)}$  channel matrix between the  $i$ th group of relays and destinations, and  $\mathbf{H}_{r(i-1),r(i)}$  denote the  $N_{r(i)} \times N_{r(i-1)}$  channel matrix between two groups of relays. The received signal at the  $i$ th group of relays ( $\mathbf{x}_i$ ) and destinations ( $\mathbf{d}$ ) for each phase can be expressed as given here.

Phase 1

$$\mathbf{x}_1 = \mathbf{H}_{s,r(1)}\mathbf{s} + \mathbf{v}_{r(1)} \quad (1)$$

$$\mathbf{d}^1 = \mathbf{H}_{s,d}\mathbf{s} + \mathbf{v}_d^1. \quad (2)$$

Phase 2

$$\mathbf{x}_2 = \mathbf{H}_{r(1),r(2)} \mathbf{A}_1 \mathbf{x}_1 + \mathbf{v}_{r(2)} \quad (3)$$

$$\mathbf{d}^2 = \mathbf{H}_{r(1),d} \mathbf{A}_1 \mathbf{x}_1 + \mathbf{v}_d^2$$

$$\vdots \quad (4)$$

Phase  $i$  ( $i = 2, 3, \dots, m-1$ )

$$\mathbf{x}_i = \mathbf{H}_{r(i-1),r(i)} \mathbf{A}_{i-1} \mathbf{x}_{i-1} + \mathbf{v}_{r(i)} \quad (5)$$

$$\mathbf{d}^i = \mathbf{H}_{r(i-1),d} \mathbf{A}_{i-1} \mathbf{x}_{i-1} + \mathbf{v}_d^i$$

$$\vdots \quad (6)$$

Phase  $m$

$$\mathbf{d}^m = \mathbf{H}_{r(m-1),d} \mathbf{A}_{m-1} \mathbf{x}_{m-1} + \mathbf{v}_d^m. \quad (7)$$

Here,  $\mathbf{v}$  is a zero-mean circularly symmetric complex additive white Gaussian noise vector with covariance matrix  $\sigma^2 \mathbf{I}$ .  $\mathbf{A}_i$  is a diagonal matrix whose elements represent the amplification coefficient of each relay of the  $i$ th group. Vectors  $\mathbf{d}^i$  and  $\mathbf{v}_d^i$  denote the received signal and noise at the destination nodes during the  $i$ th phase, respectively. At the destination nodes, the received signal can be expressed as

$$\mathbf{d} = \mathbf{H}_d \mathbf{A} \mathbf{y} + \mathbf{v}_d \quad (8)$$

where

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}^m \\ \text{---} \\ \mathbf{d}^{m-1} \\ \text{---} \\ \vdots \\ \text{---} \\ \mathbf{d}^2 \\ \text{---} \\ \mathbf{d}^1 \end{bmatrix} \quad \mathbf{v}_d = \begin{bmatrix} \mathbf{v}_d^m \\ \text{---} \\ \mathbf{v}_d^{m-1} \\ \text{---} \\ \vdots \\ \text{---} \\ \mathbf{v}_d^2 \\ \text{---} \\ \mathbf{v}_d^1 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} \mathbf{x}_{m-1} \\ \text{---} \\ \mathbf{x}_{m-2} \\ \text{---} \\ \vdots \\ \text{---} \\ \mathbf{x}_1 \\ \text{---} \\ \mathbf{s} \end{bmatrix} \quad (9)$$

$$\mathbf{H}_d = \begin{bmatrix} \mathbf{H}_{r(m-1),d} & & \cdots & \mathbf{0} \\ & \mathbf{H}_{r(m-2),d} & & \vdots \\ \vdots & & \ddots & \\ \mathbf{0} & & & \mathbf{H}_{r(1),d} \\ & & & \cdots & \mathbf{H}_{s,d} \end{bmatrix} \quad (10)$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{m-1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{A}_1 & \mathbf{I} \end{bmatrix} \quad (11)$$

Here, we use dashed lines to separate vectors  $\mathbf{d}$ ,  $\mathbf{v}_d$ , and  $\mathbf{y}$  to distinguish between transmissions to the destinations in  $m$  different time slots. Matrix  $\mathbf{H}_d$  consists of all the channels between each group of relays and destinations. Matrix  $\mathbf{A}$  consists of the amplification coefficients of all relays.

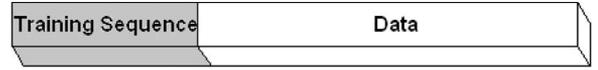


Fig. 3. Structure of the packet transmitted from source nodes and relay nodes.

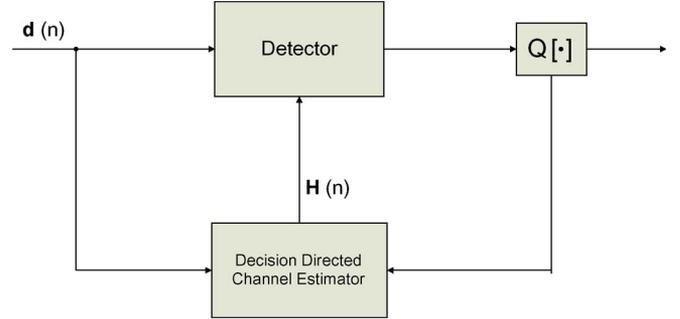


Fig. 4. Structure of the decision-directed channel estimation at the destination.

In our transmission scheme, all the data packets transmitted from the source nodes and relay nodes contain two parts: a preamble part with training sequence symbols and another part with data symbols (see Fig. 3). The source nodes transmit packets, and the relay nodes retransmit those packets that contain the identical training sequence symbols, which are known at the destination nodes. Therefore, we can make use of them for channel estimation at the destination nodes. Moreover, decision-directed channel estimation is exploited in our system by a scheme detailed in Fig. 4. We consider a minimum MSE (MMSE) detector whose formula can be expressed as  $\mathbf{W}_{\text{MMSE}}(n) = [\mathbf{H}(n)\mathbf{H}^H(n) + (\sigma_n^2/\sigma_s^2)\mathbf{I}]^{-1}\mathbf{H}(n)$  [8], where  $\mathbf{H}(n)$  is the estimated channel coefficient at time instant  $n$ , which can be received from the channel estimator. In addition, the block marked with a  $Q[\cdot]$  represents a decision device. After the training sequence, the channel estimation algorithm is switched to decision-directed mode [20], and the detected data symbols are fed to the channel estimator. It can continue to estimate and track the channel. Therefore, the channel variation can be tracked after the training phase, and this can yield better results. Furthermore, this decision-directed approach can reduce the length of the training sequence, which increases the bandwidth efficiency of the WSNs.

### III. CONVENTIONAL LEAST SQUARES AND MINIMUM MEAN SQUARE ERROR CHANNEL ESTIMATION

Consider a channel estimation problem where the output error is defined as

$$\mathbf{e} = \mathbf{r} - \mathbf{H}\mathbf{s} \quad (12)$$

where  $\mathbf{s}$  ( $N \times 1$ ) is the training sequence symbol vector,  $\mathbf{H}$  ( $M \times N$ ) is the estimated channel matrix, and  $\mathbf{r}$  ( $M \times 1$ ) is the received signal vector at the destination. Conventional channel estimation schemes seek to find channel matrix  $\mathbf{H}$  by minimizing a cost function that is a suitable objective function of the output error vector  $\mathbf{e}$ .

### A. LS Channel Estimator

The LS channel estimation minimizes the weighted sum of the squared norm of the error vector  $\|\mathbf{e}\|^2$ , which can be described as

$$\mathbf{H}_{\text{LS}}(n) = \arg \min_{\mathbf{H}(n)} \sum_{l=1}^n \lambda^{n-l} \|\mathbf{r}(l) - \mathbf{H}(n)\mathbf{s}(l)\|^2 \quad (13)$$

where  $\lambda$  denotes the forgetting factor. Computing the gradient of the argument and equating it to a zero matrix, we obtain the LS channel estimator as given by [21]

$$\mathbf{H}_{\text{LS}}(n) = \left[ \sum_{l=1}^n \lambda^{n-l} \mathbf{r}(l)\mathbf{s}^H(l) \right] \left[ \sum_{l=1}^n \lambda^{n-l} \mathbf{s}(l)\mathbf{s}^H(l) \right]^{-1} \quad (14)$$

where  $(\cdot)^H$  and  $(\cdot)^{-1}$  denote the complex-conjugate (Hermitian) transpose and the inverse, respectively. The LS estimator has a cubic cost with the number of parameters. A complexity reduction is possible by using a recursive procedure that yields the RLS algorithm with quadratic cost.

### B. MMSE Channel Estimator

The MMSE channel estimation minimizes the expected value of the squared norm of the error vector  $\|\mathbf{e}\|^2$ , which can be described as

$$\mathbf{H}_{\text{MMSE}} = \arg \min_{\mathbf{H}} E [\|\mathbf{r} - \mathbf{H}\mathbf{s}\|^2]. \quad (15)$$

After some derivation, the MMSE channel estimator is given by [21]

$$\mathbf{H}_{\text{MMSE}} = \mathbf{R} (\mathbf{S}^H E[\mathbf{H}^H \mathbf{H}] \mathbf{S} + M\sigma_n^2 \mathbf{I})^{-1} \mathbf{S}^H E[\mathbf{H}^H \mathbf{H}] \quad (16)$$

where  $\mathbf{S}$  and  $\mathbf{R}$  are the training sequence symbol matrix and received symbol matrix, respectively, during a training period. The MMSE channel estimator requires full *a priori* knowledge of the channel correlation matrix and the noise variance  $\sigma_n^2$  and a cubic cost with the number of parameters.

