Low-Complexity Variable Step-Size Mechanisms for Stochastic Gradient Algorithms in Minimum Variance CDMA Receivers

Rodrigo Caiado de Lamare and Raimundo Sampaio-Neto

***Abstract*—In this paper, the performance of blind adaptive receivers for direct sequence code division multiple access (DS-CDMA) systems that employ stochastic gradient (SG) algorithms with variable step size mechanisms is investigated. Two low complexity variable step size mechanisms are proposed for estimating the parameters of linear CDMA receivers that operate with SG algorithms. For multipath channels the novel adaptation mechanisms are also incorporated in the channel estimation algorithms, whereas for the single-path case the novel techniques are restricted to the linear receiver parameter vector estimation. Analytical expressions for the excess mean squared error (MSE) are derived and a convergence analysis of the proposed adaptation techniques is carried out for both frequency selective and flat scenarios. Finally, numerical experiments are presented for nonstationary environments, showing that the new mechanisms achieve superior performance to previously reported methods at a reduced complexity.**

***Index Terms*—Adaptive receivers, blind multiuser detection, blind variable step-size(BVSS) mechanisms, direct sequence code division multiple access (DS-CDMA), interference suppression.**

I. INTRODUCTION

CODE division multiple access (CDMA) implemented with direct sequence (DS) spread-spectrum signalling is amongst the most promising multiplexing technologies for current and future communication systems. Such services include third-generation cellular telephony, indoor wireless networks, terrestrial and satellite communication systems. The advantages of CDMA include good performance in multipath channels, flexibility in the allocation of channels, increased capacity in bursty and fading environments, and the ability to share bandwidth with narrowband communication systems without deterioration of either's systems performance [1], [9].

Demodulating a desired user in a DS-CDMA network requires processing the received signal in order to mitigate different types of interference, namely, narrowband interference (NBI), multiaccess interference (MAI), intersymbol interference (ISI), and the noise at the receiver. The major source of interference in most CDMA systems is MAI, which arises due to the fact that users communicate through the same physical channel with nonorthogonal signals. The conventional (single-user) receiver that employs a filter matched to the signature sequence does not suppress MAI and is very sensitive to differences in power between the received signals (near-far problem). Multiuser detection has been proposed as a means to suppress MAI, increasing the capacity and the performance of CDMA systems [1], [9]. The optimal multiuser detector of Verdu [3] suffers from exponential complexity and requires the knowledge of timing, amplitude and signature sequences. This fact has motivated the development of various suboptimal strategies: the linear [4] and decision feedback [5] receivers, the successive interference canceller [6] and the multistage detector [7]. In this context, adaptive signal processing methods are suitable to CDMA systems because they can track the highly dynamic conditions often encountered in such systems due to the mobility of mobile terminals, the random nature of the channel access and can also alleviate the computational complexity required for parameter estimation.

The linear minimum mean squared error (MMSE) receiver [8] implemented with an adaptive filter is one of the most prominent schemes for use in the downlink because it only requires the timing of the desired user and a training sequence. A blind adaptive linear receiver has been developed in [9] and trades off the need for a training sequence in favor of the knowledge of the desired user's spreading code. In [9], Honig *et al.* have shown that the minimum variance (MV) criterion leads to a solution identical to that obtained from the minimization of the mean squared error (MSE). A disadvantage of the original MV detector of [9] is that it suffers from the problem of signature mismatch and thus has to be modified for multipath environments. A class of detectors with good performance and based on subspace tracking with channel estimation were reported in [10], [11], but they require singular value decompositions (SVD) of large matrices, which leads to a heavy computational load. A solution to the problem of signature mismatch of [9] was attempted in [12], [13] by forcing the receiver response to delayed copies of the desired signal to zero. More successful constrained optimization solutions that combine multipath components and suppress MAI were presented in [14], [15]. Recently, blind adaptive stochastic gradient (SG) and recursive least squares (RLS) algorithms based upon the linearly constrained minimum variance (CMV) criterion of [15] and that can operate in frequency selective channels were introduced by Xu and Tsatsanis in [16]. Later improvements to the method of

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[15] and [16] include those in [17] where the covariance matrix is raised to a finite power in order to improve the channel estimation and consequently the receiver performance. Therefore, blind adaptive linear receivers have become an interesting alternative for situations where a CDMA receiver loses track of the desired user and a training sequence is not available.

A question that arises when designing an adaptive receiver for a DS-CDMA system is: What kind of algorithm should be used? In the literature of adaptive algorithms [18], [19], SG algorithms (e.g., LMS) represent simple and low complexity (linear with the number of adaptive elements) solutions that are subject to slow convergence depending on the eigenvalue spread of the covariance matrix of the received vector. Conversely, RLS techniques have fast convergence, are independent from the eigenvalue spread of the covariance matrix of the received vector but require a quadratic complexity with the number of parameters. Despite the faster response of RLS algorithms, however, it is preferable to implement adaptive receivers with SG algorithms due to complexity and cost issues, and for this reason the improvement of blind SG techniques is an important research and development topic. In this regard, the works in [9] and [16] employ standard SG algorithms with fixed step size (FSS) that are not efficient with respect to convergence and steady-state performance. Indeed, the performance of adaptive receivers for CDMA that use SG algorithms is strongly dependent on the choice of the step size [18], [19]. In wireless networks characterized by nonstationary environments, users frequently enter and exit the system, making it very difficult for the receiver to compute a predetermined step size. This suggests the deployment of mechanisms to automatically adjust the step size of an SG algorithm in order to ensure good tracking of the interference and the channel. Previous works have shown significant gains in performance due to the use of averaging methods (AV) [20], [21] or adaptive step size (ASS) [23], [24] mechanisms. The works in [20] and [21] have borrowed the idea of averaging originally developed by Polyak [22] and applied it to CDMA receivers with the MV criterion. The ASS algorithms in [23], [24] can be considered MV extensions of the papers [25]–[27], [35] where one LMS algorithm adapts the parameter vector and another LMS recursion adapts the step size. All these methods require an additional number of operations (i.e., additions and multiplications) proportional to the processing gain N and to the number of multipath components L_p. Furthermore, the techniques so far reported do not introduce any improvement for the channel estimation procedure of [16] that also employs an SG recursion.

This paper proposes two novel variable step size mechanisms for blind MV CDMA receivers in multipath channels that are used for MAI, ISI suppression, and are also incorporated in the channel estimation algorithm. The origins of these mechanisms can be traced back to the works of [28] and [29] where low complexity adaptive step size mechanisms were developed for LMS algorithms, that utilize the MMSE criterion. Other representative variable step size approaches for supervised LMS algorithms include the works in [30] and [31], which are significantly outperformed by those in [28] and [29], and the techniques in [32] and [33], that obtain marginal gains at the expense of more computational complexity. In contrast to [28] and [29], our mechanisms are designed for MV algorithms and for the complex case. The additional number of operations of the proposed techniques does not depend on the processing gain N and the number of paths of the channel L_p. A convergence analysis of the proposed adaptation techniques is carried out for both frequency selective and flat scenarios, and analytical results are derived for the computation of the excess MSE. In addition, simulation experiments are presented for stationary and nonstationary environments, showing that the new mechanisms are superior to previously reported methods and exhibit a reduced complexity.

The paper is structured as follows. Section II describes the DS-CDMA system model. The linear MV receiver design is presented in Section III for both flat and frequency selective channels. Section IV describes the SG adaptive algorithms that estimate the receiver and the channel parameters. Section V is dedicated to the proposed blind adaptive step size mechanisms. A convergence analysis of the resulting algorithms is developed in Section VI. Section VII presents and discusses the simulation results and Section VIII gives the conclusions of this paper.

II. DS-CDMA System Model

Let us consider the downlink of a synchronous DS-CDMA system with K users, N chips per symbol and L_p propagation paths. The signal broadcasted by the base station intended for user k has a baseband representation given by

$$x_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k(i) s_k(t - iT) \quad (1)$$

where $b_k(i) \in \{\pm 1\}$ denotes the ith symbol for user k, the real valued spreading waveform and the amplitude associated with user k are $s_k(t)$ and A_k, respectively. The spreading waveforms are expressed by $s_k(t) = \sum_{i=1}^{N} a_k(i)\phi(t - iT_c)$, where $a_k(i) \in \{\pm 1/\sqrt{N}\}$, $\phi(t)$ is the chip waveform, T_c is the chip duration and $N = T/T_c$ is the processing gain. Assuming that the receiver is synchronized with the main path, the coherently demodulated composite received signal is

$$r(t) = \sum_{k=1}^{K} \sum_{l=0}^{L_p-1} h_l(t) x_k(t - \tau_l) + n(t) \quad (2)$$

where $h_l(t)$ and τ_l are, respectively, the channel coefficient and the delay associated with the lth path. Assuming that $\tau_{k,l} = lT_c$, the channel is constant during each symbol interval and the spreading codes are repeated from symbol to symbol, the received signal $r(t)$ after filtering by a chip-pulse matched filter and sampled at chip rate yields the $M = N + L_p - 1$ dimensional received vector

$$\mathbf{r}(i) = \mathbf{H}(i) \sum_{k=1}^{K} A_k \mathbf{S}_k \mathbf{b}_k(i) + \mathbf{n}(i) \quad (3)$$

where $\mathbf{n}(i) = [n_1(i) \ldots n_M(i)]^T$ is the complex Gaussian noise vector with $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma^2 \mathbf{I}$, where $(.)^T$ and $(.)^H$ denotes transpose and Hermitian transpose, respectively, and $E[.]$ stands for expected value, the kth user symbol vector is given by $\mathbf{b}_k(i) = [b_k(i) \ldots b_k(i - L_s + 1)]^T$, where L_s is the ISI span, the amplitude associated with user k is A_k and the $(L_s \times N) \times L_s$ matrix $\mathbf{S}_k$ with nonoverlapping shifted versions of the signature of user k is

$$\mathbf{S}_k = \begin{bmatrix} \mathbf{s}_k & 0 & \dots & 0 \\ 0 & \mathbf{s}_k & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \mathbf{s}_k \end{bmatrix} \quad (4)$$

where the signature sequence for the k-th user is $\mathbf{s}_k = [a_k(1) \dots a_k(N)]^T$ and the $M \times (L_s \times N)$ channel matrix $\mathbf{H}(i)$ is

$$\mathbf{H}(i) = \begin{bmatrix} h_0(i) & \dots & h_{L_p-1}(i) & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & h_0(i) & \dots & h_{L_p-1}(i) \end{bmatrix} \quad (5)$$

where $h_l(i) = h_l(iT_c)$ for $l = 0, \dots, L_p - 1$. The MAI arises from the nonorthogonality between the received signature sequences, whereas the ISI span L_s depends on the length of the channel response, which is related to the length of the chip sequence. For $L_p = 1, L_s = 1$ (no ISI), for $1 < L_p \leq N, L_s = 2$, for $N < L_p \leq 2N$, $L_s = 3$ and so on.

III. Minimum Variance Linear CDMA Receivers

The linear receiver design is equivalent to determining an FIR filter $\mathbf{w}_k(i)$ with M coefficients that provide an estimate of the desired symbol, as illustrated in Fig. 1 and given by

$$\hat{b}_k(i) = sgn\left(\Re\left[\mathbf{w}_k^H(i)\mathbf{r}(i)\right]\right) \quad (6)$$

where $\Re(.)$ selects the real part, $sgn(.)$ is the signum function and the receiver parameter vector $\mathbf{w}_k$ is optimized according to the MV cost function.

A. Single-Path MV Receivers

Let us consider the received vector $\mathbf{r}(i)$ for a flat channel $(L_p = 1)$ that contains $M = N$ samples, define the parameter vector $\mathbf{w}_k = \mathbf{s}_k + \mathbf{c}_k$, where $\mathbf{c}_k$ is constrained to be orthogonal to $\mathbf{s}_k$. The design of a parameter vector $\mathbf{w}_k(i)$ using the MV criterion corresponds to the optimization of the following cost function

$$J_{\mathrm{MV}} = E\left[\left|\mathbf{w}_k^H \mathbf{r}(i)\right|^2\right] = \mathbf{w}_k^H \mathbf{R} \mathbf{w}_k \quad (7)$$

subject to the constraint given by

$$\mathbf{w}_k^H \mathbf{s}_k = 1 \quad (8)$$

where $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ and $\mathbf{s}_k^H \mathbf{s}_k = 1$. Using the method of Lagrange multipliers, as in [9], the solution can be obtained by setting the gradient terms of $J'_{\mathrm{MV}} = \mathbf{w}_k^H \mathbf{R} \mathbf{w}_k + \lambda(\mathbf{w}_k^H \mathbf{s}_k - 1)$ with respect to $\mathbf{w}_k$ equal to zero

$$\mathbf{w}_k = \arg\min_{\mathbf{w}_k} J_{\mathrm{MV}} = \frac{1}{\mathbf{s}_k^H \mathbf{R}^{-1} \mathbf{s}_k} \mathbf{R}^{-1} \mathbf{s}_k \quad (9)$$

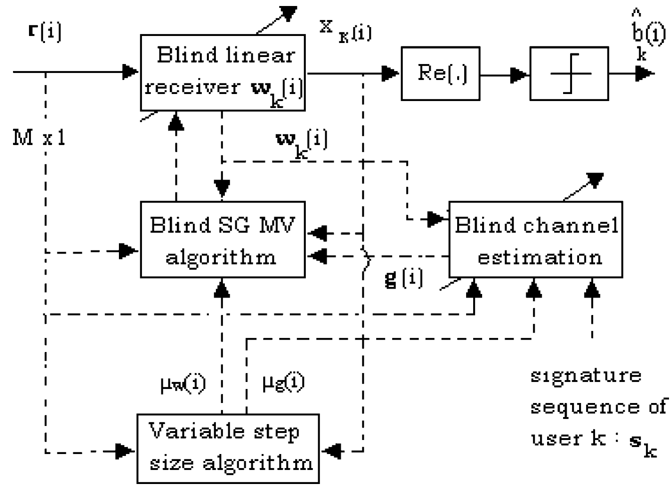

Fig. 1. Block diagram of the blind adaptive MV receiver with variable step size mechanisms.

and the resulting MV is expressed by

$$\mathrm{MV} = \mathbf{w}_k^H \mathbf{R} \mathbf{w}_k = \frac{1}{\mathbf{s}_k^H \mathbf{R}^{-1} \mathbf{s}_k}. \quad (10)$$

B. Linearly CMV Receivers

Consider the received vector $\mathbf{r}(i)$ that contains $M = N + L_p - 1$ samples, the $M \times L_p$ constraint matrix $\mathbf{C}_k$ that contains one-chip shifted versions of the signature sequence for user k and the $L_p \times 1$ constraint vector $\mathbf{g}$

$$\mathbf{C}_k = \begin{bmatrix} a_k(1) & & \mathbf{0} \\ \vdots & \ddots & a_k(1) \\ a_k(N) & & \vdots \\ \mathbf{0} & \ddots & a_k(N) \end{bmatrix}, \quad \mathbf{g}(i) = \begin{bmatrix} g_0(i) \\ \vdots \\ g_{L_p-1}(i) \end{bmatrix}. \quad (11)$$

The design of a parameter vector $\mathbf{w}_k(i)$ with M elements based on the MV criterion corresponds to the optimization of the MV cost function

$$J_{\mathrm{MV}} = \mathbf{w}_k^H \mathbf{R} \mathbf{w}_k \quad (12)$$

subject to the constraints given by

$$\mathbf{C}_k^H \mathbf{w}_k = \mathbf{g} \quad (13)$$

where $\mathbf{g}$ is the constraint vector to be determined. Using the method of Lagrange multipliers, the receiver solution [16] is

$$\mathbf{w}_k = \mathbf{R}^{-1} \mathbf{C}_k \left(\mathbf{C}_k^H \mathbf{R}^{-1} \mathbf{C}_k\right)^{-1} \mathbf{g} \quad (14)$$

where $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ and the resulting MV is expressed by

$$\mathrm{MV} = \mathbf{w}_k^H \mathbf{R} \mathbf{w}_k = \mathbf{g}^H \left(\mathbf{C}_k^H \mathbf{R}^{-1} \mathbf{C}_k\right)^{-1} \mathbf{g}. \quad (15)$$

According to (15) MV depends on $\mathbf{g}$. The optimization of the constraint vector $\mathbf{g}$ proposed in [15] and [16] maximizes (15) as described by

$$\mathbf{g} = \arg \max_{\|\mathbf{g}\|=1} \mathbf{g}^H \left(\mathbf{C}_k^H \mathbf{R}^{-1} \mathbf{C}_k\right)^{-1} \mathbf{g}. \quad (16)$$

The solution $\mathbf{g}$ is the eigenvector corresponding to the minimum eigenvalue of $\mathbf{C}_k^H \mathbf{R}^{-1} \mathbf{C}_k$ and it was shown in [15]and [16] to provide a channel estimate.

IV. Blind Adaptive SG MV Algorithms

We describe blind adaptive SG MV for estimating the parameters of the linear receiver, as depicted in Fig. 1, for both single-path and multipath scenarios.

A. Single-Path Blind Adaptive SG MV Algorithm

An SG blind algorithm, similar to the one of Honig *et al.* [9], optimizes the Lagrangian cost function described by

$$J_{\mathrm{MV}}^s = \mathbf{w}_k^H(i)\mathbf{r}(i)\mathbf{r}(i)\mathbf{w}_k(i) + \lambda\left(\mathbf{w}_k^H(i)\mathbf{s}_k - 1\right) \quad (17)$$

where λ is a scalar Lagrange multiplier and $\mathbf{w}_k(i) = \mathbf{s}_k + \mathbf{c}_k(i)$. Taking the gradient of (17) with respect to $\mathbf{c}_k(i)$ and using the orthogonal decomposition of [9] yields

$$\mathbf{c}_k(i+1) = \mathbf{c}_k(i) - \mu_w z_k^*(i)\left(\mathbf{r}(i) - z_{k,MF}(i)\mathbf{s}_k\right) \quad (18)$$

where $z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i)$ and $z_{k,MF}(i) = \mathbf{s}_k^H\mathbf{r}(i)$. A normalized version of this recursion that facilitates the setting of the convergence factor for a wide range of loads, can be derived by adding $\mathbf{s}_k$ to both sides of (18) and substituting it into the cost function, differentiating it with respect to the step size μ_w, setting the terms to zero and solving the resulting equations, as shown in Appendix I. Hence, we have $\mu_w = (\mu_0/\mathbf{r}^H(i)(\mathbf{I} - \mathbf{s}_k\mathbf{s}_k^H)\mathbf{r}(i))$ where μ_0 is the chosen fixed convergence factor. In terms of computational complexity the minimum variance SG algorithm of (18) requires $5M$ additions and $4M+1$ multiplications for the estimation of the receiver parameters.

B. Multipath Blind Adaptive SG CMV Algorithm

The SG algorithm of Tsatsatnis and Xu [16] optimizes the Lagrangian cost function described by

$$J_{\mathrm{MV}} = \mathbf{w}_k^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i) + \boldsymbol{\lambda}^H\left(\mathbf{C}_k^H\mathbf{w}_k(i) - \mathbf{g}(i)\right) + \left(\mathbf{C}_k^H\mathbf{w}_k(i) - \mathbf{g}(i)\right)^H\boldsymbol{\lambda} + \rho\left(\mathbf{g}^H(i)\mathbf{g}(i) - 1\right) \quad (19)$$

where $\boldsymbol{\lambda}$ is a vector of Lagrange multipliers and ρ is a scalar Lagrange multiplier. An SG solution to (19) can be obtained by taking the gradient terms with respect to $\mathbf{w}_k(i)$ and $\mathbf{g}(i)$ as described by [16] which yield the recursions for the blind estimation of the parameters of the receiver and the channel

$$\mathbf{w}_k(i+1) = \boldsymbol{\Pi}_k\left(\mathbf{w}_k(i) - \mu_w\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i)\right) + \mathbf{C}_k\left(\mathbf{C}_k^H\mathbf{C}_k\right)^{-1}\mathbf{g}(i). \quad (20)$$

If $a_2^2 - 4a_1a_3 \geq 0$ (else do not update)

$$\mathbf{g}(i+1) = \mu_g\rho(i)\mathbf{g}(i) + \mathbf{g}(i) - \frac{\mu_g}{\mu_w}\left(\mathbf{C}_k^H\mathbf{C}_k\right)^{-1} \times \left(\mathbf{C}_k^H\left(\mathbf{w}_k(i) - \mu_w\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i)\right) - \mathbf{g}(i)\right) \quad (21)$$

where μ_g is the step size of the channel estimation algorithm,

$$\begin{aligned}
\boldsymbol{\Pi}_k &= \mathbf{I} - \mathbf{C}_k(\mathbf{C}_k^H\mathbf{C}_k)^{-1}\mathbf{C}_k^H \\
a_1 &= \mu_g^2\|\mathbf{g}(i)\|^2 \\
a_2 &= \mu_g(\mathbf{g}^H(i)\mathbf{x}(i) + \mathbf{x}^H(i)\mathbf{g}(i)) \\
a_3 &= \mathbf{x}^H(i)\mathbf{x}(i) - 1 \\
\mathbf{x}(i) &= \mathbf{g}(i) - (\mu_g/\mu_w)(\mathbf{C}_k^H\mathbf{C}_k)^{-1}(\mathbf{C}_k^H(\mathbf{w}_k(i) \\
&\quad - \mu_w\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i)) - \mathbf{g}(i))
\end{aligned}$$

and

$$\rho(i) = (1/2a_1)(-a_2 - \sqrt{a_2^2 - 4a_1a_3}).$$

We remark that the channel estimation algorithm of (21) is convenient for a theoretical analysis such as the one that is carried out in Section V-B1. An alternative adaptive channel estimator, that is slightly less complex, can be obtained by abolishing ρ in (19) and normalizing $\mathbf{g}$ at each iteration

$$\mathbf{g}(i+1) = \mathbf{g}(i) - \frac{\mu_g}{\mu_w}\left(\mathbf{I} - \frac{\mathbf{g}(i)\mathbf{g}^H(i)}{\mathbf{g}^H(i)\mathbf{g}(i)}\right)\left(\mathbf{C}_k^H\mathbf{C}_k\right)^{-1} \times \left(\mathbf{C}_k^H\left(\mathbf{w}_k(i) - \mu_w\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i)\right) - \mathbf{g}(i)\right). \quad (22)$$

Note that the algorithms in (20) and (21) are used for the convergence analysis and for the analytical experiments. For the remaining experiments such as SINR and BER performance evaluations involving different loads, a normalized version of the channel estimation algorithm, given by (22), is preferred. A normalized version of the recursion that estimates $\mathbf{w}_k$, can be also derived by substituting (20) into the cost function, differentiating them with respect to μ_w, setting them to zero and solving the resulting equations, as shown in Appendix II. Hence, we have $\mu_w = (\mu_0/\mathbf{r}^H(i)\boldsymbol{\Pi}_k\mathbf{r}(i))$ where μ_0 is the chosen fixed convergence factor. The normalized algorithm facilitates the setting of the convergence factor for a wide range of loads.

In terms of computational complexity, the normalized constrained minimum variance (NCMV) SG algorithm requires $2M^2 + (L_p+4)M$ additions and $2M^2 + (L_p+3)M + 2$ multiplications for the estimation the parameters of the receiver, while for channel estimation using (22) it requires $L_p^3 + 3L_p^2 + (M+3)L_p$ additions and $L_p^3 + 3L_p^2 + (M+3)L_p + 1$ multiplications, where $M = N + L_p - 1$.

V. Blind Variable Step Size (BVSS) Mechanisms

This section describes the proposed low complexity BVSS mechanisms for CDMA receivers that adjust the step size μ_w of the update equation of the receiver and the step size μ_g of the algorithm that estimates the channel, as shown in Fig. 1. A convergence analysis of the mechanisms is carried out and approximate expressions relating the mean convergence factor

$E[\mu_{w|g}]$, the mean square convergence factor $E[\mu_{w|g}^2]$ and the minimum variance are derived. It is worth noting that for both mechanisms, μ_w or μ_g ($\mu_{w|g}$) is truncated between $\{\mu_{max_{w|g}}, \mu_{\min_{w|g}}\}$. In addition, the computational complexity of the novel mechanisms is presented in terms of additions and multiplications and compared to existent ones.

A. BVSS Mechanism

The first proposed BVSS mechanism employs the instantaneous output energy, is denoted BVSS and uses the update rule

$$\mu_{w|g}(i+1) = \alpha\mu_{w|g}(i) + \gamma\left|\mathbf{w}_k^H(i)\mathbf{r}(i)\right|^2 \tag{23}$$

where $0 < \alpha < 1$, $\gamma > 0$ and $\mathbf{w}_k$ is the parameter vector of the receiver. The motivation for the BVSS is that for large output energies the code-aided algorithm will employ larger step sizes whereas small output energies (that are also associated with satisfactory MAI suppression) will result in a decrease of the step size, yielding smaller misadjustment. Furthermore, it is worth pointing out that other rules have been experimented and the BVSS is a result of several attempts to devise a simple and yet effective mechanism. Indeed, the mechanism is simple to implement and a detailed analysis of the algorithm is possible under a few assumptions commonly made in the literature. The additional computational complexity required for the BVSS is only four operations. Another possibility for (23) would be to consider energy preserving coefficients such as α and $1-\alpha$. In order to control the input's energy and to scale it at different levels, we used the independent variable γ. From a practitioner's point of view, however, it might be easier to design the memory of the recursion by the choice of one parameter (α) and then scale the overall signal by a decoupled second parameter (γ), which can be performed as follows: $\gamma(i) = (1-\alpha)\gamma(i-1)$. The same proposed BVSS rule is applied to the SG channel estimation procedure introduced in [16] and described here in Section III. For the sake of simplicity we will drop the index g and proceed with the analysis for the μ_w even though it is still valid for the adaptive channel estimator.

Assumption 1: Let us consider that for the algorithms in (18) and (23)

$$\begin{aligned}E[\mu_w z_k(i)(\mathbf{r}(i) - z_{k,MF}(i)\mathbf{s}_k)] \\ = E[\mu_w]E[z_k(i)(\mathbf{r}(i) - z_{k,MF}(i)\mathbf{s}_k)]\end{aligned}$$

and

$$E[\mu_w\mathbf{r}(i)\mathbf{r}^H(i)]\mathbf{w}_k = E[\mu_w]\mathbf{R}\mathbf{w}_k$$

respectively.