## IV. SET-MEMBERSHIP CHANNEL ESTIMATION

In contrast with the two conventional channel estimation methods introduced in Section III, SM channel estimation specifies an upper bound  $\gamma$  on the norm of the estimation error vector over a model space of interest, which is denoted as  $S$ , comprising all possible received signal pairs  $(\mathbf{s}, \mathbf{r})$ . The SM criterion corresponds to finding  $\mathbf{H}$  that satisfies

$$\|\mathbf{e}(\mathbf{H})\|^2 \leq \gamma^2 \quad \forall (\mathbf{s}, \mathbf{r}) \in S. \quad (17)$$

The set of all possible  $\mathbf{H}$  that satisfy (17) is referred to as the feasibility set and can be expressed as

$$\Theta = \bigcap_{(\mathbf{s}, \mathbf{r}) \in S} \{\mathbf{H} \in C^{M \times N} : \|\mathbf{r} - \mathbf{H}\mathbf{s}\| \leq \gamma\}. \quad (18)$$

At time instant  $n$ , the constraint set  $C_n$  is defined as the set of all  $\mathbf{H}(n)$  that satisfy (17) for the received signal pairs  $(\mathbf{s}(n), \mathbf{r}(n))$

$$C_n = \{\mathbf{H}(n) \in C^{M \times N} : \|\mathbf{r}(n) - \mathbf{H}(n)\mathbf{s}(n)\| \leq \gamma\}. \quad (19)$$

The idea behind the SM channel estimation is that, if the estimated channel at a time instant lies outside the constraint set  $C_n$ , the estimated channel at the next time instant will lie on the closest boundary of  $C_n$ . Otherwise, there is no need to compute, and the power consumption can be significantly reduced. This SM approach makes the estimator adapt only in the direction that is necessary.

### A. Proposed SM-NLMS Channel Estimation

The basic update in the LMS channel estimation can be written as

$$\mathbf{H}(n+1) = \mathbf{H}(n) + \mu(n)\mathbf{e}(n)\mathbf{s}^H(n) \quad (20)$$

where  $\mathbf{e}(n) = \mathbf{r}(n) - \mathbf{H}(n)\mathbf{s}(n)$  denotes the *a priori* error vector at time instant  $n$ , and  $\mu(n)$  is the time-dependent step size. Then, we can get a posterior error vector

$$\mathbf{g}(n) = \mathbf{r}(n) - \mathbf{H}(n+1)\mathbf{s}(n). \quad (21)$$

By substituting (20) into (21), we have

$$\begin{aligned} \mathbf{g}(n) &= \mathbf{r}(n) - (\mathbf{H}(n) + \mu(n)\mathbf{e}(n)\mathbf{s}^H(n))\mathbf{s}(n) \\ &= (\mathbf{r}(n) - \mathbf{H}(n)\mathbf{s}(n)) - \mu(n)\mathbf{e}(n)\mathbf{s}^H(n)\mathbf{s}(n) \\ &= \mathbf{e}(n) - \mu(n)\mathbf{e}(n)\mathbf{s}^H(n)\mathbf{s}(n). \end{aligned} \quad (22)$$

The constraint set is described as

$$\|\mathbf{g}(n)\| = \|\mathbf{e}(n) - \mu(n)\mathbf{e}(n)\mathbf{s}^H(n)\mathbf{s}(n)\| \leq \gamma. \quad (23)$$

If  $\|\mathbf{e}(n)\| > \gamma$ , then the previous solution lies outside the constraint set. We can choose the constraint value  $\|\mathbf{g}(n)\|$  equal to  $\gamma$ , so that the new solution lies on the closest boundary of the constraint set. Therefore

$$\|\mathbf{g}(n)\| = \|\mathbf{e}(n)\| |1 - \mu(n)\mathbf{s}^H(n)\mathbf{s}(n)| = \gamma. \quad (24)$$

Hence, the step size at the  $n$ th iteration  $\mu(n)$  can be expressed as

$$\mu(n) = \frac{1}{\mathbf{s}^H(n)\mathbf{s}(n)} \left(1 - \frac{\gamma}{\|\mathbf{e}(n)\|}\right). \quad (25)$$

Finally, we can write the update equation as

$$\mathbf{H}(n+1) = \mathbf{H}(n) + \mu(n)\mathbf{e}(n)\mathbf{s}^H(n) \quad (26)$$

where

$$\mu(n) = \begin{cases} \frac{1}{\mathbf{s}^H(n)\mathbf{s}(n)} \left(1 - \frac{\gamma}{\|\mathbf{e}(n)\|}\right), & \text{if } \|\mathbf{e}(n)\| > \gamma \\ 0, & \text{otherwise.} \end{cases} \quad (27)$$

Equation (27) shows that the estimated channel matrix updates with a specified step size only when the norm of the estimation error vector is larger than a fixed error bound, which we set. Otherwise, the step sizes are zeros, which means that there is no update at these time instants.

### B. Proposed BEACON Channel Estimation

The proposed BEACON channel estimation method can be considered as the following optimization problem:

$$\begin{aligned} & \text{minimize} && \sum_{l=1}^{n-1} \lambda(n)^{n-l} \|\mathbf{r}(l) - \mathbf{H}(n)\mathbf{s}(l)\|^2 \\ & \text{subject to} && \|\mathbf{r}(n) - \mathbf{H}(n)\mathbf{s}(n)\|^2 = \gamma^2. \end{aligned} \quad (28)$$

To solve this constrained optimization problem, we can modify the LS cost function using the method of Lagrange multipliers, which yields the following Lagrangian function:

$$\begin{aligned} \mathcal{L} = & \sum_{l=1}^{n-1} \lambda(n)^{n-l} \|\mathbf{r}(l) - \mathbf{H}(n)\mathbf{s}(l)\|^2 \\ & + \lambda(n) \left[ \|\mathbf{r}(n) - \mathbf{H}(n)\mathbf{s}(n)\|^2 - \gamma^2 \right] \end{aligned} \quad (29)$$

where  $\lambda(n)$  plays the role of both the Lagrange multiplier and the forgetting factor of the LS cost function. By setting the gradient of  $\mathcal{L}$  with respect to  $\mathbf{H}(n)$  equal to zero, after some mathematical manipulations (see Appendix A), we get the desired recursive equation for updating the channel matrix  $\mathbf{H}(n)$ , i.e.,

$$\mathbf{H}(n) = \mathbf{H}(n-1) + \lambda(n)\boldsymbol{\epsilon}(n)\mathbf{k}(n) \quad (30)$$

where  $\boldsymbol{\epsilon}(n) = \mathbf{r}(n) - \mathbf{H}(n-1)\mathbf{s}(n)$  denotes the prediction error vector at time instant  $n$ , and the recursive equation for updating the gain vector  $\mathbf{k}(n)$  is

$$\mathbf{k}(n) = \frac{\mathbf{s}^H(n)\mathbf{P}(n-1)}{1 + \lambda(n)\mathbf{s}^H(n)\mathbf{P}(n-1)\mathbf{s}(n)} \quad (31)$$

where

$$\mathbf{P}(n) = \mathbf{P}(n-1) - \lambda(n)\mathbf{P}(n-1)\mathbf{s}(n)\mathbf{k}(n). \quad (32)$$

The error vector is

$$\mathbf{e}(n) = \mathbf{r}(n) - \mathbf{H}(n)\mathbf{s}(n). \quad (33)$$

By substituting (30) into (33), we have

$$\begin{aligned} \mathbf{e}(n) &= \mathbf{r}(n) - [\mathbf{H}(n-1) + \lambda(n)\boldsymbol{\epsilon}(n)\mathbf{k}(n)]\mathbf{s}(n) \\ &= \mathbf{r}(n) - \mathbf{H}(n-1)\mathbf{s}(n) - \lambda(n)\boldsymbol{\epsilon}(n)\mathbf{k}(n)\mathbf{s}(n) \\ &= \boldsymbol{\epsilon}(n) - \lambda(n)\boldsymbol{\epsilon}(n) \frac{\mathbf{s}^H(n)\mathbf{P}(n-1)\mathbf{s}(n)}{1 + \lambda(n)\mathbf{s}^H(n)\mathbf{P}(n-1)\mathbf{s}(n)} \\ &= \boldsymbol{\epsilon}(n) - \lambda(n)\boldsymbol{\epsilon}(n) \frac{G(n)}{1 + \lambda(n)G(n)} \\ &= \boldsymbol{\epsilon}(n) \left[ 1 - \frac{\lambda(n)G(n)}{1 + \lambda(n)G(n)} \right] \\ &= \boldsymbol{\epsilon}(n) \frac{1}{1 + \lambda(n)G(n)} \end{aligned} \quad (34)$$

where  $G(n) = \mathbf{s}^H(n)\mathbf{P}(n-1)\mathbf{s}(n)$ . The constraint set is described as

$$\|\mathbf{e}(n)\| = \left\| \boldsymbol{\epsilon}(n) \frac{1}{1 + \lambda(n)G(n)} \right\| \leq \gamma. \quad (35)$$

TABLE I  
SUMMARY OF THE BEACON CHANNEL ESTIMATION ALGORITHM

Initialize the algorithm by setting	
$\mathbf{H}(0) = \mathbf{0}$	
$\mathbf{P}(0) = \mathbf{I}$	
For each instant of time, $n=1, 2, \dots$ , compute	
$\boldsymbol{\epsilon}(n) = \mathbf{r}(n) - \mathbf{H}(n-1)\mathbf{s}(n)$	
$\lambda(n) = \begin{cases} \frac{1}{G(n)} \left( \frac{\ \boldsymbol{\epsilon}(n)\ }{\gamma} - 1 \right), & \text{if } \ \boldsymbol{\epsilon}(n)\  > \gamma, \\ 0, & \text{otherwise.} \end{cases}$	$\left. \begin{aligned} & \text{where } G(n) = \mathbf{s}^H(n)\mathbf{P}(n-1)\mathbf{s}(n) \\ & \mathbf{k}(n) = \frac{\mathbf{s}^H(n)\mathbf{P}(n-1)}{1 + \lambda(n)G(n)} \\ & \mathbf{H}(n) = \mathbf{H}(n-1) + \lambda(n)\boldsymbol{\epsilon}(n)\mathbf{k}(n) \\ & \mathbf{P}(n) = \mathbf{P}(n-1) - \lambda(n)\mathbf{P}(n-1)\mathbf{s}(n)\mathbf{k}(n) \end{aligned} \right\}$

If  $\|\boldsymbol{\epsilon}(n)\| > \gamma$ , then the previous solution lies outside the constraint set. We can choose the constraint value  $\|\boldsymbol{\epsilon}(n)\|$  equal to  $\gamma$ , so that the new solution lies on the closest boundary of the constraint set. Therefore

$$\|\mathbf{e}(n)\| = \|\boldsymbol{\epsilon}(n)\| \frac{1}{|1 + \lambda(n)G(n)|} = \gamma. \quad (36)$$

Hence, the optimal forgetting factor at the  $n$ th iteration can be expressed as

$$\lambda(n) = \frac{1}{G(n)} \left( \frac{\|\boldsymbol{\epsilon}(n)\|}{\gamma} - 1 \right). \quad (37)$$

Table I shows a summary of the BEACON channel estimation algorithm, which will be used for the simulations.