This assumption holds if μ_w is a constant and we claim that it is approximately true if γ is small and also because α should be close to one (as will be shown in the simulations), because μ_w will vary slowly around its mean value. By writing

$$\begin{aligned}&E\left[\mu_w z_k(i)\left(\mathbf{r}(i) - z_{k,MF}(i)\mathbf{s}_k\right)\right] \\ &\quad = E[\mu_w]E\left[z_k(i)\left(\mathbf{r}(i) - z_{k,MF}(i)\mathbf{s}_k\right)\right] \\ &\quad\quad + E\left[\left(\mu_w - E[\mu_w]\right)z_k(i)\left(\mathbf{r}(i) - z_{k,MF}(i)\mathbf{s}_k\right)\right]\end{aligned} \tag{24}$$

and

$$\begin{aligned}E\left[\mu_w\mathbf{r}(i)\mathbf{r}^H(i)\right]\mathbf{w}_k = E[\mu_w]\mathbf{R}\mathbf{w}_k \\ + E\left[\left(\mu_w - E\left[\mu_w\mathbf{r}(i)\mathbf{r}^H(i)\right]\right]\mathbf{w}_k\end{aligned} \tag{25}$$

we note that for γ sufficiently small, the second term on the right-hand side (RHS) of the (24) and (25) will be small compared to the first one. *Assumption 1* help us to proceed with the analysis. We point out that this approach can be also used for analyzing the channel estimator.

Let us also define the first-($E[\mu_w]$) and second-order ($E[\mu_w^2]$) statistics of the proposed BVSS mechanism

$$E\left[\mu_w(i+1)\right] = \alpha E\left[\mu_w(i)\right] + \gamma E\left[\left|\mathbf{w}_k^H(i)\mathbf{r}(i)\right|^2\right] \tag{26}$$

where $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$. By computing the square of μ_w, we obtain $\mu_w^2(i+1) = \alpha^2\mu_w^2(i) + 2\alpha\gamma\mu_w(i)\mathbf{w}_k^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i) + \gamma^2|\mathbf{w}_k^H(i)\mathbf{r}(i)|^4$. Since γ^2 is small, the last term of the previous equation is negligible as compared to the other terms, thus, with the help of *Assumption 1* we assume that the expected value of $E[\mu_w^2(i+1)]$ is approximately

$$\begin{aligned}E\left[\mu_w^2(i+1)\right] \approx \alpha^2 E\left[\mu_w^2(i)\right] \\ + 2\alpha\gamma E\left[\mu_w(i)\right]E\left[\left|\mathbf{w}_k^H(i)\mathbf{r}(i)\right|^2\right].\end{aligned} \tag{27}$$

If we consider the steady-state values of $E[\mu_w(i+1)]$ and $E[\mu_w^2(i+1)]$ by making $\lim_{i\to\infty} E[\mu_w(i+1)] = \lim_{i\to\infty} E[\mu_w(i)] = E[\mu_w(\infty)]$ and $\lim_{i\to\infty} E[\mu_w^2(i+1)] = \lim_{i\to\infty} E[\mu_w^2(i)] = E[\mu_w^2(\infty)]$, and using $\lim_{i\to\infty} E[|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2] = (\xi_{\min} + \xi_{\mathrm{ex}}(\infty))$ [9] we have the following:

$$E\left[\mu_w(\infty)\right] \approx \frac{\gamma\left(\xi_{\min} + \xi_{\mathrm{ex}}(\infty)\right)}{1-\alpha} \tag{28}$$

$$E\left[\mu_w^2(\infty)\right] \approx \frac{2\alpha\gamma^2\left(\xi_{\min} + \xi_{\mathrm{ex}}(\infty)\right)^2}{(1-\alpha)^2(1+\alpha)} \tag{29}$$

where the minimum variance provided by the optimum solution is given by $\xi_{\min} = (1/\mathbf{s}_k^H\mathbf{R}^{-1}\mathbf{s}_k)$ for the single-path and $\xi_{\min} = \mathbf{g}^H(\mathbf{C}_k^H\mathbf{R}^{-1}\mathbf{C}_k)^{-1}\mathbf{g}$ for the multipath case, and ξ_{ex} is the excess MSE. At this point we reached the expressions in (28) and (29) that still have the inconvenient term ξ_{ex} on the right. To further simplify those expressions, let us consider another assumption.

Assumption 2: Let us consider that for (28) and (29), $(\xi_{\min} + \xi_{\mathrm{ex}}(\infty)) \approx \xi_{\min}$ and $(\xi_{\min} + \xi_{\mathrm{ex}}(\infty))^2 \approx \xi_{\min}^2$, respectively.

This assumption holds if $\xi_{\min} \gg \xi_{\mathrm{ex}}(\infty)$ and we claim that it is approximately true when the SG adaptive algorithm is close to optimum solution and $\xi_{\mathrm{ex}}(\infty)$ is a small fraction of $\xi_{\min}$.

By using *Assumption 2* we have the following:

$$E\left[\mu_w(\infty)\right] \approx \frac{\gamma\xi_{\min}}{1-\alpha} \tag{30}$$

$$E\left[\mu_w^2(\infty)\right] \approx \frac{2\alpha\gamma^2\xi_{\min}^2}{(1-\alpha)^2(1+\alpha)}. \tag{31}$$

Note that (30) and (31) will be used for the computational of the excess MSE of the algorithms. It is worth pointing out that (30) and (31) can be further simplified, even though our studies reveal that (30) and (31) have proven to be valid and useful for predicting the steady-state performance of the BVSS mechanism. We also remark that the above analysis is analogous for the BVSS mechanism μ_g when used for channel estimation.

B. Blind Recursive Variable Step Size (BRVSS) Mechanism

The second mechanism employs a time average estimate of the correlation of $|\mathbf{w}_k^H(i)\mathbf{r}(i)|$ and $|\mathbf{w}_k^H(i-1)\mathbf{r}(i-1)|$. We call it BRVSS. It employs the recursion

$$\mu_{w|g}(i+1) = \alpha\mu_{w|g}(i) + \gamma v^2(i) \tag{32}$$

where $v(i) = \beta v(i-1) + (1-\beta)|\mathbf{w}_k^H(i)\mathbf{r}(i)||\mathbf{w}_k^H(i-1)\mathbf{r}(i-1)|$ and $0 < \beta < 1$. The BRVSS is an alternative mechanism that employs an exponential weighting parameter that controls the quality of the estimation of the minimum variance. We remark that β should be slightly smaller than 1 so that it is able to adapt to the current statistics. The BRVSS mechanism is also simple to implement and a detailed analysis of the algorithm is possible under a few assumptions commonly made in the literature. The additional computational complexity required for the BRVSS is only eight operations. The BRVSS rule is also applied to the SG channel estimation procedure and we will drop the index g and proceed with the analysis for the μ_w even though it is still valid for the adaptive channel estimator.

Computing the term $\mu_w^2(i+1) = \alpha^2\mu_w^2(i) + 2\alpha\gamma\mu_w(i)v^2(i) + \gamma^2 v^4(i)$ and using the fact that γ^2 is small, the last term is negligible as compared to the others. Thus, $\mu_w^2(i+1) \approx \alpha^2\mu_w^2(i) + 2\alpha\gamma\mu_w(i)v^2(i)$. The estimate $v(i) = \beta v(i-1) + (1-\beta)|\mathbf{w}_k^H(i)\mathbf{r}(i)||\mathbf{w}_k^H(i-1)\mathbf{r}(i-1)|$ can be alternatively written as

$$v(i) = (1-\beta)\sum_{n=0}^{i-1}\beta^n \left|\mathbf{w}_k^H(i-n)\mathbf{r}(i-n)\right| \times \left|\mathbf{w}_k^H(i-n-1)\mathbf{r}(i-n-1)\right| \tag{33}$$

and

$$v^2(i) = (1-\beta)^2\sum_{n=0}^{i-1}\sum_{j=0}^{i-1}\beta^n\beta^j \left|\mathbf{w}_k^H(i-n)\mathbf{r}(i-n)\right| \times \left|\mathbf{w}_k^H(i-j)\mathbf{r}(i-j)\right| \left|\mathbf{w}_k^H(i-n-1)\mathbf{r}(i-n-1)\right| \times \left|\mathbf{w}_k^H(i-j-1)\mathbf{r}(i-j-1)\right|. \tag{34}$$

In the analysis of the BRVSS mechanism, we consider its steady-state performance. Therefore, we assume that the algorithm has converged. In this case, the samples of $|\mathbf{w}_k^H(i)\mathbf{r}(i)|$ can be assumed uncorrelated, i.e., $E[|\mathbf{w}_k^H(i-n)\mathbf{r}(i-n)||\mathbf{w}_k^H(i-j)\mathbf{r}(i-j)|] = 0 \forall n \neq j$. Taking the expectation of μ_w and μ_w^2 and using *Assumption 1* and (32), the mean and the mean-square behavior of the mechanism upon convergence are

$$E\left[\mu_w(i+1)\right] = \alpha E\left[\mu_w(i)\right] + \gamma E\left[v^2(i)\right] \tag{35}$$

$$E\left[\mu_w^2(i+1)\right] \approx \alpha^2 E\left[\mu_w^2(i)\right] + 2\alpha\gamma E\left[\mu_w(i)\right] E\left[v^2(i)\right] \tag{36}$$

where $E[v^2(i)] = (1-\beta)^2\sum_{n=0}^{i-1}\beta^{2n}E[|\mathbf{w}_k^H(i-n)\mathbf{r}(i-n)|^2]E[|\mathbf{w}_k^H(i-n-1)\mathbf{r}(i-n-1)|^2]$. If we consider the steady-state values of $E[\mu_w(i+1)]$, $E[\mu_w^2(i+1)]$ and $E[v^2(i)]$ by making $\lim_{i\to\infty} E[\mu_w(i+1)] = \lim_{i\to\infty} E[\mu_w(i)] = E[\mu_w(\infty)]$, $\lim_{i\to\infty} E[\mu_w^2(i+1)] = \lim_{i\to\infty} E[\mu_w^2(i)] = E[\mu_w^2(\infty)]$ and $\lim_{i\to\infty} E[v^2(i)] = E[v^2(\infty)]$, and using $\lim_{i\to\infty} E[\mathbf{w}^H(i)\mathbf{R}(i)\mathbf{w}(i)] = (\xi_{\min} + \xi_{\text{ex}}(\infty))$ [9] we have the following:

$$E\left[v^2(\infty)\right] = \frac{(1-\beta)\left(\xi_{\min} + \xi_{\text{ex}}(\infty)\right)^2}{(1+\beta)} \tag{37}$$

$$E\left[\mu_w(\infty)\right] \approx \frac{\gamma(1-\beta)\left(\xi_{\min} + \xi_{\text{ex}}(\infty)\right)^2}{(1-\alpha)(1+\beta)} \tag{38}$$

$$E\left[\mu_w^2(\infty)\right] \approx \frac{2\alpha\gamma^2(1-\beta)^2\left(\xi_{\min} + \xi_{\text{ex}}(\infty)\right)^4}{(1-\alpha)^2(1+\alpha)(1+\beta)^2} \tag{39}$$

where the minimum variance provided by the optimum solution is given by $\xi_{\min} = (1/\mathbf{s}_k^H\mathbf{R}^{-1}\mathbf{s}_k)$ for the single-path and $\xi_{\min} = \mathbf{g}^H(\mathbf{C}_k^H\mathbf{R}^{-1}\mathbf{C}_k)^{-1}\mathbf{g}$ for the multipath case, and ξ_{ex} is the excess MSE. To further simplify (37)–(39), we employ *Assumption 2* and extend it by using the approximation $(\xi_{\min} + \xi_{\text{ex}}(\infty))^4 \approx \xi_{\min}^4$, which is approximately true if $\xi_{\min} \gg \xi_{\text{ex}}(\infty)$. Thus, we obtain the following:

$$E\left[\mu_w(\infty)\right] \approx \frac{\gamma(1-\beta)\xi_{\min}^2}{(1-\alpha)(1+\beta)} \tag{40}$$

$$E\left[\mu_w^2(\infty)\right] \approx \frac{2\alpha\gamma^2(1-\beta)^2\xi_{\min}^4}{(1-\alpha)^2(1+\alpha)(1+\beta)^2}. \tag{41}$$

Note that (40) and (41) will be used for the computational of the excess MSE (ξ_{ex}) of the algorithms.

C. Computational Complexity

In this section, we show the computational complexity of the proposed BVSS mechanisms and the other analyzed methods. In Table I, we show the additional computational complexity of the proposed variable step size mechanisms, BVSS and BRVSS, and other recently reported methods for single-path channels:

TABLE I
ADDITIONAL COMPUTATIONAL COMPLEXITY OF VARIABLE STEP SIZE MECHANISMS FOR SINGLE-PATH CHANNELS

Mechanism	Number of operations per symbol	
	Additions	Multiplications
AV	$M+2$	$2M$
ASS	$3M+3$	$5M+4$
BVSS	1	3
BRVSS	2	6

TABLE II
ADDITIONAL COMPUTATIONAL COMPLEXITY OF VARIABLE STEP SIZE MECHANISMS FOR MULTIPATH CHANNELS

Mechanism	Number of operations per symbol	
	Additions	Multiplications
AV	$M+2$	$2M$
ASS	M^2+2M+2	M^2+2M+3
BVSS	1	3
BRVSS	2	6
BVSS (rec and chanmel)	2	5
BRVSS (rec and channel)	3	8

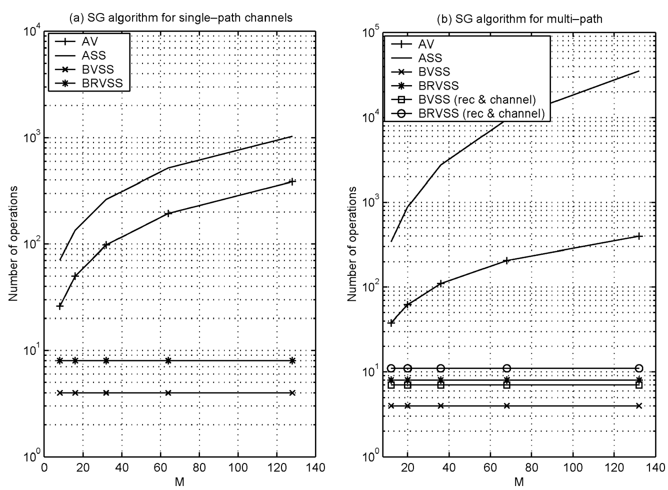

Fig. 2. Complexity in terms of arithmetic operations per symbol for the variable step size mechanisms in (a) single-path and (b) multipath environments. (a) SG algorithm for single-path channels. (b) SG algorithm for multipath.

AV [20] and ASS [23], where $M = N + L_p - 1$. The additional computational complexity of the proposed variable step size mechanisms, BVSS and BRVSS, the AV and the ASS [24] methods for multipath environments is depicted in Table II.

We remark that the algorithms presented in [23] and [24] use the same principle of adaptation although they differ in the number of operations. Specifically, the method in [23] can only operate in single path channels, whereas the technique reported in [24] can work in multipath environments.

An important advantage of the proposed adaptation rules is that they require only a few fixed number of operations while the other existing techniques have additional complexity proportional to the processing gain N and to the number of propagation paths L_p, as shown in Fig. 2. Note that we estimated the number of arithmetic operations by taking into account the number of complex additions and multiplications required by the mechanisms. A small reduction in complexity can be obtained by using real additions and multiplications in certain situations where the quantities are no longer complex and shifting operations if the quantities α, γ and β are chosen as powers of two.

VI. CONVERGENCE ANALYSIS

In this section, we investigate the convergence behavior of our mechanisms when used in MV-based algorithms in terms of trajectory of the mean receiver vector and the steady-state excess MSE. We remark that global convergence of the method has been established in [16] and here we will focus on the analysis of the mechanisms BVSS and BRVSS.

A. Single-Path Case

Here, we focus on the single-path case SG minimum variance algorithm and rely on the analysis carried out for a fixed step size in [9]. Note that we will include most steps of the analysis in [9] for completeness and then we will take into account the new BVSS mechanisms.

1) Trajectory of Mean Receiver Vector: To study the trajectory of the mean parameter vector and constraint vector for the SG algorithm $\mathbf{c}_k(i+1) = \mathbf{c}_k(i) - \mu_w(i) z_k(i)(\mathbf{r}(i) - z_{k,MF}(i)\mathbf{s}_k)$ given by (18) with variable step size $\mu_w(i)$, let us add $\mathbf{s}_k$ to both sides of the equation and define the receiver error vector $\mathbf{e}_w(i)$ at time i

$$\begin{aligned}\mathbf{w}_k(i+1) &= \mathbf{w}_k(i) - \mu_w(i)\mathbf{r}^H\mathbf{w}_k(i)\left(\mathbf{I} - \mathbf{s}_k\mathbf{s}_k^H\right)\mathbf{r}(i)\\ &= \left(\mathbf{I} - \mu_w(i)\mathbf{v}(i)\mathbf{r}^H\right)\mathbf{w}_k(i)\\ \mathbf{e}_w(i+1) &= \mathbf{w}_k(i+1) - \mathbf{w}_{\rm opt}\\ &= \left(\mathbf{I} - \mu_w(i)\mathbf{v}(i)\mathbf{r}^H(i)\right)\mathbf{e}_w(i)\\ &\quad - \mu_w\mathbf{v}(i)\mathbf{r}^H(i)\mathbf{w}_{\rm opt}\end{aligned} \tag{42}$$

where $\mathbf{v}(i) = (\mathbf{I} - \mathbf{s}_k\mathbf{s}_k^H)\mathbf{r}(i)$. By taking expectations on both sides of (42) and using Assumption 1 we have

$$E\left[\mathbf{e}_w(i+1)\right] = \left(\mathbf{I} - E\left[\mu_w(i)\right]\mathbf{R}_{vr}(i)\right)E\left[\mathbf{e}_w(i)\right] \tag{43}$$

where $\mathbf{R}_{vr}(i) = E[\mathbf{v}(i)\mathbf{r}^H(i)] = (\mathbf{I} - \mathbf{s}_k\mathbf{s}_k^H)\mathbf{R}(i)$ and $\mathbf{R}_{vr}\mathbf{w}_{\rm opt} = \mathbf{0}$. Therefore, it can be concluded that $\mathbf{w}_k$ converges to $\mathbf{w}_{\rm opt}$ and (43) is stable if and only if $\prod_{i=0}^{\infty}(\mathbf{I} - E[\mu_w(i)]\mathbf{R}_{vr}) \rightarrow 0$, which is a necessary and sufficient condition for $\lim_{i\rightarrow\infty} E[\mathbf{e}_w(i)] = \mathbf{0}$ and $E[\mathbf{w}_k(i)] \rightarrow \mathbf{w}_{\rm opt}$. For stability, a sufficient condition for (43) to hold implies that

$$0 \leq E\left[\mu_w(\infty)\right] < \min_k \frac{2}{|\lambda_k^{vr}|} \tag{44}$$

where λ_k^{vr} is the kth eigenvalue of $\mathbf{R}_{vr}$, that is not real since $\mathbf{R}_{vr}$ is not symmetric.

2) Trajectory of Excess MSE: Now let us consider an analysis of the steady-state excess MSE that follows the general steps presented in [9] for variable step size algorithms. Let us define the MSE at time $i+1$ using the fact that $\boldsymbol{e}_w(i+1) = \mathbf{w}(i+1) - \mathbf{w}_{\rm opt}$ and $\xi(i) = E[\mathbf{w}_k^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i)]$

$$\begin{aligned}\epsilon(i+1) &= E\left[\left|b_k(i) - \mathbf{w}_k^H(i)\mathbf{r}(i)\right|^2\right]\\ &= \epsilon_{\min} + \xi(i) - \xi_{\min} - E\left[\mathbf{e}_w^H(i)\right]\mathbf{s}_k\\ &\quad - \mathbf{s}_k^H E\left[\mathbf{e}_w(i)\right]\\ &= \epsilon_{\min} + \xi_{\rm ex}(i) - E\left[\mathbf{e}_w^H(i)\right]\mathbf{s}_k\\ &\quad - \mathbf{s}_k^H E\left[\mathbf{e}_w(i)\right]\end{aligned} \tag{45}$$

where

$$\xi(i) = E[\mathbf{w}_k^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i)] \\ = E[\mathbf{w}_k^H(i)\mathbf{R}(i)\mathbf{w}_k(i)] \\ \epsilon_{\min} = E[|b(i+1) - \mathbf{w}_{\mathrm{opt}}^H\mathbf{r}(i+1)|^2]$$

is the MSE with $\mathbf{w}_{\mathrm{opt}} = \xi_{\min}\mathbf{R}^{-1}\mathbf{s}_k$ where $\xi_{\min} = 1/(\mathbf{s}^k\mathbf{R}^{-1}\mathbf{s}_k)$ is the minimum variance, and $\xi_{\mathrm{ex}}(i) = \xi(i) - \xi_{\min}$ is the excess MSE due to the adaptation process at the time instant i. Since $\lim_{i\to\infty} E[\mathbf{e}_w(i)] = 0$ we have

$$\lim_{i\to\infty} \epsilon(i+1) = \epsilon_{\min} + \lim_{i\to\infty} \xi(i) \equiv \epsilon_{\min} + \xi_{\mathrm{ex}}(\infty) \quad (46)$$

where the third term in (46) is the steady-state excess MSE resulted from the adaptation process. Thus, we conclude as in [9] that the asymptotic excess MSE is equal to the asymptotic excess minimum variance plus the minimum variance $\xi_{\min}$ obtained with $\mathbf{w}_{\mathrm{opt}}$, which is used in the expressions of the BVSS and BRVSS. To analyze the trajectory of $\xi(i)$, let us rewrite it similarly to [9]

$$\xi(i) = E\left[\mathbf{w}^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}(i)\right] = tr\, E\left[\mathbf{R}_{\mathbf{w}}(i)\mathbf{R}\right] \quad (47)$$

where $\mathbf{R}_w(i) = \mathbf{w}_{\mathrm{opt}}\mathbf{w}_{\mathrm{opt}}^H + E[\mathbf{e}_w(i)\mathbf{w}_{\mathrm{opt}}^H + \mathbf{w}_{\mathrm{opt}}E[\mathbf{e}_w^H(i)] + \mathbf{R}_{e_w}(i)$ [9]. To proceed with the analysis, we must define the quantities $\mathbf{R} = \mathbf{\Phi}\mathbf{\Lambda}\mathbf{\Phi}^H$, where the columns of $\mathbf{\Phi}$ are the eigenvectors of the symmetric and positive semidefinite matrix $\mathbf{R}$ and $\mathbf{\Lambda}$ is the diagonal matrix of corresponding eigenvalues, $\mathbf{R}_e(i) = E[\mathbf{e}_w(i)\mathbf{e}_w^H(i)]$, the rotated tap error vector $\tilde{\mathbf{e}}_w(i) = \mathbf{\Phi}^H\mathbf{e}_w(i)$, the rotated signal vectors $\tilde{\mathbf{r}}(i) = \mathbf{\Phi}^H\mathbf{r}(i)$, $\tilde{\mathbf{s}}_k = \mathbf{\Phi}^H\mathbf{s}_k$ and $\mathbf{R}_{\tilde{\mathbf{e}}_w} = E[\tilde{\mathbf{e}}_w\tilde{\mathbf{e}}_w^H] = \mathbf{\Phi}^H\mathbf{R}_{\mathbf{e}_w}\mathbf{\Phi}$. Rewriting (47) in terms of the above transformed quantities we have

$$\xi(i) = tr\, E[\mathbf{\Lambda}\mathbf{\Phi}^H\mathbf{R}_w\mathbf{\Phi}] \\ = \xi_{\min} + tr\left[E\left[\tilde{\mathbf{e}}_w(i)\right]\tilde{\mathbf{s}}_k^H + \tilde{\mathbf{s}}_k E\left[\tilde{\mathbf{e}}_w^H(i)\right] + \Lambda\mathbf{R}_{\tilde{\mathbf{e}}_w}(i)\right]. \quad (48)$$

Because $\lim_{i\to\infty} E[\tilde{\mathbf{e}}(i)] = 0$, then $\xi_{\mathrm{ex}}(\infty) = \lim_{i\to\infty}\xi(i) = tr[\Lambda\mathbf{R}_{\tilde{\mathbf{e}}_w}]$. Thus, it becomes clear that to assess the evolution of ξ_{ex} it is sufficient to study $\mathbf{R}_{\tilde{\mathbf{e}}_w}$. Using the results of [9], (43), Assumption 1 and incorporating a variable step size mechanism we have

$$\mathbf{R}_{\tilde{\mathbf{e}}_w}(i) \approx \mathbf{R}_{\tilde{\mathbf{e}}_w}(i-1) - E\left[\mu_w(i-1)\right]\left(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H\right) \\ \times \mathbf{\Lambda}\mathbf{R}_{\tilde{\mathbf{e}}_w}(i-1) - E\left[\mu_w(i-1)\right]\mathbf{R}_{\tilde{\mathbf{e}}_w}(i-1) \\ \times \mathbf{\Lambda}\left(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H\right) + E\left[\mu_w^2(i-1)\right]\left(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H\right) \\ \times \mathbf{\Lambda}\left(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H\right) - E\left[\mu_w^2(i-1)\right]\left(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H\right) \\ \times \mathbf{\Lambda}\left(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H\right)\left(tr\left[\mathbf{R}_{\tilde{\mathbf{e}}_w}(i-1)\mathbf{\Lambda}\right] + 2\xi_{\min}E\left[\tilde{\mathbf{e}}(i-1)\right]\tilde{\mathbf{s}}_k\right). \quad (49)$$

Remark that if the signal vectors were approximately orthogonal, then the first K eigenvectors of $\mathbf{R}$ can be approximated as $\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K$. Similarly to [9], we make the approximation that the matrix $\mathbf{s}_k\mathbf{s}_k^H$ is diagonal, so that $\mathbf{R}_{\tilde{\mathbf{e}}_w}$ is approximately diagonal. Let us now define the $M\times 1$ vector $\mathbf{r}_{\tilde{\mathbf{e}}_w}$ with elements equal to the diagonal elements of $\mathbf{R}_{\tilde{\mathbf{e}}_w}$ and with some manipulations (49) can be rewritten as:

$$\mathbf{r}_{\tilde{\mathbf{e}}_w}(i) \approx \mathbf{r}_{\tilde{\mathbf{e}}_w}(i-1) - 2E\left[\mu_w(i-1)\right]\left(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H\right) \\ \times \mathbf{\Lambda}\mathbf{r}_{\tilde{\mathbf{e}}_w}(i-1) + E\left[\mu_w^2(i-1)\right]\left(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H\right)^2 \\ \times \boldsymbol{\lambda}\boldsymbol{\lambda}^H\mathbf{r}_{\tilde{\mathbf{e}}_w}(i-1) + E\left[\mu_w^2(i-1)\right]\xi_{\min} \\ \times \left(2E\left(\tilde{\mathbf{e}}(i-1)\tilde{\mathbf{s}}_k + 1\right)\left(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H\right)^2\boldsymbol{\lambda}\right. \quad (50)$$

where $\boldsymbol{\lambda}$ is the $M\times 1$ vector containing the eigenvalues of $\mathbf{R}$. Since $E[\tilde{\mathbf{e}}_w(i)] = 0$ converges to zero as $i\to\infty$, to guarantee stability it is sufficient that all eigenvalues of $\mathbf{B} = (\mathbf{I} - 2E[\mu_w(\infty)](\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H)\mathbf{\Lambda} + E[\mu_w^2(\infty)](\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H)^2\boldsymbol{\lambda}\boldsymbol{\lambda}^H)$ have magnitude less than one. This is true if the row sums of $\mathbf{B}$ are less than one, which implies that for stability

$$\frac{E\left[\mu_w^2(\infty)\right]}{E\left[\mu_w(\infty)\right]} < \frac{2}{\sum_{k=1}^M \lambda_k} = \frac{2}{\sum_{k=1}^K A_k^2 + M\sigma^2}. \quad (51)$$