### C. Time-Varying Bound

To obtain the optimal error bound at each time instant, in this section, we introduce an error bound function that can automatically adjust the error bound with the update of the channel estimate. A similar bound for the SM filtering techniques has been described in [9]. For channel estimation, the bound is heuristic and employs the CSI parameter matrix and the noise variance that should be related with the estimates of interest. It can be expressed as

$$\gamma(n+1) = (1 - \beta)\gamma(n) + \beta\sqrt{\alpha \|\mathbf{H}(n)\|^2 \sigma^2} \quad (38)$$

where  $\beta$  is the forgetting factor,  $\alpha$  is the tuning parameter, and  $\sigma^2$  is the variance of the noise, which is assumed to be known at the destinations. This time-varying bound (TVB) is recursive, so that it can be used to avoid too high or low values of  $\|\mathbf{H}(n)\|^2$ .

## V. ANALYSIS OF THE PROPOSED ALGORITHMS

### A. Steady-State Output MSE Analysis

In this section, we investigate the output MSE in the SM-NLMS and the BEACON channel estimation. The received signal at time instant  $n$  is given by

$$\mathbf{r}(n) = \mathbf{H}_0\mathbf{s}(n) + \mathbf{n}(n) \quad (39)$$

where  $\mathbf{H}_0$  ( $M \times N$ ) is the channel matrix needed to be estimated, and  $\mathbf{n}(n)$  is measurement noise that is assumed here

to be Gaussian with zero mean and variance  $\sigma_n^2$ . Defining the channel estimation error matrix as

$$\Delta\mathbf{H}(n) = \mathbf{H}_0 - \mathbf{H}(n) \quad (40)$$

we can express the output error vector as

$$\begin{aligned} \mathbf{e}(n) &= \mathbf{r}(n) - \mathbf{H}(n)\mathbf{s}(n) \\ &= \mathbf{r}(n) - [\mathbf{H}_0 - \Delta\mathbf{H}(n)]\mathbf{s}(n) \\ &= \mathbf{r}(n) - \mathbf{H}_0\mathbf{s}(n) + \Delta\mathbf{H}(n)\mathbf{s}(n) \\ &= \mathbf{n}(n) + \Delta\mathbf{H}(n)\mathbf{s}(n). \end{aligned} \quad (41)$$

Therefore, the output MSE expression can be derived as

$$\begin{aligned} J(n) &= E[\|\mathbf{e}(n)\|^2] \\ &= E\left\{[\mathbf{n}(n) + \mathbf{s}(n)\Delta\mathbf{H}(n)]^H [\mathbf{n}(n) + \Delta\mathbf{H}(n)\mathbf{s}(n)]\right\} \\ &= E[\|\mathbf{n}(n)\|^2] + E[\mathbf{s}^H(n)\Delta\mathbf{H}^H(n)\Delta\mathbf{H}(n)\mathbf{s}(n)] \\ &= M\sigma_n^2 + \text{tr}\{E[\mathbf{s}^H(n)\Delta\mathbf{H}^H(n)\Delta\mathbf{H}(n)\mathbf{s}(n)]\} \end{aligned} \quad (42)$$

where  $\text{tr}(\cdot)$  denotes the trace of a matrix. The property of the matrix trace  $\text{tr}(\mathbf{X}\mathbf{Y}) = \text{tr}(\mathbf{Y}\mathbf{X})$  will be used in the following derivation. From (42), we can define the output excess MSE as

$$\begin{aligned} J_{\text{ex}}(n) &= \text{tr}\{E[\mathbf{s}^H(n)\Delta\mathbf{H}^H(n)\Delta\mathbf{H}(n)\mathbf{s}(n)]\} \\ &= \text{tr}\{E[\mathbf{s}(n)\mathbf{s}^H(n)\Delta\mathbf{H}^H(n)\Delta\mathbf{H}(n)]\}. \end{aligned} \quad (43)$$

1) *For the SM-NLMS:* The update equations for the SM-NLMS channel estimation are given by (26) and (27). In (27),  $\mathbf{s}^H(n)\mathbf{s}(n)$  is equal to  $N\sigma_s^2$ , where  $\sigma_s^2$  is the variance of the pilot signal. By substituting (27) into (26), we can achieve an alternative update equation

$$\mathbf{H}(n+1) = \mathbf{H}(n) + \frac{1}{N\sigma_s^2} \left(1 - \frac{\gamma}{\|\mathbf{e}_0(n)\|}\right) \mathbf{e}(n)\mathbf{s}^H(n) \quad (44)$$

where

$$\|\mathbf{e}_0(n)\| = \begin{cases} \|\mathbf{e}(n)\|, & \text{if } \|\mathbf{e}(n)\| > \gamma \\ \gamma, & \text{otherwise.} \end{cases} \quad (45)$$

As a consequence, the update equation of the channel estimation error can be expressed as

$$\begin{aligned} \Delta\mathbf{H}(n+1) &= \Delta\mathbf{H}(n) - \frac{1}{N\sigma_s^2} \left(1 - \frac{\gamma}{\|\mathbf{e}_0(n)\|}\right) \mathbf{e}(n)\mathbf{s}^H(n) \\ &= \Delta\mathbf{H}(n) - \frac{1}{N\sigma_s^2} \mathbf{e}(n)\mathbf{s}^H(n) \\ &\quad + \frac{\gamma}{N\sigma_s^2} \frac{\mathbf{e}(n)}{\|\mathbf{e}_0(n)\|} \mathbf{s}^H(n). \end{aligned} \quad (46)$$

Then, we can use (46) to derive the update equation of the output excess MSE in (43) (see Appendix B)

$$\begin{aligned} J_{\text{ex}}(n+1) &= M\sigma_n^2 + 2\gamma E\left[\frac{1}{\|\mathbf{e}_0(n)\|}\right] J_{\text{ex}}(n) \\ &\quad - 2\gamma E\left[\frac{\|\mathbf{e}(n)\|^2}{\|\mathbf{e}_0(n)\|}\right] + \gamma^2 E\left[\frac{\|\mathbf{e}(n)\|^2}{\|\mathbf{e}_0(n)\|^2}\right]. \end{aligned} \quad (47)$$

From (45), the three expected values in (47) can be expressed as

$$E\left[\frac{1}{\|\mathbf{e}_0(n)\|}\right] = E\left[\frac{1}{\|\mathbf{e}(n)\|} \mid \|\mathbf{e}(n)\| > \gamma\right] P_{\text{up}} + \frac{1}{\gamma} (1 - P_{\text{up}}) \quad (48)$$

$$\begin{aligned} E\left[\frac{\|\mathbf{e}(n)\|^2}{\|\mathbf{e}_0(n)\|}\right] &= E[\|\mathbf{e}(n)\| \mid \|\mathbf{e}(n)\| > \gamma] P_{\text{up}} \\ &\quad + \frac{1}{\gamma} E[\|\mathbf{e}(n)\|^2 \mid \|\mathbf{e}(n)\| \leq \gamma] (1 - P_{\text{up}}) \end{aligned} \quad (49)$$

$$E\left[\frac{\|\mathbf{e}(n)\|^2}{\|\mathbf{e}_0(n)\|^2}\right] = P_{\text{up}} + \frac{1}{\gamma^2} E[\|\mathbf{e}(n)\|^2 \mid \|\mathbf{e}(n)\| \leq \gamma] (1 - P_{\text{up}}) \quad (50)$$

where  $E[\cdot|\cdot]$  denotes the conditional expected value, and  $P_{\text{up}}$  stands for the probability of update in each recursion. Let

$$X_1 = E\left[\frac{1}{\|\mathbf{e}(n)\|} \mid \|\mathbf{e}(n)\| > \gamma\right] \quad (51)$$

$$Y_1 = E[\|\mathbf{e}(n)\| \mid \|\mathbf{e}(n)\| > \gamma] \quad (52)$$

$$Z_1 = E[\|\mathbf{e}(n)\|^2 \mid \|\mathbf{e}(n)\| \leq \gamma]. \quad (53)$$

Equation (47) becomes

$$\begin{aligned} J_{\text{ex}}(n+1) &= M\sigma_n^2 + [2\gamma X_1 P_{\text{up}} + 2(1 - P_{\text{up}})] J_{\text{ex}}(n) \\ &\quad - 2\gamma Y_1 P_{\text{up}} - 2Z_1(1 - P_{\text{up}}) + \gamma^2 P_{\text{up}} \\ &\quad + Z_1(1 - P_{\text{up}}) \\ &= (2\gamma X_1 P_{\text{up}} + 2 - 2P_{\text{up}}) J_{\text{ex}}(n) - 2\gamma Y_1 P_{\text{up}} \\ &\quad - Z_1(1 - P_{\text{up}}) + M\sigma_n^2 + \gamma^2 P_{\text{up}}. \end{aligned} \quad (54)$$

During the steady state,  $J_{\text{ex}}(n+1) \rightarrow J_{\text{ex}}(n)$ . Therefore, the steady-state output excess MSE expression of the SM-NLMS channel estimation is

$$J_{\text{ex}}(n) = \frac{2\gamma Y_1 P_{\text{up}} + Z_1(1 - P_{\text{up}}) - M\sigma_n^2 - \gamma^2 P_{\text{up}}}{2\gamma X_1 P_{\text{up}} - 2P_{\text{up}} + 1}. \quad (55)$$