If we apply limits on both sides of (50) and let $i\to\infty$, we obtain $\lim_{i\to\infty}\mathbf{r}_{\tilde{\mathbf{e}}_w}(i) = \lim_{i\to\infty}\mathbf{r}_{\tilde{\mathbf{e}}_w}(i-1) = \mathbf{r}_{\tilde{\mathbf{e}}_w}(\infty)$, $\lim_{i\to\infty} E[\mu_w(i)] = \lim_{i\to\infty} E[\mu_w(i-1)] = E[\mu_w(\infty)]$, $\lim_{i\to\infty} E[\mu_w^2(i)] = \lim_{i\to\infty} E[\mu_w^2(i-1)] = E[\mu_w^2(\infty)]$ and because $\lim_{i\to\infty} E[\tilde{\mathbf{e}}(i-1)] = 0$ the last term of (50) associated with $E[\tilde{\mathbf{e}}(i-1)]$ is eliminated, yielding

$$\mathbf{r}_{\tilde{\mathbf{e}}_w}(\infty) \approx \mathbf{r}_{\tilde{\mathbf{e}}_w}(\infty) - 2E\left[\mu_w(\infty)\right]\left(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H\right)\mathbf{\Lambda}\mathbf{r}_{\tilde{\mathbf{e}}_w}(\infty) \\ + E\left[\mu_w^2(\infty)\right]\left(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H\right)^2\boldsymbol{\lambda}\left(\boldsymbol{\lambda}^H\mathbf{r}_{\tilde{\mathbf{e}}_w}(\infty) + \xi_{\min}\right). \quad (52)$$

Using the fact that $\lim_{i\to\infty}\boldsymbol{\lambda}^H\mathbf{r}_{\tilde{\mathbf{e}}_w}(i) = \xi_{\mathrm{ex}}(\infty)$, that $\mathbf{1}^H(\mathbf{I} - \tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H)^2\boldsymbol{\lambda} \approx tr\mathbf{R}_{vr}$ [9] and rearranging the terms we obtain

$$\xi_{\mathrm{ex}}(\infty) \approx \frac{E\left[\mu_w^2(\infty)\right]}{E\left[\mu_w(\infty)\right]}\left(\xi_{\min} + \xi_{\mathrm{ex}}(\infty)\right) tr\mathbf{R}_{vr}. \quad (53)$$

By employing *Assumption 2*, substituting (30) and (31), we obtain for the BVSS

$$\xi_{\mathrm{ex}}(\infty) \approx \xi_{\min}^2\frac{2\alpha\gamma tr\mathbf{R}_{vr}}{(1-\alpha^2)} \quad (54)$$

whereas replacing (40) and (41), for the BRVSS we get

$$\xi_{\mathrm{ex}}(\infty) \approx \xi_{\min}^3\frac{2\alpha\gamma(1-\beta)tr\mathbf{R}_{vr}}{(1-\alpha^2)(1+\beta)}. \quad (55)$$

The parameters α, γ and β are selected to achieve a small excess MSE, while accelerating the convergence of the algorithm and maintaining its stability. The constant α provides exponential forgetting similarly to the leaky LMS [18], [19], reducing the excess MSE. A larger γ results in a larger step size in the initial stages of adaptation, ensuring faster convergence. A smaller γ provides a smaller level of misadjustment at the expense of a slower convergence speed. The choice of β is very important to achieve a good performance and tracking in a nonstationary environment because it can cope with the time-varying statistics of a dynamic channel and the interference. The value of β should be slightly less than one in nonstationary environments.

B. Multipath Case

Here, we focus on the multipath case SG minimum variance algorithm and rely on the analysis carried out for a fixed step size in [16]. We remark that we will follow the analysis in [16] for completeness and then we will take into account the new BVSS mechanisms.

1) Trajectory of Mean Receiver Vector: To study the trajectory of the mean parameter vector and constraint vector for the multipath SG algorithm, let us drop the user index k for ease of presentation and define the receiver error vector $\mathbf{e}_w(i)$ and the constraint error vector $\mathbf{e}_g(i)$ at time i

$$\mathbf{e}_w(i) = \mathbf{w}(i) - \mathbf{w}_{\text{opt}}, \quad \mathbf{e}_g(i) = \mathbf{g}(i) - \mathbf{g}_{\text{opt}} \tag{56}$$

where $\mathbf{g}_{\text{opt}}$ is the optimal constraint vector which is the eigenvector of matrix $(\mathbf{C}^H\mathbf{R}^{-1}\mathbf{C})^{-1}$ that corresponds to its maximum eigenvalue $\xi_{\max}$ and $\mathbf{w}_{\text{opt}} = \mathbf{R}^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{g}_{\text{opt}} = \xi_{\max}\mathbf{R}^{-1}\mathbf{C}\mathbf{g}_{\text{opt}}$.

The error vectors $\mathbf{e}_w(i)$ and $\mathbf{e}_g(i)$ must be considered together due to the joint optimization procedure. By using the fact that $\mathbf{C}^H\mathbf{w}_{\text{opt}} = \mathbf{g}_{\text{opt}}$ and replacing $\mathbf{R}$ by $\mathbf{r}\mathbf{r}^H$, similarly to [16] we can write

$$\begin{aligned}\mathbf{e}_w(i+1) = &\,\mathbf{\Pi}_k(\mathbf{I} - \mu_w(i)\mathbf{r}(i)\mathbf{r}^H(i))\mathbf{e}_w(i)\\ &+ \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{e}_g(i) - \mu_w(i)\mathbf{\Pi}_k\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_{\text{opt}}.\end{aligned}$$

Taking expectations on both sides, using *Assumption 1* and the independence theory [18], we obtain

$$\begin{aligned}E\left[\mathbf{e}_w(i+1)\right] = &\,\mathbf{\Pi}_k\left(\mathbf{I} - E\left[\mu_w(i)\right]\mathbf{R}\right)E\left[\mathbf{e}_w(i)\right]\\ &+\mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{e}_g(i)\end{aligned} \tag{57}$$

where the term $\mu_w(i)\mathbf{\Pi}_k\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_{\text{opt}}$ has been eliminated. Now, let us consider the constraint error vector

$$\begin{aligned}\mathbf{e}_g(i+1) = &\,[(1+\mu_g(i)\rho(i))\mathbf{I} + (\mu_g(i)/\mu_w(i))(\mathbf{C}^H\mathbf{C})^{-1}]\\ &\times \mathbf{e}_g(i) + [\mu_g(i)\rho(i)\mathbf{I}\\ &+ (\mu_g(i)/\mu_w(i))(\mathbf{C}^H\mathbf{C})^{-1}]\mathbf{g}_{\text{opt}}\\ &+ (\mu_g(i)/\mu_w(i))(\mathbf{C}^H\mathbf{C})^{-1}\\ &\times \mathbf{C}^H(\mu_w(i)\mathbf{r}(i)\mathbf{r}^H(i) - \mathbf{I})\mathbf{w}(i).\end{aligned}$$

After taking expectations on both sides, the mean constraint error vector is expressed by

$$\begin{aligned}E\left[\mathbf{e}_g(i+1)\right] = &\left[(1+E\left[\mu_g(i)\right]\bar{\rho})\mathbf{I}\right.\\ &\left.+ E\left[\frac{\mu_g(i)}{\mu_w(i)}\right](\mathbf{C}^H\mathbf{C})^{-1}\right]E\left[\mathbf{e}_g(i)\right]\\ &+ E\left[\mu_g(i)\right](\xi_{\max}+\bar{\rho})\mathbf{g}_{\text{opt}}\\ &+ (\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\\ &\times\left(E\left[\mu_g(i)\right]\mathbf{R} - E\left[\frac{\mu_g(i)}{\mu_w(i)}\right]\mathbf{I}\right)\\ &\times \mathbf{e}_w(i)\end{aligned} \tag{58}$$

where $\bar{\rho(i)} = E[\rho(i)]$. By combining (57) and (58) the trajectory of the error vectors is given by the following:

$$\begin{bmatrix}E\left[\mathbf{e}_w(i+1)\right]\\ E\left[\mathbf{e}_g(i+1)\right]\end{bmatrix} = \mathbf{\Gamma}\begin{bmatrix}E\left[\mathbf{e}_w(i)\right]\\ E\left[\mathbf{e}_g(i)\right]\end{bmatrix} + \begin{bmatrix}\mathbf{0}\\ E\left[\mu_g(i)\right](\xi_{\max}+\bar{\rho}\mathbf{g}_{\text{opt}})\end{bmatrix} \tag{59}$$

where

$$\begin{aligned}\mathbf{\Gamma} &= \begin{bmatrix}\mathbf{\Gamma}_{11} & \mathbf{\Gamma}_{12}\\ \mathbf{\Gamma}_{21} & \mathbf{\Gamma}_{22}\end{bmatrix}\\ \mathbf{\Gamma}_{11} &= \mathbf{\Pi}_k(\mathbf{I} - E[\mu_w(i)]\mathbf{R})\\ \mathbf{\Gamma}_{12} &= \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\\ \mathbf{\Gamma}_{21} &= (\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H(E[\mu_g(i)]\mathbf{R} - E[(\mu_g(i)/\mu_w(i))]\mathbf{I})\end{aligned}$$

and

$$\mathbf{\Gamma}_{22} = [(1+E[\mu_g(i)]\bar{\rho})\mathbf{I} + E[(\mu_g(i)/\mu_w(i))](\mathbf{C}^H\mathbf{C})^{-1}].$$

By examining the previous expression we see that the stability of the algorithm depends on the matrix $\mathbf{\Gamma}$ and the study of its trajectory is intractable, as pointed out in [16]. Specifically, we will focus on the variable step size mechanisms, whose maximum values allowed for adaptation must be chosen such that the eigenvalues of $\mathbf{\Gamma}\mathbf{\Gamma}^H$ have magnitude less than one.

2) Trajectory of Excess MSE: Now let us consider an analysis of the steady-state excess MSE taking into account the novel variable step size mechanisms. The analysis follows the general steps presented for the single-path case and was first reported in [16] for fixed step size (FSS) algorithms. Let us define the MSE at time $i+1$

$$\epsilon(i+1) = E\left[\left|b(i+1) - \mathbf{w}^H(i+1)\mathbf{r}(i+1)\right|^2\right]. \tag{60}$$

By using the fact that $\boldsymbol{e}_w(i+1) = \mathbf{w}(i+1) - \mathbf{w}_{\text{opt}}$ and employing the independence assumption the MSE can be written as

$$\begin{aligned}\epsilon(i+1) = &\,\epsilon_{\min} + E\left[\mathbf{g}\mathbf{C}^H\mathbf{e}_w(i+1) + \mathbf{e}_w^H(i+1)\mathbf{C}\mathbf{g}\right]\\ &+ tr\left[E\left[\mathbf{R}\mathbf{w}_{\text{opt}}\mathbf{e}_w^H(i+1)\right]\right]\\ &+ tr\left[E\left[\mathbf{R}\mathbf{e}_w(i+1)\mathbf{w}_{\text{opt}}^H\right]\right]\\ &+ tr\left[E\left[\mathbf{R}\mathbf{e}_w(i+1)\mathbf{e}_w^H(i+1)\right]\right]\end{aligned} \tag{61}$$

where $\epsilon_{\min} = E[|b(i+1) - \mathbf{w}_{\text{opt}}^H\mathbf{r}(i+1)|^2]$. Since $\lim_{i\to\infty} E[\mathbf{e}_w(i)] = 0$ we have

$$\begin{aligned}\lim_{i\to\infty}\epsilon(i+1) &= \epsilon_{\min} + \lim_{i\to\infty} tr\left[E\left[\mathbf{R}\mathbf{e}_w(i+1)\mathbf{e}_w^H(i+1)\right]\right]\\ &= \epsilon_{\min} + \xi_{\text{ex}}(\infty)\end{aligned} \tag{62}$$

where the second term in (62) is the steady-state excess MSE resulted from the adaptation process. Let us define $\mathbf{R}_e(i) = E[\mathbf{e}_w(i)\mathbf{e}_w^H(i)]$, $\mathbf{R}_e = \lim_{i\to\infty}\mathbf{R}_e(i)$ and use the property of trace $tr(\mathbf{R}\mathbf{R}_e) = vec^H(\mathbf{R})vec(\mathbf{R}_e)$ to express the steady-state excess MSE as

$$\xi_{\text{ex}} = tr(\mathbf{R}\mathbf{R}_e) = vec^H(\mathbf{R})vec(\mathbf{R}_e). \tag{63}$$