2) *For the BEACON:* According to Table I, we can get the update equation of the channel estimation error for the BEACON channel estimation, which is very similar to (46), i.e.,

$$\begin{aligned} \Delta\mathbf{H}(n) &= \Delta\mathbf{H}(n-1) - \frac{\boldsymbol{\epsilon}(n)\mathbf{s}^H(n)\mathbf{P}(n-1)}{G(n)} \\ &\quad + \gamma \frac{\boldsymbol{\epsilon}(n)}{\|\boldsymbol{\epsilon}_0(n)\|} \frac{\mathbf{s}^H(n)\mathbf{P}(n-1)}{G(n)} \end{aligned} \quad (56)$$

where

$$\|\boldsymbol{\epsilon}_0(n)\| = \begin{cases} \|\boldsymbol{\epsilon}(n)\|, & \text{if } \|\boldsymbol{\epsilon}(n)\| > \gamma \\ \gamma, & \text{otherwise.} \end{cases} \quad (57)$$

Following the same steps described for the SM-NLMS channel estimation in the Appendix, we find that the steady-state output

TABLE II  
COMPUTATIONAL COMPLEXITY PER UPDATE

Algorithm	Multiplication	Addition	Division
NLMS	$2MN + N + \min\{M, N\}$	$2MN + N - 1$	1
SM-NLMS	$MN + M + P_{up}(MN + N + \min\{M, N\})$	$MN + M - 1 + P_{up}(MN + N)$	2
RLS	$4N^2 + 2MN + N$	$3N^2 + 2MN - N$	2
BEACON	$N^2 + MN + M + N + P_{up}(2N^2 + MN + N + \min\{M, N\})$	$N^2 + MN + M - 2 + P_{up}(2N^2 + MN - N + 2)$	2

excess MSE expression of the BEACON channel estimation has the same style as (55), i.e.,

$$J_{\text{ex}}(n) = \frac{2\gamma Y_2 P_{\text{up}} + Z_2(1 - P_{\text{up}}) - M\sigma_n^2 - \gamma^2 P_{\text{up}}}{2\gamma X_2 P_{\text{up}} - 2P_{\text{up}} + 1} \quad (58)$$

where

$$X_2 = E \left[ \frac{1}{\|\boldsymbol{\epsilon}(n)\|} \middle| \|\boldsymbol{\epsilon}(n)\| > \gamma \right] \quad (59)$$

$$Y_2 = E [\|\boldsymbol{\epsilon}(n)\| \mid \|\boldsymbol{\epsilon}(n)\| > \gamma] \quad (60)$$

$$Z_2 = E [\|\boldsymbol{\epsilon}(n)\|^2 \mid \|\boldsymbol{\epsilon}(n)\| \leq \gamma]. \quad (61)$$

3) *Probability of Update  $P_{\text{up}}$* : From (27), we can get the relation about the probability of update of the SM-NLMS channel estimation

$$P_{\text{up}} = \Pr \{\|\boldsymbol{e}(n)\| > \gamma\} = \Pr \{\|\boldsymbol{e}(n)\|^2 > \gamma^2\}. \quad (62)$$

Similarly, for the BEACON channel estimation, we just need to use  $\boldsymbol{\epsilon}(n)$ , instead of  $\boldsymbol{e}(n)$ . It is easy to see that  $P_{\text{up}}$  depends on the distribution of  $\|\boldsymbol{e}(n)\|^2$ . For the estimated channel matrix  $\mathbf{H}_0$  with size  $M \times N$

$$\begin{aligned} \|\boldsymbol{e}(n)\|^2 &= \sum_{i=1}^M (\Re^2[e_i(n)] + \Im^2[e_i(n)]) \\ &= \frac{\sigma_n^2}{2} \sum_{i=1}^M \left( \frac{\Re^2[e_i(n)]}{\sigma_n^2/2} + \frac{\Im^2[e_i(n)]}{\sigma_n^2/2} \right). \end{aligned} \quad (63)$$

During the steady state, assuming  $\Delta\mathbf{H}(n) \rightarrow 0$ , the linear relationship between  $\boldsymbol{e}(n)$ ,  $\Delta\mathbf{H}(n)$ , and  $\mathbf{n}(n)$  in (41) shows that the distribution of  $\boldsymbol{e}(n)$  is typically Gaussian, unless a jamming or other interference signal with another distribution is present. Therefore, we can get that the elements of the error vector  $\boldsymbol{e}(n)$  have the same distribution with the elements of the noise vector  $\mathbf{n}(n)$ . Recalling that  $\Re[n_i(n)]$  and  $\Im[n_i(n)] \sim \mathcal{N}(0, \sigma_n^2/2)$ , we can express the distribution of (63) by a chi-square random variable with  $2M$  degree of freedom as follows:

$$\|\boldsymbol{e}(n)\|^2 \sim \frac{\sigma_n^2}{2} \chi_{2M}^2. \quad (64)$$

Therefore, (62) becomes

$$\begin{aligned} P_{\text{up}} &= \Pr \left\{ \sum_{i=1}^M \left( \frac{\Re^2[e_i(n)]}{\sigma_n^2/2} + \frac{\Im^2[e_i(n)]}{\sigma_n^2/2} \right) > \gamma^2 \frac{2}{\sigma_n^2} \right\} \\ &= 1 - \Pr \left\{ \sum_{i=1}^M \left( \frac{\Re^2[e_i(n)]}{\sigma_n^2/2} + \frac{\Im^2[e_i(n)]}{\sigma_n^2/2} \right) \leq \gamma^2 \frac{2}{\sigma_n^2} \right\} \\ &= 1 - F \left( \gamma^2 \frac{2}{\sigma_n^2}; 2M \right) \end{aligned} \quad (65)$$

where  $F(\cdot)$  is the chi-square cumulative distribution function [22] defined by

$$F(x; l) = \frac{\Gamma_L(l/2, x/2)}{\Gamma(l/2)}. \quad (66)$$

In (66),  $\Gamma_L(s, x)$  is the lower incomplete Gamma function

$$\Gamma_L(s, x) = \int_0^x t^{s-1} e^{-t} dt \quad (67)$$

and  $\Gamma(x)$  is the gamma function

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt. \quad (68)$$

By substituting (67) and (68) into (66), we can finally obtain

$$F(x; l) = \frac{\int_0^{\frac{x}{2}} t^{\frac{l}{2}-1} e^{-t} dt}{\int_0^\infty t^{\frac{l}{2}-1} e^{-t} dt} \quad (69)$$

where  $l$  denotes the number of degrees of freedom.

### B. Computational Complexity Analysis

Table II lists the computational complexity per update in terms of the number of multiplications, additions, and divisions for the SM-NLMS and BEACON algorithms and their competing algorithms. The size of the estimated channel matrix is  $M \times N$ . For our cooperative WSN system model, when  $\mathbf{H}_d$  is chosen as the estimated channel, we can get

$$M = mN_d \quad (70)$$

$$N = N_r + N_s. \quad (71)$$

Because the multiplication dominates the computational complexity of the algorithms, to compare the computational complexity of our proposed algorithms with their competition algorithms, the number of multiplications versus the size of the channel matrix performance for each update is shown in Fig. 5. For the purpose of illustration, we set  $M$  to be equal to  $N$ . It can be seen that our proposed SM-NLMS and BEACON channel estimation algorithms have a significant complexity reduction compared with the conventional NLMS and RLS channel estimation algorithms. Obviously, a lower  $P_{\text{up}}$  will cause lower computational complexity. Furthermore, assuming that the linear MMSE detectors are used in the destination nodes, which require cubic complexity, we can come to the conclusion that the power used for our proposed channel estimation is only a small fraction of the power budget of these nodes.

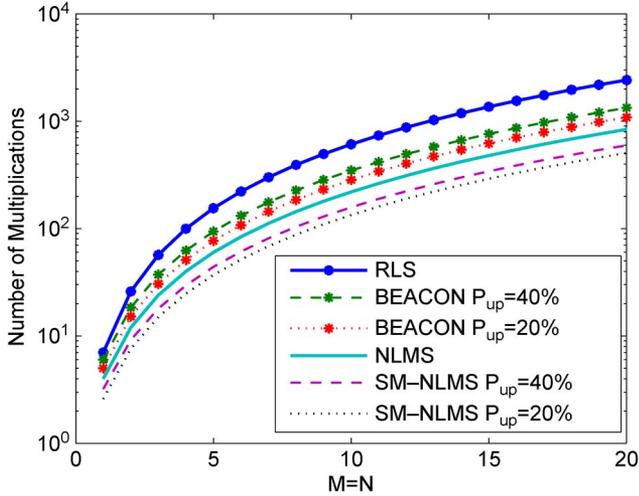


Fig. 5. Number of multiplications versus the size of the channel matrix.

## VI. SIMULATIONS

In this section, we numerically study the performance of our two proposed SM estimation methods, as well as the design of the optimal error bound. We consider a three-hop ( $m = 3$ ) WSN. The number of sources ( $N_s$ ), two groups of relays ( $N_{r(1)}, N_{r(2)}$ ), and destinations ( $N_d$ ) are 2, 4, 4, and 3, respectively. We consider an AF cooperation protocol, and the amplification coefficient of each relay is set to 1 for the purpose of simplification. We choose  $\mathbf{H}_d$  as our estimated channel because it is the most significant and most complex channel among all channels of the WSN system. The quasi-static fading channel (block-fading channel) is considered in our simulations whose elements are Rayleigh random variables (with zero mean and unit variance) and assumed to be invariant during the transmission of each packet. In addition, to test our proposed channel estimation algorithms in a time-varying environment, we consider a typical fading channel for wireless communications systems, i.e., a Rayleigh fading channel that is modeled according to Clarke's Model [23]. According to the transmission scheme introduced in Section II, during each phase, the sources and each group of relays transmit the quadrature-phase-shift-keying modulated packets with  $n_p$  symbols, among which,  $n_t$  are training symbols, and  $n_d$  are data symbols (note that  $n_p = n_t + n_d$ ).  $n_p$ ,  $n_t$ , and  $n_d$  will be specified in the succeeding simulations. The noise at the destination nodes is modeled as circularly symmetric complex Gaussian random variables with zero mean. The SNR is fixed at 10 dB.