Thus, it becomes clear that to assess ξ_{ex} is sufficient to study $\mathbf{R}_e$, which depends on the tap error vector $\mathbf{e}_w$. To simplify the analysis let us assume that $\mathbf{C}^H\mathbf{e}_w \approx \mathbf{e}_g$, which is true when adaptation is close to the steady state, as pointed out in [16]. Now, we can rewrite $\mathbf{e}_w$ as

$$\mathbf{e}_w(i+1) \approx \mathbf{P}\mathbf{e}_w(i) - \mu_w(i)\mathbf{\Pi}\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_{\text{opt}}(i) \tag{64}$$

where $\mathbf{P}(i) = \mathbf{I} - \mu_w(i)\mathbf{\Pi}\mathbf{r}(i)\mathbf{r}^H(i)$. By substituting $\mathbf{P}(i)$ into (64), taking expectation on both sides and using *Assumption 1* we obtain

$$\begin{aligned}\mathbf{R}_e(i+1) &= E\left[\mathbf{e}_w(i+1)\mathbf{e}_w^H(i+1)\right]\\ &\approx \mathbf{R}_e(i) - E\left[\mu_w(i)\right]\mathbf{\Pi}\mathbf{R}(i)\mathbf{R}_e\\ &\quad - E\left[\mu_w(i)\right]\mathbf{R}_e(i)\mathbf{R}(i)\mathbf{\Pi} - E\left[\mu_w(i)\right]\\ &\quad \times E\left[\left(\mathbf{I} - \mu_w(i)\mathbf{\Pi}\mathbf{r}(i)\mathbf{r}^H(i)\right)\right.\\ &\qquad \left. \times\, \mathbf{e}_w(i)\mathbf{w}_{\text{opt}}^H\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{\Pi}\right] - E\left[\mu_w(i)\right]\\ &\quad \times E\left[\mathbf{\Pi}(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_{\text{opt}}\mathbf{e}_w^H(i)\right.\\ &\qquad \left. \times \left(\mathbf{I} - E\left[\mu_w(i)\right]\mathbf{\Pi}(i)\mathbf{r}(i)\mathbf{r}^H(i)\right)^H\right]\\ &\quad \times E\left[\mu_w^2(i)\right] E\left[\mathbf{\Pi}\mathbf{r}(i)\mathbf{r}^H(i)\right.\\ &\qquad \times \left(\mathbf{w}_{\text{opt}}\mathbf{w}_{\text{opt}}^H + \mathbf{e}_w(i)\mathbf{e}_w^H(i)\right)\\ &\qquad \left. \times\, \mathbf{r}(i)\mathbf{r}^H(i)\mathbf{\Pi}\right]. \end{aligned} \tag{65}$$

By making $i \rightarrow \infty$, we have $\lim_{i\rightarrow\infty}\mathbf{R}_e(i+1) = \lim_{i\rightarrow\infty}\mathbf{R}_e(i+1) = \mathbf{R}_e$ and $\lim_{i\rightarrow\infty} E[\mathbf{e}_w(i)] = 0$, and then using these limits on both sides of (65), we arrive at

$$\begin{aligned}\mathbf{\Pi}\mathbf{R}\mathbf{R}_e + \mathbf{R}_e\mathbf{R}\mathbf{\Pi} \approx\ & \frac{E\left[\mu_w^2(\infty)\right]}{E\left[\mu_w(\infty)\right]}\\ & \times E\left[\mathbf{\Pi}\mathbf{r}\mathbf{r}^H\left(\mathbf{w}_{\text{opt}}\mathbf{w}_{\text{opt}}^H + \mathbf{R}_e\right)\mathbf{r}\mathbf{r}^H\mathbf{\Pi}\right]. \end{aligned} \tag{66}$$

At this point we can use the "vec" operation, i.e., we arrange all elements of a matrix into a vector along column-wise, and the property of trace $tr(\mathbf{R}\mathbf{R}_e) = vec^H(\mathbf{R})vec(\mathbf{R}_e)$ to express the steady-state excess MSE as

$$\xi_{\text{ex}} = \frac{E\left[\mu_w^2(\infty)\right]}{E\left[\mu_w(\infty)\right]} vec^H(\mathbf{R})\mathbf{\Theta}^{-1}\mathbf{a} \tag{67}$$

where

$$\begin{aligned}\mathbf{\Theta} &= \mathbf{A} - (E[\mu_w^2(\infty)]/E[\mu_w(\infty)])\mathbf{B}\\ \mathbf{a} &= \mathbf{B}vec(\mathbf{w}_{\text{opt}}\mathbf{w}_{\text{opt}}^H)\\ \mathbf{A} &= (\mathbf{R}\mathbf{\Pi})^T \otimes \mathbf{I} + \mathbf{I} \otimes (\mathbf{\Pi}\mathbf{R})\\ \mathbf{B} &= [(\mathbf{\Pi})^T \otimes \mathbf{\Pi}]E[(\mathbf{r}(i)\mathbf{r}^H(i))^T \otimes (\mathbf{r}(i)\mathbf{r}^H(i))]\end{aligned}$$

and $\otimes$ accounts for the Kronecker product.

By using *Assumption 2* for the BVSS, the substitution of (30) and (31) into (67) yields

$$\begin{aligned}\xi_{\text{ex}} = vec^H(\mathbf{R})&\left(\mathbf{A} - \frac{2\alpha\gamma\xi_{\min}}{(1-\alpha^2)}\mathbf{B}\right)^{-1}\\ &\times \frac{2\alpha\gamma\xi_{\min}}{(1-\alpha^2)}\mathbf{B}vec\left(\mathbf{w}_{\text{opt}}\mathbf{w}_{\text{opt}}^H\right)\end{aligned} \tag{68}$$

where $\xi_{\min} = \mathbf{w}_{\text{opt}}^H\mathbf{r}\mathbf{r}^H\mathbf{w}_{\text{opt}}$. By proceeding similarly for the BRVSS, using *Assumption 2* (i.e., $\xi_{\min} \gg \xi_{\text{ex}}(\infty)$) and replacing (40) and (41) into (67) we obtain

$$\begin{aligned}\xi_{\text{ex}} = vec^H(\mathbf{R})&\left(\mathbf{A} - \frac{2\alpha\gamma(1-\beta)\xi_{\min}^2}{(1-\alpha^2)(1+\beta)}\mathbf{B}\right)^{-1}\\ &\times \frac{2\alpha\gamma(1-\beta)\xi_{\min}^2}{(1-\alpha^2)(1+\beta)}\mathbf{B}vec\left(\mathbf{w}_{\text{opt}}\mathbf{w}_{\text{opt}}^H\right).\end{aligned} \tag{69}$$

It is worth noting that the expressions in (68) and (69) reduce to the one given in [16] if we employ a fixed step size rather than a variable step size. Despite they involve fourth-order statistics, it can be seen that ξ_{ex} increases almost linearly with $E[\mu_w]$, similarly to the single-path case. Another aspect that should be mentioned is that a variable step size approach is able to deal with the tradeoff between excess error and convergence rate, by automatically tuning the convergence factor so that larger step sizes are used for improving convergence rate and smaller step sizes are employed for ensuring a small ξ_{ex}. With regard to stability, it can be guaranteed provided the designer chooses adequate values for the maximum and the minimum step sizes allowed by truncation, as described in Section V.

VII. Simulation Results

In this section, we investigate the effectiveness of the proposed variable step size mechanisms through simulations and verify the validity of the convergence analysis undertaken for predicting the MSE obtained by the BVSS and BRVSS methods. We have conducted experiments under stationary and nonstationary scenarios to assess the convergence performance in terms of SINR of the proposed VSS and RVSS mechanisms and compared them with other recently reported techniques, namely the ASS [23], [24] and the AV [20], [21]. Moreover, the BER performance of the receivers employing the different analyzed mechanisms is assessed for different loads, processing gains (N), channel paths (L_p) and profiles, and fading rates. The spreading sequences used in the DS-CDMA system are indicated for each experiment, and chosen amongst random and Gold sequences. All simulations are averaged over 100 experiments and when the proposed mechanisms are employed for both receiver and channel estimation, the legends show (rec and channel). For the remaining cases and for the single-path algorithm, the examined adaptation techniques are used only for receiver estimation. For all algorithms the step size $\mu_{w|g}(i)$ is truncated between $\{\mu_{\max_{w|g}}, \mu_{\min_{w|g}}\}$.

All channels are normalized so that

$$\sum_{l=1}^{L_p} p_l^2 = 1.$$

For fading channels, the sequence of channel coefficients $h_l(i) = p_l\alpha_l(i)$ $(l = 0, 1, 2)$, where $\alpha_l(i)$, is obtained with Clarke's model [34]. The phase ambiguity derived from channel estimation is eliminated in our simulations by using the phase of $\mathbf{g}(0)$ as a reference to remove the ambiguity and for fading channels we assume ideal phase tracking and express the results in terms of the normalized Doppler frequency f_dT (cycles/symbol).

A. MSE Performance: Analytical Results

Here, we verify that the results (54), (55), (68), and (69) of the section on convergence analysis of the mechanisms can provide a means of estimating the excess MSE. The steady-state MSE between the desired and the estimated symbol obtained through simulation is compared with the steady-state MSE computed via the expressions derived in Section VI. In order to illustrate the usefulness of our analysis we have carried out some experiments. To semianalytically compute the MSE for the

TABLE III
PARAMETERS FOR THE VARIABLE STEP SIZE MECHANISMS: CONVERGENCE TO THE SAME MSE

	Single-Path	Multi-path
FSS	$\mu_w = 10^{-4}$	$\mu_w = 10^{-4}$
BVSS	$\mu_w^{(0)} = 10^{-4}$,$\gamma = 0.0025$ $\alpha = 0.98$,$\mu_{max_w} = 10^{-2}$ $\mu_{min_w} = 10^{-4}$	$\alpha = 0.98$,$\gamma_w = 0.0025$ $\gamma_g = 5 \times 10^{-4}$,$\mu_w^{(0)} = 10^{-3}$ $\mu_g^{(0)} = 10^{-2}$,$\mu_{min_w} = 10^{-4}$ $\mu_{max_w} = 10^{-2}$
BRVSS	$\alpha = 0.98$,$\mu_{min_w} = 10^{-4}$ $\gamma = 5 \times 10^{-3}$,$\mu_w^{(0)} = 10^{-3}$ $\beta = 0.9$,$\mu_{max_w} = 10^{-2}$	$\alpha = 0.98$,$\gamma_g = 5 \times 10^{-4}$ $\mu_{max_w} = 10^{-2}$,$\gamma_w = 0.0035$ $\mu_w^{(0)} = 10^{-3}$,$\mu_g^{(0)} = 10^{-2}$ $\beta = 0.9$,$\mu_{min_w} = 10^{-4}$

TABLE IV
OPTIMIZED PARAMETERS FOR THE VARIABLE STEP SIZE MECHANISMS: CONVERGENCE TO THE SMALLEST MSE

	Single-Path	Multi-path
FSS	$\mu_w = 10^{-3}$	$\mu_w = 10^{-3}$
BVSS	$\alpha = 0.98$,$\gamma = 10^{-3}$ $\mu_w^{(0)} = 10^{-3}$,$\mu_{max_w} = 10^{-2}$ $\mu_{min_w} = 10^{-4}$	$\alpha = 0.98$,$\gamma_w = 10^{-3}$ $\gamma_g = 10^{-4}$,$\mu_w^{(0)} = 10^{-3}$ $\mu_g^{(0)} = 10^{-2}$,$\mu_{min_w} = 10^{-4}$ $\mu_{max_w} = 10^{-2}$
BRVSS	$\gamma = 10^{-3}$,$\mu_{min_w} = 10^{-4}$ $\alpha = 0.98$,$\mu_w^{(0)} = 10^{-3}$ $\beta = 0.99$,$\mu_{max_w} = 10^{-2}$	$\alpha = 0.98$,$\gamma_g = 10^{-4}$ $\beta = 0.99$, $\gamma_w = 10^{-3}$ $\mu_w^{(0)} = 10^{-3}$,$\mu_{min_w} = 10^{-4}$ $\mu_{max_w} = 10^{-2}$,$\mu_g^{(0)} = 10^{-2}$

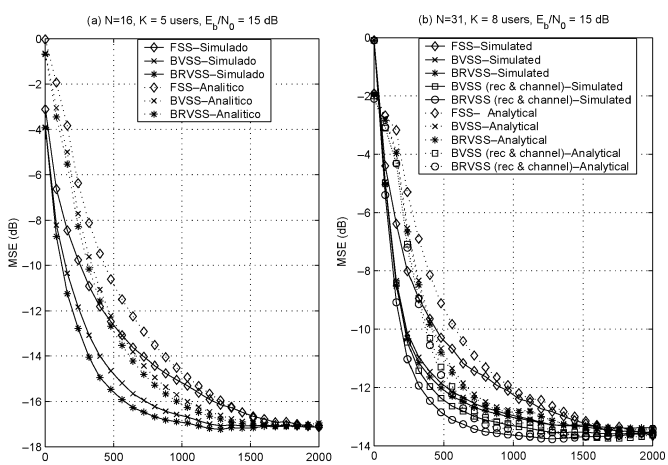

Fig. 3 MSE analytical versus simulated performance for the variable step size mechanisms in (a) single-path channels with $K = 5$ users. (b) In multipath channels with $K = 8$ users. (a) $\mathrm{N} = 16, \mathrm{K} = 5$ users, $\mathrm{E_b/N_0} = 15$ dB. (b) $\mathrm{N} = 31, \mathrm{K} = 8$ users, $\mathrm{E_b/N_0} = 15$ dB.

single-path recursion, we have used (46), $\xi_{\min} \approx A_k^2 + \sigma^2\mathbf{I}$ [9], (54), (55), $\epsilon_{\min} = 1 - \mathbf{p}_k^H(i)\mathbf{R}^{-1}(i)\mathbf{p}_k(i)$, $\mathbf{R}_{vr}(i) = (\mathbf{I} - \mathbf{s}_k\mathbf{s}_k^H)\mathbf{R}(i)$, where $\mathbf{R}(i) = 1/i\sum_{n=1}^{i}\mathbf{r}(n)\mathbf{r}^H(n)$ and $\mathbf{p}_k(i) = 1/i\sum_{n=1}^{i} b_k^*(n)\mathbf{r}(n)$. For the multipath case and taking into account the variable step size mechanism for channel estimation, we employed a semianalytical approach with $\xi_{\min}(i) = \mathbf{g}_{\mathrm{est}}^H(i)(\mathbf{C}_k^H\mathbf{R}^{-1}(i)\mathbf{C}_k)^{-1}\mathbf{g}_{\mathrm{est}}(i)$ and $\mathbf{w}_{\mathrm{opt}}(i) = \mathbf{R}^{-1}(i)\mathbf{p}_k(i)$, where $\mathbf{g}_{\mathrm{est}}$ was obtained from the SG channel estimator with the different mechanisms. The channel parameters for these experiments are $p_0 = 1$, $p_1 = 0.5$ and $p_2 = 0.5$ (or alternatively 0, −6, and −6 dB, respectively).