### A. MSE Performance

Figs. 6 and 7 show the channel matrix MSE performance of our proposed SM-NLMS and BEACON channel estimation methods for the quasi-static fading channel and compare them with the conventional NLMS and RLS channel estimation algorithms. For the SM-NLMS estimator, we transmit packets with 1000 ( $n_p$ ) symbols, among which 100 ( $n_t$ ) are training symbols and 900 ( $n_d$ ) are data symbols. We choose five fixed error bounds ( $\gamma$ ) ranging from 0.3 to 1.1. It can be seen that

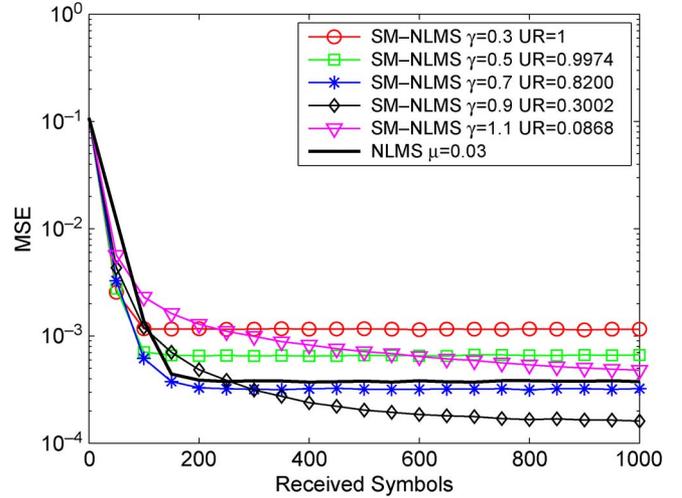


Fig. 6. MSE performance of the SM-NLMS channel estimation of  $\mathbf{H}_d$  for quasi-static fading channel, compared with the NLMS channel estimation.  $n_p = 1000$ ,  $n_t = 100$ , and  $n_d = 900$ .

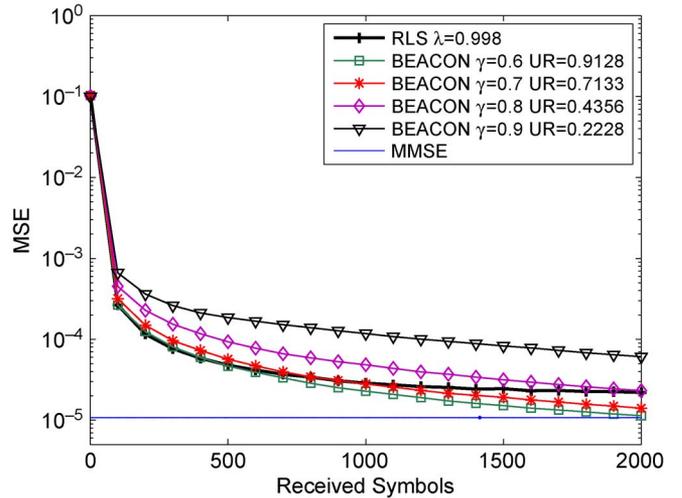


Fig. 7. MSE performance of the BEACON channel estimation of  $\mathbf{H}_d$  for quasi-static fading channel, compared with the RLS channel estimation.  $n_p = 2000$ ,  $n_t = 100$ , and  $n_d = 1900$ .

increasing the error bound makes the UR decrease. It means that the update is selective, which can reduce the computational complexity and power consumption. In the case of an error bound equal to 1.1, the UR can dramatically fall to 0.0868. The optimal error bound appears between 0.7 and 0.9. In that situation, the SM-NLMS channel estimation achieves the fastest convergence speed and lowest steady states. Otherwise, the performance degrades due to overbounding or underbounding. For the BEACON estimator, we transmit packets with 2000 ( $n_p$ ) symbols, among which, 100 ( $n_t$ ) are training symbols, and 1900 ( $n_d$ ) are data symbols. We choose four fixed error bounds ranging from 0.6 to 0.9. In addition, the MMSE channel estimator, which requires full *a priori* knowledge of the channel correlation matrix and the noise variance, is used here for reference. It can be seen that a higher value of  $\gamma$  results in worse MSE performance but a lower UR. In the case of an error bound equal to 0.6, the BEACON algorithm outperforms the conventional RLS algorithm (with a forgetting factor of 0.998)

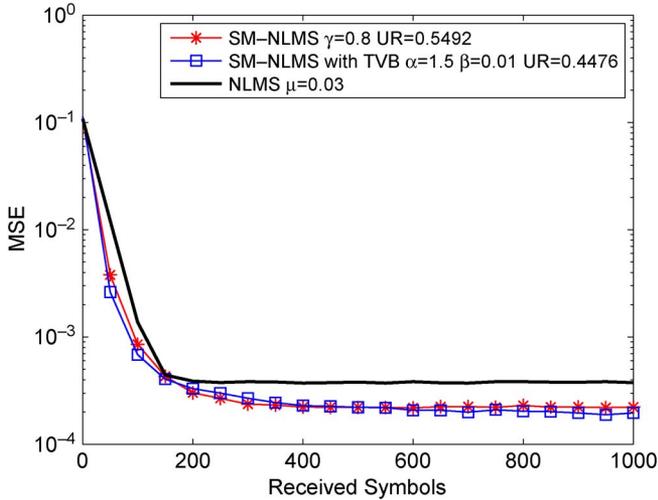


Fig. 8. MSE performance of the SM-NLMS channel estimation with a TVB for quasi-static fading channel.  $n_p = 1000$ ,  $n_t = 100$ , and  $n_d = 900$ .

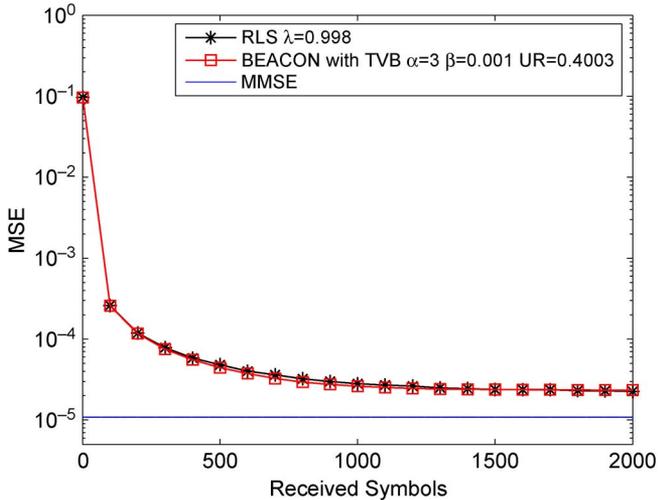


Fig. 9. MSE performance of the BEACON channel estimation with a TVB for quasi-static fading channel.  $n_p = 2000$ ,  $n_t = 100$ , and  $n_d = 1900$ .

in terms of convergence speed and steady state with a slightly reduced UR (0.9128). When the error bound is increased to 0.8, although its convergence speed is slower than RLS channel estimation, the final MSE is comparable with a much lower UR (0.4356). Figs. 8 and 9 show the performance when we apply the TVB into the SM-NLMS and BEACON channel estimation. For the SM-NLMS estimator, we set  $\alpha$  to 1.5 and  $\beta$  to 0.01. The curve of our proposed algorithm lies on the optimal position that is very close to the curve of the SM-NLMS with fixed error bound 0.8. In addition, its UR further decreases, which is our expectation. For the BEACON estimator, we set  $\alpha$  to 3 and  $\beta$  to 0.001. Our proposed algorithm can achieve very similar performance to the conventional RLS channel estimation with a substantial reduction in the UR. Therefore, the computational complexity is significantly reduced. The MSE versus SNR performance of the SM-NLMS and BEACON channel estimation methods are displayed with fixed error bounds and the proposed time-varying error bounds in Figs. 10 and 11. In the cases

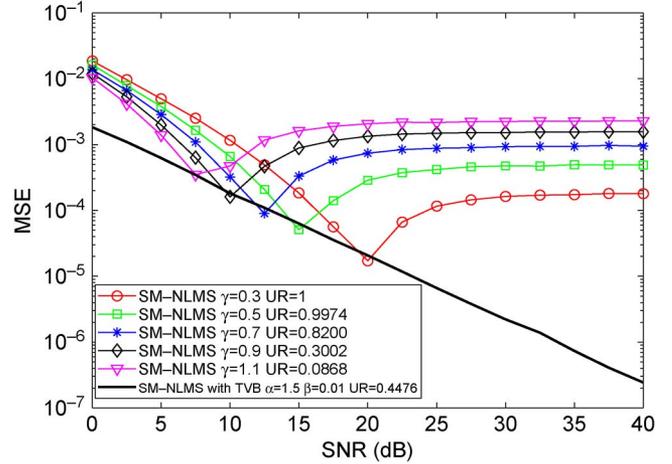


Fig. 10. SM-NLMS channel estimation MSEs versus SNR for both the fixed bound and TVB for quasi-static fading channel.  $n_p = 1000$ ,  $n_t = 100$ , and  $n_d = 900$ .

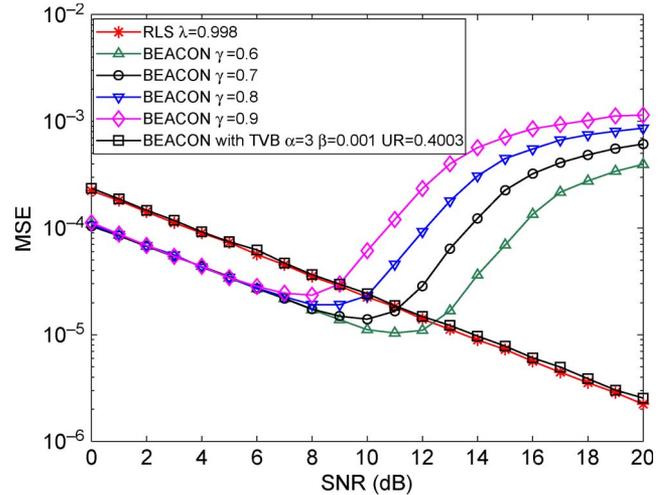


Fig. 11. BEACON channel estimation MSEs versus SNR for both the fixed bound and TVB for quasi-static fading channel.  $n_p = 2000$ ,  $n_t = 100$ , and  $n_d = 1900$ .

of fixed error bounds, the MSE is lower bounded at different values for different error bounds. For the SM-NLMS estimator, a higher SNR needs a specified lower error bound to achieve the optimal MSE performance. When the time-varying error bound is applied, the MSE remains very close to the optimal values for all SNRs. For the BEACON estimator, when the SNR is larger than a specified value, its MSE will become worse. However, when the time-varying error bound is applied, it can be observed that the MSE keeps on decreasing along with the increase in the SNR. We can notice from Fig. 11 that, when the SNR is low, setting fixed bounds can achieve better performance than setting TVB. Therefore, it would be possible to devise a “hybrid” BEACON channel estimation that switches between fixed bound and TVB, depending on the SNR. These two figures show the robustness to the SNR variation of our proposed algorithms for the quasi-static fading channel. To test our proposed channel estimation algorithms in a time-varying environment, we consider a typical fading channel for

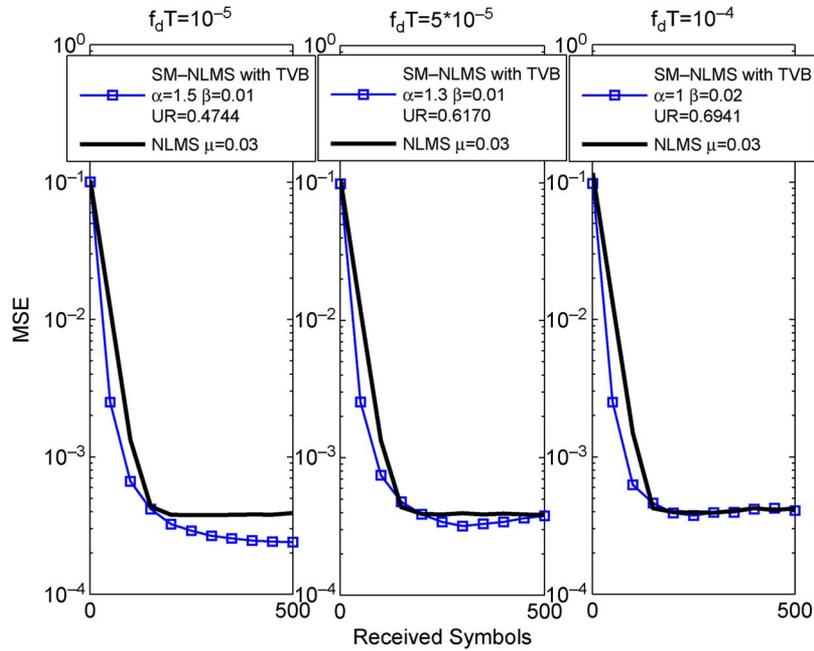


Fig. 12. MSE performance of the SM-NLMS channel estimation for Rayleigh fading channels, compared with the NLMS channel estimation.  $n_p = 500$ ,  $n_t = 50$ , and  $n_d = 450$ .

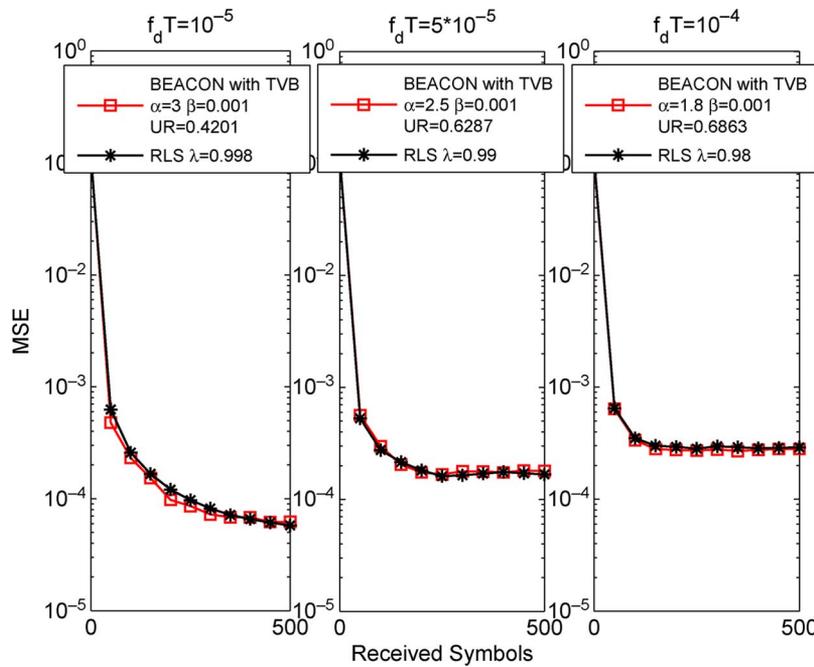


Fig. 13. MSE performance of the BEACON channel estimation for Rayleigh fading channels, compared with the RLS channel estimation.  $n_p = 500$ ,  $n_t = 50$ , and  $n_d = 450$ .

wireless systems, i.e., a Rayleigh fading channel, which is modeled according to Clarke’s Model [23]. Figs. 12 and 13 show the MSE performance of our proposed channel estimation algorithms for the time-varying fading channel and three different fading rates (normalized Doppler frequency  $f_d T$ , where  $T$  is the symbol duration) are used in the simulations:  $10^{-5}$ ,  $5 \times 10^{-5}$ , and  $10^{-4}$ . Because of the requirements of low power consumption and the fact that a fast convergence speed of the proposed algorithms might help in reducing the need for long training sequences for the WSNs, we focus on the performance

of packets with 500 ( $n_p$ ) symbols, among which, 50 ( $n_t$ ) are training symbols, and 450 ( $n_d$ ) are data symbols. For the SM-NLMS estimator, our proposed algorithm can achieve better performance than the conventional NLMS algorithms for all the three fading rates. Along with the increase in fading rate, the advantage becomes less pronounced, and the UR becomes higher. For the BEACON estimator, our proposed algorithm can achieve very similar performance to the conventional RLS algorithms for all three fading rates. (Note that, for the conventional RLS algorithms, when increasing the fading rate, we have

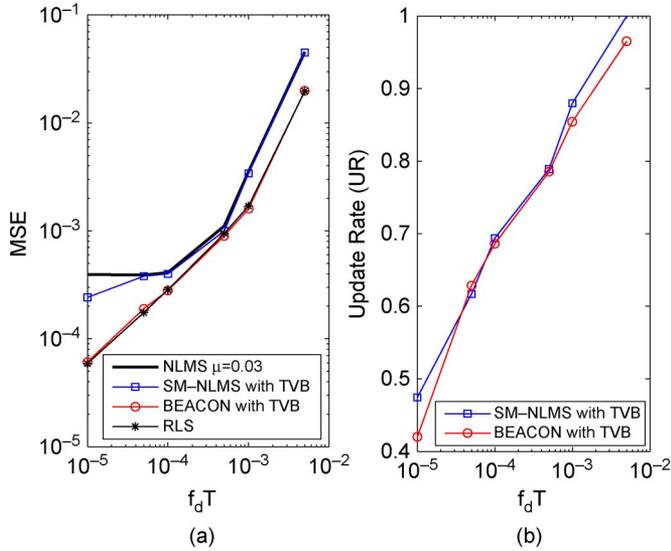


Fig. 14. (a) MSE performance versus  $f_d T$  and (b) UR versus  $f_d T$  of SM-NLMS and BEACON channel estimation for Rayleigh fading channels.

to lower the forgetting factor to get the optimal performance.) Along with the increase in fading rate, the UR becomes higher. To show the performance tendency for higher  $f_d T$ , we extend its range up to  $5 \times 10^{-3}$ . The performance curves are shown in Fig. 14, which includes the MSE performance versus  $f_d T$  and UR versus  $f_d T$  of SM-NLMS and BEACON channel estimation for Rayleigh fading channels. (Note that the MSE values we used in this figure are chosen from the MSE when receiving 500 symbols.) This figure indicates that the performance of our proposed algorithms is comparable with the existing NLMS and RLS algorithms, even for the fast fading channels. Therefore, we can conclude that our proposed channel estimation algorithms can work well for the time-varying fading channel and for a wide range of values of  $f_d T$ .

**B. BER Performance**

The MSE performance is very useful in giving designers an idea of how well channel estimators perform, whereas bit error rate (BER) performance is meaningful in practice. Therefore, in this section, we focus on the BER performance of our proposed algorithms. We consider a simulation where the data packets transmitted at the sources nodes have 1000 ( $n_p$ ) symbols and trained with 100 ( $n_t$ ) symbols. Linear MMSE detectors are used in the destination nodes. We choose  $\mathbf{H}_d$  as our estimated channel, and other channels are assumed to be known. Quasi-static fading channels and Rayleigh fading channels are considered. In addition, for the Rayleigh fading channel, the SNR is fixed at 5 dB. It can be seen from Fig. 15 that our two proposed SM channel estimation algorithms with TVB can achieve a similar BER performance to their competing algorithms, regardless of whether they are in a Quasi-static fading channel or a Rayleigh fading channel with a wide range values of fading rate. In addition, the BEACON channel estimator has lower BER than the SM-NLMS channel estimator due to the higher computational complexity and the use of second-order statistics.

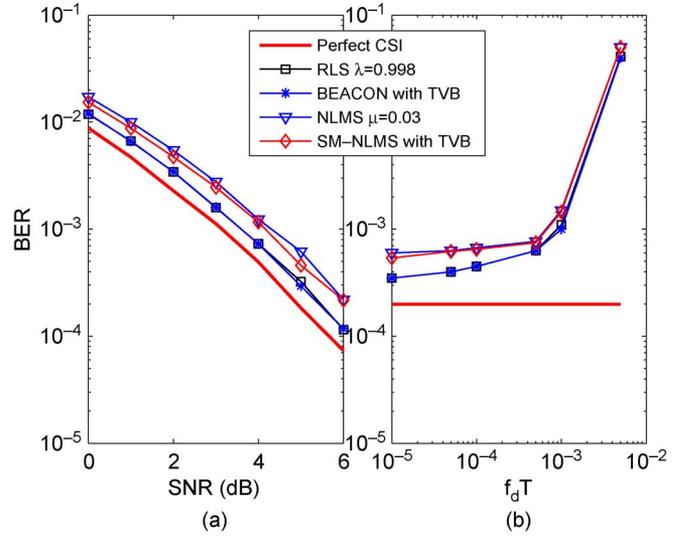


Fig. 15. (a) BER performance versus SNR for quasi-static fading channel and (b) BER performance versus  $f_d T$  (SNR = 5 dB) for Rayleigh fading channels.  $n_p = 1000$ ,  $n_t = 100$ , and  $n_d = 900$ .