In the first experiment, we have tuned the parameters of the mechanisms, shown in Table III, in order to achieve the same steady-state MSE upon convergence for the algorithms. The results are shown in Fig. 3, for the single and multipath cases, respectively, and indicate that the BVSS and BRVSS proposed mechanisms enjoy a significantly faster convergence than the FSS approach. For the multipath case, we note that when BVSS and BRVSS are employed for channel estimation the performance is further improved. By comparing the curves, it can be seen that as the number of received symbols increase and the simulated MSE converges, the analytical curves obtained converge to about the same steady-state MSE, showing the usefulness of our analysis and assumptions.

In the second experiment, we have tuned the parameters of the mechanisms, shown in Table IV, in order to achieve the smallest steady-state MSE upon convergence. The results are shown in Fig. 4, for the single and multipath cases, respectively, and indicate that the BVSS and BRVSS proposed mechanisms achieve a significant improvement over the FSS. For the multipath case, the variable step size mechanisms incorporated in the channel estimation improve the performance of the algorithm. Again, a comparison of the curves indicates that the analytical curves match the simulated ones upon convergence, verifying the validity of our analysis. The parameters shown in Table IV are used for the remaining experiments in this paper.

B. SINR Convergence Performance

The SINR at the receiver end has been chosen as the performance index to evaluate the convergence performance in the remaining situations. In the following experiments we will assess the SINR performance of the analyzed mechanisms, namely, FSS, ASS, AV, BVSS, and BRVSS. We remark that the parameters of the FSS, ASS, and AV techniques have been tuned in

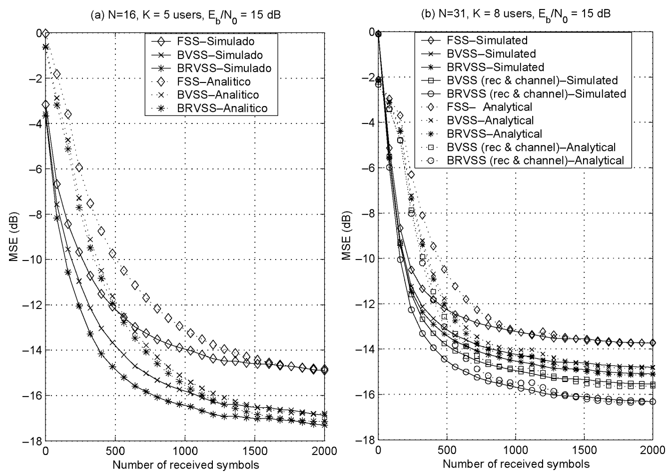

Fig. 4. MSE analytical versus simulated performance for the variable step size mechanisms in (a) single-path channels with $K = 5$ users (b) multipath channels with $K = 8$ users. (a) $\mathrm{N} = 16, \mathrm{K} = 5$ users, $\mathrm{E_b/N_0} = 15$ dB. (b) $\mathrm{N} = 31, \mathrm{K} = 8$ users, $\mathrm{E_b/N_0} = 15$ dB.

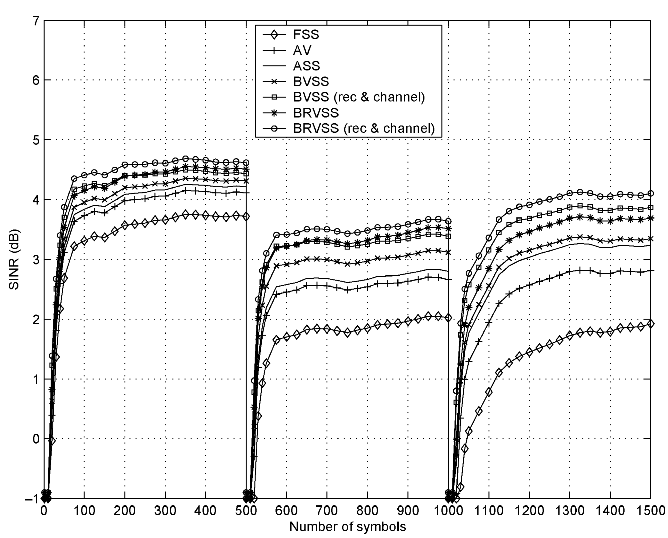

Fig. 5. SINR performance for the variable step size mechanisms with fading ($f_dT = 0.001$) and $E_b/N_0 = 15$ dB.

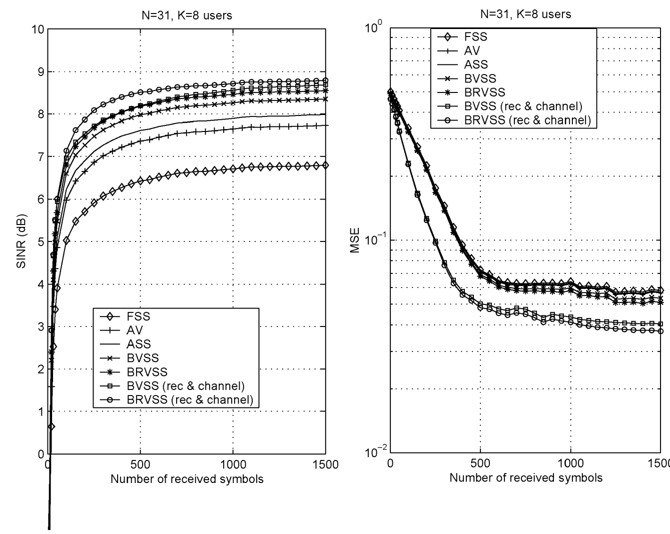

Fig. 6. (a) SINR and (b) MSE performance of the channel estimators for the variable step size mechanisms without fading and $E_b/N_0 = 15$ dB.

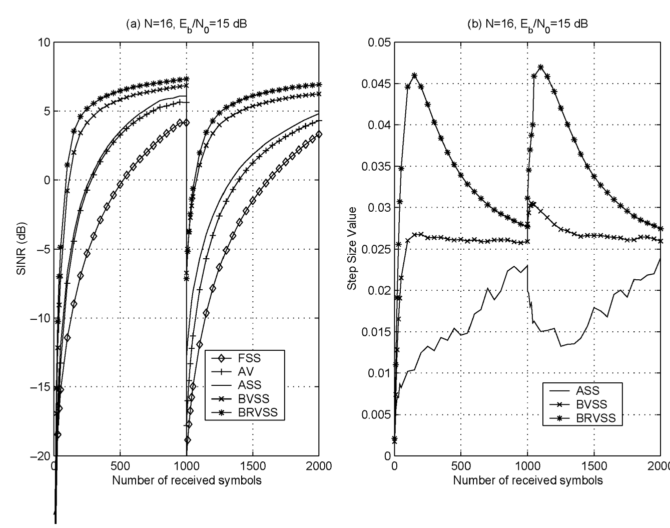

Fig. 7. (a) SINR performance and (b) step size values for the variable step size mechanisms in nonstationary environment with AWGN channel and $E_b/N_0 = 15$ dB. (a) $\mathrm{N} = 16, \mathrm{E_b/N_0} = 15$ dB; (b) $\mathrm{N} = 16, \mathrm{E_b/N_0} = 15$ dB.

order to optimize performance, allowing for a fair comparison with the new mechanisms. The normalization described in Appendix I and II is utilized in order to facilitate the setting of the step sizes for different loads.

In the first experiment, shown in Fig. 5, for a fading channel the system starts with 4 interferers with 7 dB above the desired user's power level and 1 interferer with the same power level of the desired one, which corresponds to $E_b/N_0 = 15$ dB. At 500 symbols, 2 interferers with 10 dB above the desired signal power level and 2 with the same power level enter the system, whereas 2 interferers with 7 dB above the desired signal power level leave it. At 1000 symbols, 1 interferer with 10 dB above, 1 interferer with 7 dB above, and 2 interferers with the same power level of the desired signal exit the system, while 1 interferer with 15 above the desired user enters the system. The channel parameters are $p_0 = 1, p_1 = 0.5$, and $p_2 = 0.3$ (or alternatively 0, -6, and -10 dB, respectively).

In the second experiment, shown in Fig. 6, we illustrate the performance in terms of SINR of the analyzed algorithms and their respective channel estimation performance in terms of MSE. The channel parameters to be estimated are $p_0 = 1$, $p_1 = 0.7$, and $p_2 = 0.5$ (or alternatively 0, -3, and -6 dB, respectively) and the system has 6 users, where 1 interferer operates with 7 dB above the desired user's power level, 1 interferer 10 dB above the desired signal and the remaining interferers work with the same power level of the desired one, which corresponds to $E_b/N_0 = 15$ dB.

In the third and fourth experiments, shown in Figs. 7 and 8, we illustrate the performance in terms of SINR of the analyzed algorithms and their respective step size values as a function of the received symbols in a nonstationary setup. In Fig. 7, the system starts with 4 users, where 1 interferer operates with 7 dB above the desired user's power level, and the remaining interferers work with the same power level of the desired one, which corresponds to $E_b/N_0 = 15$ dB. At 1000 symbols, one interferer with 10 dB above the desired user signal enters the system.

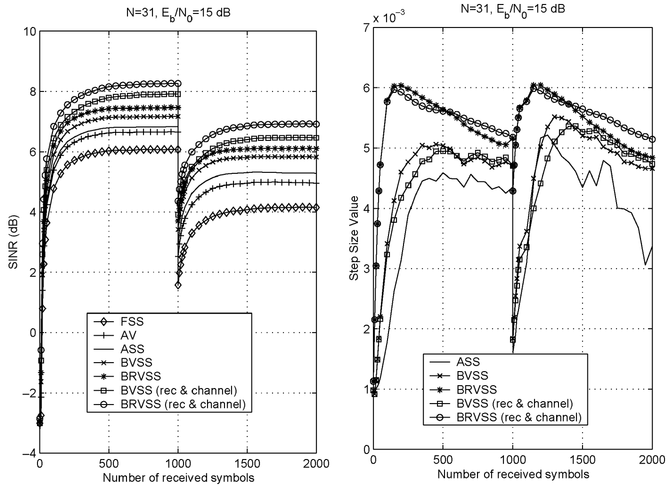

Fig. 8. (a) SINR performance and (b) step size values for the variable step size mechanisms in nonstationary environment with channel $p_0 = 1, p_1 = 0.5$ and $p_2 = 0.5$ and $E_b/N_0 = 15$ dB.

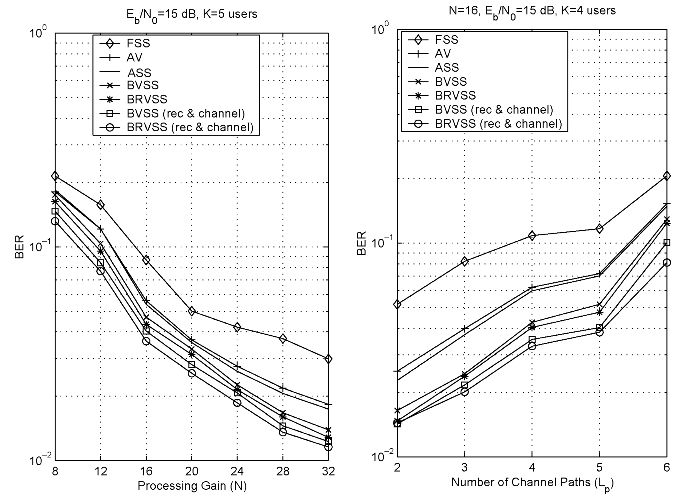

Fig. 9. BER performance versus (a) processing gain (N), $L_p = 3$ and (b) number of channel paths (L_p) for the variable step size mechanisms without fading using random generated spreading sequences.

In Fig. 8, we employ the mechanisms with the multipath algorithm, where the channel parameters to be estimated are $p_0 = 1$, $p_1 = 0.5$ and $p_2 = 0.5$. The system starts with 8 users, where 1 interferer operates with 7 dB above the desired user's power level, 1 interferer operates with 10 dB above the desired user and the remaining interferers work with the same power level of the desired one, which corresponds to $E_b/N_0 = 15$ dB. At 1000 symbols, two interferers with 15 dB above the desired user signal enter the system. The curves show that the entrance of a near-far user degrades the performance of the receiver and the variable step size mechanisms are superior to the FSS in dealing with nonstationary environments, exhibiting a faster convergence improvement than the FSS. Moreover, the novel adaptation mechanisms, the BVSS and the BRVSS, exhibit faster convergence than the ASS even though they are less complex.