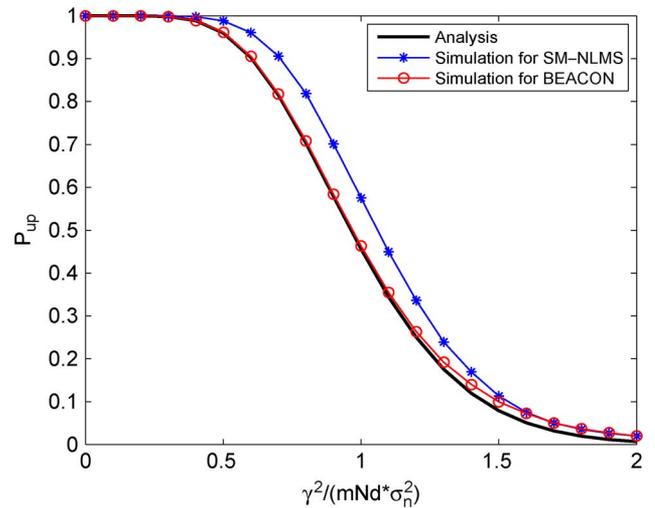


Fig. 16. Analysis of the probability of update  $P_{up}$ .

**C. Verification of the Analysis**

In this section, experiments were conducted to validate our analysis of the SM-NLMS and BEACON algorithms. The basic idea is to evaluate the formulas derived in Section V by comparing the analytical results with that obtained by computer simulations. From (70) and (71), the two variables  $M$  and  $N$  used in Section V can be obtained, i.e.,  $M = 9$ , and  $N = 10$ . First, the analysis of the probability of update is verified using (65). It can be seen from Fig. 16 that the  $P_{up}$  in simulations of the SM-NLMS and BEACON channel estimation is close to and lower bounded by the  $P_{up}$  from our analysis. The gap between the analytical curve and the simulations of two SM channel estimation is due to the approximation made in the analysis. In Section V, we assume that the channel matrix error  $\Delta \mathbf{H}$  approaches zero during the steady state. However, for the SM algorithms, it is not accurate because the bound set for the output estimation error would enlarge  $\Delta \mathbf{H}$ . During the steady state, the SM-NLMS channel estimation has a larger  $\Delta \mathbf{H}$  than

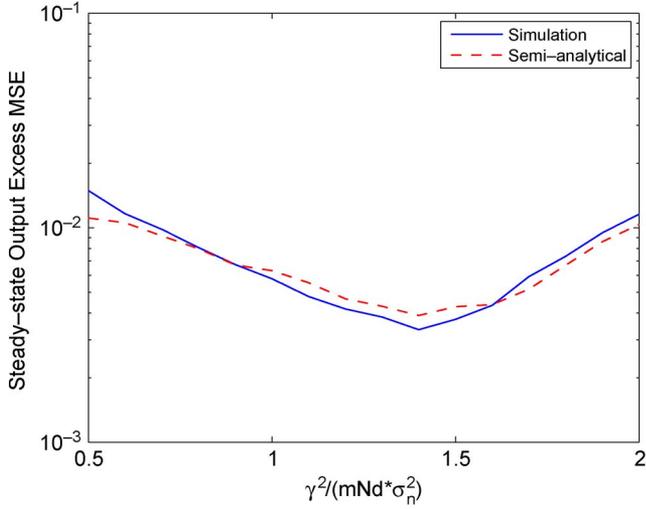


Fig. 17. Steady-state excess MSE analysis for the SM-NLMS channel estimation.

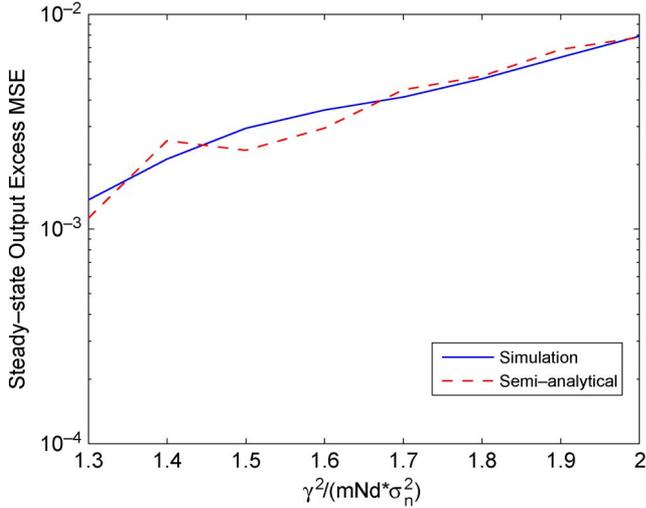


Fig. 18. Steady-state excess MSE analysis for the BEACON channel estimation.

the BEACON channel estimation, which therefore causes a larger gap between the analytical curve and the simulation. After that, we continue to verify the analysis of the steady-state output excess MSE using (55) and (58). Because it is difficult to obtain the full-analytical expressions of the conditional expected values  $X_1$ ,  $Y_1$ ,  $Z_1$ ,  $X_2$ ,  $Y_2$ , and  $Z_2$ , a semi-analytical method is used here. It means that the data from the simulations is used to calculate these conditional expected values in (55) and (58). To lower the effect of the difference between the analytical  $P_{\text{up}}$  and the simulation  $P_{\text{up}}$  of the SM-NLMS channel estimation,  $1.1\sigma_n^2$  is chosen approximately to take the place of  $\sigma_n^2$  in (65), which would produce a more accurate  $\Delta\mathbf{H}$  and  $P_{\text{up}}$  for the SM-NLMS channel estimation. Figs. 17 and 18 show the steady-state output excess MSE versus  $\gamma^2/(mN_d\sigma_n^2)$  of the two channel estimation algorithms. From the figures, it can be seen that the semi-analytical curves can match the simulation curves well. Therefore, it can be stated that our analysis is able to predict accurately the output steady-state excess MSE for different choices of bound  $\gamma$ .

## VII. CONCLUSION

Two SM channel estimation methods have been proposed based on TVB for cooperative WSNs. It has been shown that our proposed methods can achieve better or similar performance to conventional NLMS and RLS channel estimation, offering reduced computational complexity. Analyses of the steady-state MSE and computational complexity have been presented for the two channel estimation and closed-form expressions of the excess MSE, and the probability of update has been provided. Furthermore, the incorporation of the TVB function makes it robust to changes in the environment. These features are desirable for WSNs and bring about a significant reduction in energy consumption. Possible extensions to this work may include other advanced parameter estimation [24], [25] and nonlinear detection and iterative processing algorithms [26], [27].

## APPENDIX A

### DERIVATION OF THE PROPOSED BEACON CHANNEL ESTIMATION ALGORITHM

By setting the gradient of  $\mathcal{L}$  in (29) with respect to  $\mathbf{H}(n)$  equal to zero, we have

$$\frac{\partial \mathcal{L}}{\partial \mathbf{H}(n)} = 2 \sum_{l=1}^{n-1} \lambda(n)^{n-l} [\mathbf{r}(l) - \mathbf{H}(n)\mathbf{s}(l)] [-\mathbf{s}^H(l)] + 2\lambda(n) [\|\mathbf{r}(n) - \mathbf{H}(n)\mathbf{s}(n)\|] [-\mathbf{s}^H(n)] = 0. \quad (72)$$

Then, we can get

$$\mathbf{H}(n) = \left[ \sum_{l=1}^{n-1} \lambda(n)^{n-l} \mathbf{r}(l)\mathbf{s}^H(l) + \lambda(n)\mathbf{r}(n)\mathbf{s}^H(n) \right] \cdot \left[ \sum_{l=1}^{n-1} \lambda(n)^{n-l} \mathbf{s}(l)\mathbf{s}^H(l) + \lambda(n)\mathbf{s}(n)\mathbf{s}^H(n) \right]^{-1}. \quad (73)$$

Let

$$\phi(n) = \sum_{l=1}^{n-1} \lambda(n)^{n-l} \mathbf{s}(l)\mathbf{s}^H(l) + \lambda(n)\mathbf{s}(n)\mathbf{s}^H(n) \quad (74)$$

$$\mathbf{Z}(n) = \sum_{l=1}^{n-1} \lambda(n)^{n-l} \mathbf{r}(l)\mathbf{s}^H(l) + \lambda(n)\mathbf{r}(n)\mathbf{s}^H(n). \quad (75)$$

Equation (73) becomes

$$\mathbf{H}(n) = \mathbf{Z}(n)\phi^{-1}(n). \quad (76)$$

Isolating the term corresponding to  $l = n - 1$  from the rest of the summation on the right-hand side of (74), we may write

$$\phi(n) = \left[ \sum_{l=1}^{n-2} \lambda(n)^{n-l} \mathbf{s}(l)\mathbf{s}^H(l) + \lambda(n)\mathbf{s}(n-1)\mathbf{s}^H(n-1) \right] + \lambda(n)\mathbf{s}(n)\mathbf{s}^H(n). \quad (77)$$

The expression inside the brackets on the right-hand side of (77) is equal to  $\phi(n-1)$ , assuming the forgetting factor of the cost

function is close to 1. Hence, we have the following recursion for updating the value of  $\phi(n)$ :

$$\phi(n) = \phi(n-1) + \lambda(n)\mathbf{s}(n)\mathbf{s}^H(n). \quad (78)$$

Similarly, we may use (75) to derive the following recursion for updating  $\mathbf{Z}(n)$ :

$$\mathbf{Z}(n) = \mathbf{Z}(n-1) + \lambda(n)\mathbf{r}(n)\mathbf{s}^H(n). \quad (79)$$

Then, using the matrix inversion lemma [8], we obtain the following recursive equation for the inverse of  $\phi(n)$ :

$$\begin{aligned} \phi^{-1}(n) &= \phi^{-1}(n-1) \\ &- \frac{\lambda(n)\phi^{-1}(n-1)\mathbf{s}(n)\mathbf{s}^H(n)\lambda(n)\phi^{-1}(n-1)}{1 + \lambda(n)\mathbf{s}^H(n)\phi^{-1}(n-1)\mathbf{s}(n)}. \end{aligned} \quad (80)$$

For convenience of computation, let

$$\mathbf{P}(n) = \phi^{-1}(n) \quad (81)$$

$$\mathbf{k}(n) = \frac{\mathbf{s}^H(n)\mathbf{P}(n-1)}{1 + \lambda(n)\mathbf{s}^H(n)\mathbf{P}(n-1)\mathbf{s}(n)}. \quad (82)$$

Therefore, we may rewrite (76) and (80) as

$$\mathbf{H}(n) = \mathbf{Z}(n)\mathbf{P}(n) \quad (83)$$

$$\mathbf{P}(n) = \mathbf{P}(n-1) - \lambda(n)\mathbf{P}(n-1)\mathbf{s}(n)\mathbf{k}(n). \quad (84)$$

Then, we substitute (79) and (84) into (83) to obtain a recursive equation for updating the channel matrix  $\mathbf{H}(n)$ , i.e.,

$$\begin{aligned} \mathbf{H}(n) &= \mathbf{H}(n-1) - \lambda(n)\mathbf{H}(n-1)\mathbf{s}(n)\mathbf{k}(n) \\ &+ \lambda(n)\mathbf{r}(n)\mathbf{s}^H(n)\mathbf{P}(n). \end{aligned} \quad (85)$$

By rearranging (82), we can get

$$\begin{aligned} \mathbf{k}(n) &= \mathbf{s}^H(n)\mathbf{P}(n-1) - \lambda(n)\mathbf{s}^H(n)\mathbf{P}(n-1)\mathbf{s}(n)\mathbf{k}(n) \\ &= \mathbf{s}^H(n) [\mathbf{P}(n-1) - \lambda(n)\mathbf{P}(n-1)\mathbf{s}(n)\mathbf{k}(n)] \\ &= \mathbf{s}^H(n)\mathbf{P}(n). \end{aligned} \quad (86)$$

Using (86), we get the desired recursive equation for updating the channel matrix  $\mathbf{H}(n)$ , i.e.,

$$\begin{aligned} \mathbf{H}(n) &= \mathbf{H}(n-1) - \lambda(n)\mathbf{H}(n-1)\mathbf{s}(n)\mathbf{k}(n) + \lambda(n)\mathbf{r}(n)\mathbf{k}(n) \\ &= \mathbf{H}(n-1) + \lambda(n) [\mathbf{r}(n) - \mathbf{H}(n-1)\mathbf{s}(n)] \mathbf{k}(n) \\ &= \mathbf{H}(n-1) + \lambda(n)\boldsymbol{\epsilon}(n)\mathbf{k}(n) \end{aligned} \quad (87)$$

where  $\boldsymbol{\epsilon}(n) = \mathbf{r}(n) - \mathbf{H}(n-1)\mathbf{s}(n)$  denotes the prediction error vector at time instant  $n$ .

## APPENDIX B

### ANALYSIS OF THE PROPOSED SM-NLMS CHANNEL ESTIMATION ALGORITHM

From (46), the update equation of the channel estimation error is

$$\Delta\mathbf{H}(n+1) = \Delta\mathbf{H}(n) - \frac{1}{N\sigma_s^2}\mathbf{e}(n)\mathbf{s}^H(n) + \frac{\gamma}{N\sigma_s^2} \frac{\mathbf{e}(n)}{\|\mathbf{e}_0(n)\|} \mathbf{s}^H(n). \quad (88)$$

Let

$$\mathbf{A} = \Delta\mathbf{H}(n) - \frac{1}{N\sigma_s^2}\mathbf{e}(n)\mathbf{s}^H(n) \quad (89)$$

$$\mathbf{B} = \frac{\gamma}{N\sigma_s^2} \frac{\mathbf{e}(n)}{\|\mathbf{e}_0(n)\|} \mathbf{s}^H(n). \quad (90)$$

Equation (88) becomes

$$\Delta\mathbf{H}(n+1) = \mathbf{A} + \mathbf{B}. \quad (91)$$

From (43), we can get the output excess MSE at time instant  $n+1$ , i.e.,

$$\begin{aligned} J_{\text{ex}}(n+1) &= \text{tr} \left\{ E \left[ \mathbf{s}(n+1)\mathbf{s}^H(n+1) \right. \right. \\ &\quad \left. \left. \cdot \Delta\mathbf{H}^H(n+1)\Delta\mathbf{H}(n+1) \right] \right\} \\ &= \text{tr} \left\{ E \left[ \mathbf{s}(n)\mathbf{s}^H(n)\Delta\mathbf{H}^H(n+1)\Delta\mathbf{H}(n+1) \right] \right\} \\ &= \psi_1 + \psi_2 + \psi_3 + \psi_4. \end{aligned} \quad (92)$$

Then, we separately analyze each term

$$\psi_1 = \text{tr} \left\{ E \left[ \mathbf{s}(n)\mathbf{s}^H(n)\mathbf{A}^H\mathbf{A} \right] \right\} = \rho_1 + \rho_2 \quad (93)$$

$$\rho_1 = J_{\text{ex}}(n) - 2N\sigma_s^2 \frac{1}{N\sigma_s^2} J_{\text{ex}}(n) + N^2\sigma_s^4 \frac{1}{N^2\sigma_s^4} J_{\text{ex}}(n) = 0 \quad (94)$$

$$\rho_2 = N^2\sigma_s^4 M\sigma_n^2 \frac{1}{N^2\sigma_s^4} = M\sigma_n^2 \quad (95)$$

$$\begin{aligned} \psi_2 &= \text{tr} \left\{ E \left[ \mathbf{s}(n)\mathbf{s}^H(n)\mathbf{A}^H\mathbf{B} \right] \right\} \\ &= \text{tr} \left\{ E \left[ \mathbf{s}(n)\mathbf{s}^H(n)\Delta\mathbf{H}^H(n) \frac{\gamma}{N\sigma_s^2} \frac{\mathbf{e}(n)}{\|\mathbf{e}_0(n)\|} \mathbf{s}^H(n) \right] \right\} \\ &\quad - \text{tr} \left\{ E \left[ \mathbf{s}(n)\mathbf{s}^H(n) \frac{\gamma}{N^2\sigma_s^4} \mathbf{s}(n) \frac{\mathbf{e}^H(n)\mathbf{e}(n)}{\|\mathbf{e}_0(n)\|} \mathbf{s}^H(n) \right] \right\} \\ &= \gamma \text{tr} \left\{ E \left[ \mathbf{s}^H(n)\Delta\mathbf{H}^H(n) \frac{\mathbf{e}(n)}{\|\mathbf{e}_0(n)\|} \right] \right\} - \gamma E \left[ \frac{\|\mathbf{e}(n)\|^2}{\|\mathbf{e}_0(n)\|} \right] \\ &= \gamma \text{tr} \left\{ E \left[ \mathbf{s}^H(n)\Delta\mathbf{H}^H(n) \frac{\mathbf{n}(n) + \Delta\mathbf{H}(n)\mathbf{s}(n)}{\|\mathbf{e}_0(n)\|} \right] \right\} \\ &\quad - \gamma E \left[ \frac{\|\mathbf{e}(n)\|^2}{\|\mathbf{e}_0(n)\|} \right] \\ &= \gamma \text{tr} \left\{ E \left[ \mathbf{s}^H(n)\Delta\mathbf{H}^H(n) \frac{\Delta\mathbf{H}(n)\mathbf{s}(n)}{\|\mathbf{e}_0(n)\|} \right] \right\} - \gamma E \left[ \frac{\|\mathbf{e}(n)\|^2}{\|\mathbf{e}_0(n)\|} \right] \\ &= \gamma E \left[ \frac{1}{\|\mathbf{e}_0(n)\|} \right] J_{\text{ex}}(n) - \gamma E \left[ \frac{\|\mathbf{e}(n)\|^2}{\|\mathbf{e}_0(n)\|} \right] \end{aligned} \quad (96)$$

$$\psi_3 = \text{tr} \left\{ E \left[ \mathbf{s}(n)\mathbf{s}^H(n)\mathbf{B}^H\mathbf{A} \right] \right\} = \psi_2 \quad (97)$$

$$\begin{aligned} \psi_4 &= \text{tr} \left\{ E \left[ \mathbf{s}(n)\mathbf{s}^H(n)\mathbf{B}^H\mathbf{B} \right] \right\} \\ &= \text{tr} \left\{ E \left[ \mathbf{s}(n)\mathbf{s}^H(n) \frac{\gamma^2}{N^2\sigma_s^4} \mathbf{s}(n) \frac{\mathbf{e}^H(n)\mathbf{e}(n)}{\|\mathbf{e}_0(n)\|^2} \mathbf{s}^H(n) \right] \right\} \\ &= \gamma^2 E \left[ \frac{\|\mathbf{e}(n)\|^2}{\|\mathbf{e}_0(n)\|^2} \right]. \end{aligned} \quad (98)$$

Finally, we can obtain the update equation of the output excess MSE, i.e.,

$$\begin{aligned} J_{\text{ex}}(n+1) &= M\sigma_n^2 + 2\gamma E \left[ \frac{1}{\|\mathbf{e}_0(n)\|} \right] J_{\text{ex}}(n) \\ &\quad - 2\gamma E \left[ \frac{\|\mathbf{e}(n)\|^2}{\|\mathbf{e}_0(n)\|} \right] + \gamma^2 E \left[ \frac{\|\mathbf{e}(n)\|^2}{\|\mathbf{e}_0(n)\|^2} \right]. \end{aligned} \quad (99)$$

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