C. BER Performance

In the next experiments, we evaluate the BER performance of the variable step size mechanisms versus E_b/N_0, the processing gain (N), the number of channel paths (L_p) and versus the number of users (K), as shown in Figs. 9 to 13. The receivers process 2000 symbols, averaged over 100 independent runs for all BER simulations and employ the normalization described in the appendices in order to facilitate the setting of the step sizes for different loads.

In Fig. 9 we depict a scenario where the BER versus N and L_p is assessed. The channel parameters are randomly generated using uniform random variables (r. v.) and normalized so that $\sum_{l=1}^{L_p} p_l^2 = 1$ and the received power of the interferers are log-normal r. v. with associated standard deviation 3 dB. The curves show that the proposed mechanisms BVSS and BRVSS outperform the FSS, ASS, and AV methods. When the new mechanisms are incorporated in the channel estimator the BER performance is further improved. In addition, we notice that as N is increased so is the BER performance and the resistance against multipath effects, while an increase in L_p degrades BER performance, as expected.

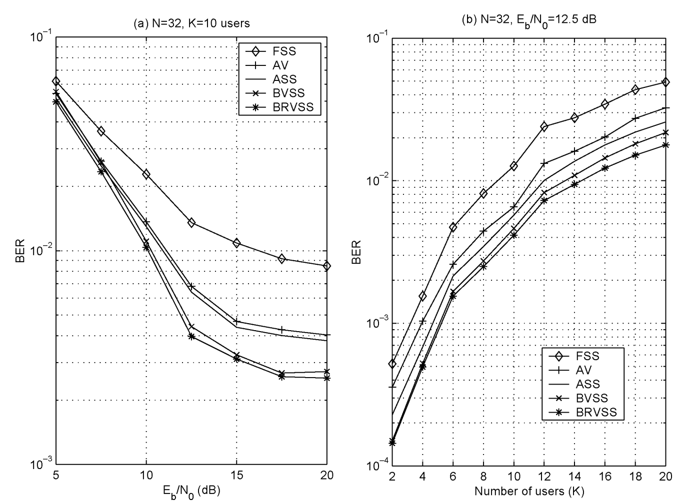

Fig. 10. BER performance versus (a) E_b/N_0 and (b) number of users (K) for the variable step size mechanisms with AWGN channel using random spreading sequences. (a) $\mathrm{N} = 32, \mathrm{K} = 10$ users; (b) $\mathrm{N} = 32, \mathrm{E_b/N_0} = 12.5$ dB.

In Fig. 10, we assess the BER performance versus E_b/N_0 and number of users (K) in a flat channel environment, where 1 interferer operates with 7 dB above and 1 interferer with 10 dB above the desired signal, which corresponds to $E_b/N_0 = 12.5$ dB, for $K > 2$. The results show that new mechanisms can afford significant gains in BER performance over the FSS, the AV, and the ASS at a low additional complexity.

In Fig. 11, we evaluate the BER performance versus E_b/N_0 and number of users (K) in a multipath channel environment, where 1 interferer operates with 7 dB above and 1 interferer with 10 dB above the desired signal, which corresponds to $E_b/N_0 = 15$ dB, for $K > 2$. The channel parameters are randomly generated using uniform r.v., normalized so that $\sum_{l=1}^{L_p} p_l^2 = 1$ and the received power of the interferers are log-normal r. v. with associated standard deviation 3 dB. The curves depicted in Fig. 11

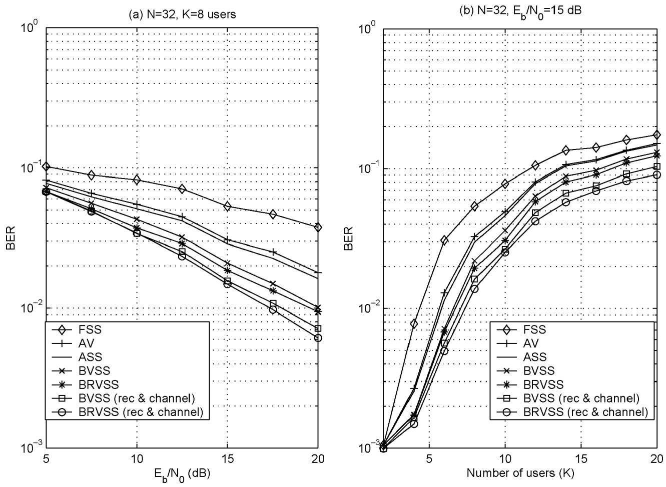

Fig. 11. BER performance versus (a) E_b/N_0 and (b) number of users (K) for the variable step size mechanisms without fading using random spreading sequences. (a) $\mathrm{N} = 32, \mathrm{K} = 8$ users; (b) $\mathrm{N} = 32, \mathrm{E_b/N_0} = 15\ \mathrm{dB}$.

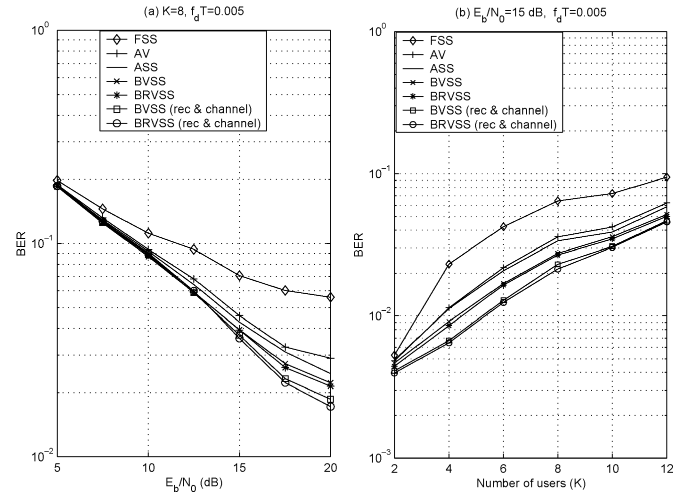

Fig. 13. BER performance versus (a) E_b/N_0 and (b) number of users (K) for the variable step size mechanisms with fading using Gold spreading sequences with $N = 31$. (a) $\mathrm{N} = 8, \mathrm{f_d T} = 0.005$. (b) $\mathrm{E_b/N_0} = 15\ \mathrm{dB}, \mathrm{f_d T} = 0.005$.

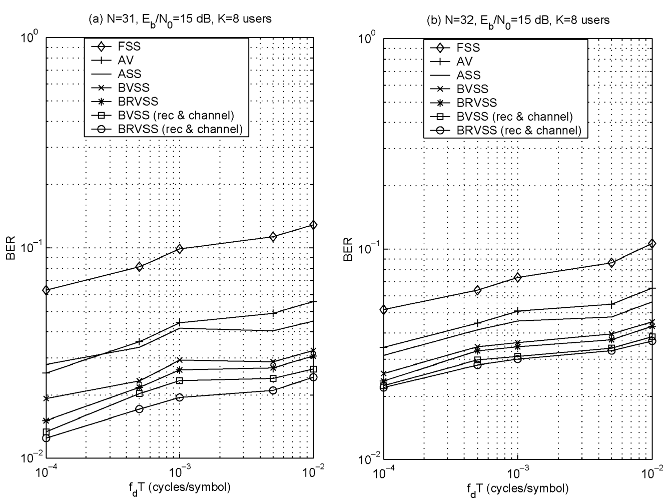

Fig. 12. BER performance versus $f_d T$ with (a) Gold spreading sequences and two interferers with 5 and 10 dB above the desired signal and (b) random generated spreading sequences. (a) $\mathrm{N} = 31, \mathrm{E_b/N_0} = 15\ \mathrm{dB}, \mathrm{K} = 8$ users; (b) $\mathrm{N} = 32, \mathrm{E_b/N_0} = 15\ \mathrm{dB}, \mathrm{K} = 8$ users.

indicate that the new mechanisms offer superior BER performance to the FSS, the AV, and the ASS, as observed for the single-path case. It can also be noted that the incorporations of these mechanisms for channel estimation can further improve the BER performance.

In Figs. 12 and 13, we consider scenarios with multipath fading and assess the BER performance of the techniques so far analyzed. The channel has a power profile given by 0, -3, and -6 dB, respectively, where in each run the second path delay (τ_2) is given by a discrete uniform r. v. between 1 and 4 chips, the third path delay is taken from a discrete uniform r. v. between 1 and $5-\tau_2$ chips and the received power of the interferers given by log-normal r. v. with associated standard deviation 3 dB.

In Fig. 12, the BER performance versus $f_d T$ is assessed and it can be noted that again the novel mechanisms achieve the best performances. It should be remarked that advantages of the BVSS and BRVSS techniques are more pronounced for more severe near-far situations as in Fig. 12(a), where a larger BER performance gap is noticed. In Fig. 13, the BER performance versus E_b/N_0 and number of users (K) is depicted. The results indicate once more that the BVSS and the BRVSS mechanisms are highly effective for use with SG algorithms in minimum variance receivers.

VIII. Concluding Remarks

We have investigated the performance of blind adaptive receivers for DS-CDMA systems that employ SG algorithms with variable step size mechanisms. Two low complexity variable step size mechanisms have been proposed and analyzed for estimating the parameters of linear CDMA receivers that operate with SG algorithms in both single-path and multipath channels. For multipath channels, the new blind adaptation mechanisms, namely the BVSS and the BRVSS, were incorporated in the channel estimation algorithms, showing significant improvements over the conventional channel estimation method with fixed step sizes. We have also derived analytical expressions for predicting the excess MSE of the adaptive receivers in steady state using approximations and assumptions. The analytical results have been compared with simulations and show that our analysis is consistent and capable of predicting the steady-state MSE for nonfading scenarios with AWGN. Finally, experiments for typical mobile channels show that the new mechanisms outperform existent methods at a lower complexity.

Appendix I
Derivation of Normalized Step Size: Single-Path Case

To derive a normalized step size for the algorithm in (18) let us add $\mathbf{s}_k$ to both sides of it and write the minimum variance cost function $J = |\mathbf{w}_k^H \mathbf{r}|^2$ as

$$J = \left| \left(\mathbf{w}_k - \mu_w \left(\mathbf{w}_k^H \mathbf{r} \right) \mathbf{v}_k \right)^H \mathbf{r} \right|^2 \quad (70)$$

where $\mathbf{v}_k = (\mathbf{I} - \mathbf{s}_k\mathbf{s}_k^H)\mathbf{r}$. If we take the gradient of J with respect to μ_w and equal it to zero, we have

$$\nabla J_{\mu_w} = 2\left|\mathbf{w}_k^H\mathbf{r} - \mu_w\left(\mathbf{w}_k^H\mathbf{r}\right)\mathbf{v}_k\right)^H\mathbf{r}\right| \times \left(\left(-\mathbf{w}_k^H\mathbf{r}\right)\mathbf{v}_k\right)^H\mathbf{r} = 0 \quad (71)$$

and the solution is $\mu_w = (1/\mathbf{r}^H(\mathbf{I}-\mathbf{s}_k\mathbf{s}_k^H)\mathbf{r})$. Note that we introduce a convergence factor μ_0 so that the algorithm can operate with adequate step sizes that are usually small to ensure good performance, and, thus, we have $\mu_w = (\mu_0/\mathbf{r}^H(\mathbf{I} - \mathbf{s}_k\mathbf{s}_k^H)\mathbf{r})$.

Appendix II
Derivation of Normalized Step Size: Multipath Case

To derive a normalized step size for the multipath case in (19) let us write the minimum variance cost function $J = |\mathbf{w}_k^H\mathbf{r}|^2$ as

$$J = \left|\left(\mathbf{\Pi}_k(\mathbf{w}_k - \mu_w\mathbf{r}\mathbf{r}^H\mathbf{w}_k)^H\mathbf{r} + \mathbf{C}_k\left(\mathbf{C}_k^H\mathbf{C}_k\right)^{-1}\mathbf{g}\right)^H\mathbf{r}\right|^2. \quad (72)$$

If we take the gradient of J with respect to μ_w and equal it to zero, we have

$$\nabla J_{\mu_w} = 2\left|(\mathbf{\Pi}_k\mathbf{w}_k)^H\mathbf{r} - \mu_w(\mathbf{\Pi}_k\mathbf{r}\mathbf{r}^H\mathbf{w}_k)^H\mathbf{r} + \left(\mathbf{C}_k\left(\mathbf{C}_k^H\mathbf{C}_k\right)^{-1}\mathbf{g}\right)^H\mathbf{r}\right|(-\mathbf{\Pi}_k\mathbf{r}\mathbf{r}^H\mathbf{w}_k)^H\mathbf{r} = 0. \quad (73)$$

If we substitute $\mathbf{\Pi}_k = \mathbf{I} - (\mathbf{C}_k(\mathbf{C}_k^H\mathbf{C}_k)^{-1}\mathbf{C}^H$ into the first term of (73) and use $\mathbf{C}_k\mathbf{w}_k = \mathbf{g}$ we can eliminate the third term of (73) obtain the solution $\mu_w = (1/\mathbf{r}^H\mathbf{\Pi}_k\mathbf{r})$. Note that we introduce again a convergence factor μ_0 so that the algorithm can operate with adequate step sizes that are usually small to ensure good performance, and, thus, we have $\mu_w = (\mu_0/\mathbf{r}^H\mathbf{\Pi}_k\mathbf{r})$.

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