Blind Adaptive MIMO Receivers for Space-Time Block-Coded DS-CDMA Systems in Multipath Channels Using the Constant Modulus Criterion

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***Abstract*—We propose blind adaptive multi-input multi-output (MIMO) linear receivers for DS-CDMA systems using multiple transmit antennas and space-time block codes (STBC) in multipath channels. A space-time code-constrained constant modulus (CCM) design criterion based on constrained optimization techniques is considered and recursive least squares (RLS) adaptive algorithms are developed for estimating the parameters of the linear receivers. A blind space-time channel estimation method for MIMO DS-CDMA systems with STBC based on a subspace approach is also proposed along with an efficient RLS algorithm. Simulations for a downlink scenario assess the proposed algorithms in several situations against existing methods.**

***Index Terms*—DS-CDMA systems, MIMO systems, space-time block codes, blind adaptive algorithms, interference suppression.**

I. Introduction

THE ever-increasing demand for performance and capacity in wireless networks has motivated the development of numerous signal processing and communications techniques for utilizing these resources efficiently. Recent results on information theory have shown that higher spectral efficiency [1], [2] and diversity [3], [4] can be achieved with multiple antennas at both transmitter and receiver. Space-time coding (STC) techniques can exploit spatial and temporal transmit diversity [3], [4], [5]. The problem of receiver design for DS-CDMA systems using multiple transmit antennas and STBC has been considered in recent works [6]-[15], however, there are still some open problems. One key issue is the amount of training required by MIMO channels which motivates the use of blind techniques. In addition, the existing blind MIMO schemes [8], [9], [12], [14], [15] are susceptible to the problem of mismatch resulting from imperfect channel knowledge and this calls for a robust approach. Prior work on blind techniques for MIMO CDMA systems is limited to subspace [9] and constrained minimum variance (CMV) approaches [8], [12], [14], [15], which are susceptible to the problem of signature mismatch. The code-constrained constant modulus (CCM) approach has demonstrated increased robustness and better performance than CMV techniques [16]-[22] for single-antenna systems although it has not been considered for MIMO systems.

The goal of this work is to propose blind adaptive MIMO receivers for DS-CDMA systems using multiple transmit antennas and STBC based on the CCM design in multipath channels. In the proposed scheme, we exploit the unique structure of the spreading codes and STBC to derive efficient blind receivers based on the CCM design and develop computationally efficient RLS algorithms. The proposed design approach for MIMO receivers requires the knowledge of the space-time channel. In order to blindly estimate the channel, we present a subspace approach that exploits the STBC structure present in the received signal and derive an adaptive RLS type channel estimator. We also establish the necessary and sufficient conditions for the channel identifiability of the method. The only requirement for the receivers is the knowledge of the signature sequences for the desired user.

II. Space-Time DS-CDMA System Model

Consider the downlink of a symbol synchronous QPSK DS-CDMA system shown in Fig. 1 with K users, N chips per symbol, N_t antennas at the transmitter, N_r antennas at the receiver and L_p propagation paths. For simplicity, we assume that the transmitter (Tx) employs only $N_t = 2$ antennas and adopts Alamouti's STBC scheme [3], although other STBC can be used. In this scheme, for $i = 1, \ldots, P$ two symbols $b_k(2i-1)$ and $b_k(2i)$ are transmitted from Tx1 and Tx2, respectively, during the $(2i-1)th$ symbol interval and, during the next symbol interval, $-b_k^*(2i)$ and $b_k^*(2i-1)$ are transmitted from Tx1 and Tx2, respectively. Each user is assigned a unique spreading code for each Tx, which may be constructed in different ways [7], [8]. We assume that the receiver (Rx) is synchronized with the main path, the delays of the channel paths are multiples of the chip rate, the channel is constant during two symbol intervals and the spreading codes are repeated from symbol to symbol. The received signal at antenna m after chip-pulse matched filtering and sampling at chip rate over two consecutive symbols yields the M-dimensional received vectors

$$\begin{aligned}\mathbf{r}(2i-1) &= \sum_{k=1}^{K} A_k b_k(2i-1)\mathbf{C}_k^1\mathbf{h}_m^1 + A_k b_k(2i)\mathbf{C}_k^2\mathbf{h}_m^2 \\ &\quad + \boldsymbol{\eta}_{k,m}(2i-1) + \mathbf{n}_m(2i-1) \\ \mathbf{r}(2i) &= \sum_{k=1}^{K} A_k b_k^*(2i-1)\mathbf{C}_k^2\mathbf{h}_m^2 - A_k b_k^*(2i)\mathbf{C}_k^1\mathbf{h}_m^1 \\ &\quad + \boldsymbol{\eta}_{k,m}(2i) + \mathbf{n}_m(2i) \\ & i = 1, \ \ldots, P, \qquad m = 1, \ \ldots, N_r\end{aligned} \quad (1)$$

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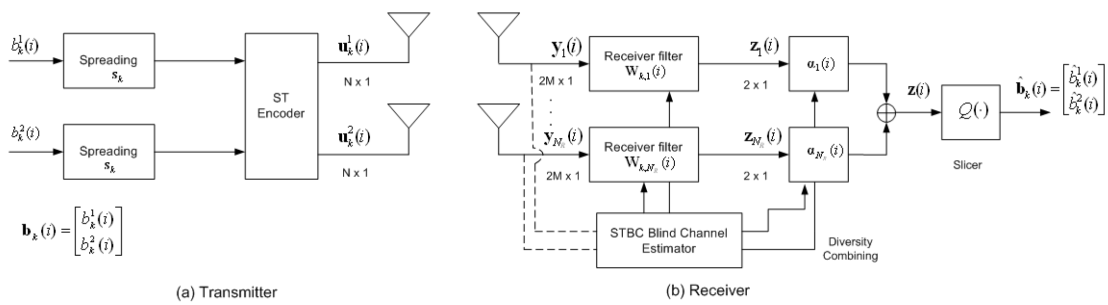

Fig. 1. Proposed space-time system: schematic of the kth user (a) Transmitter and (b) Receiver .

where $M = N + L_p - 1$, $\mathbf{n}_m(i) = [n_1(i) \ \ldots \ n_M(i)]^T$ is the complex Gaussian noise vector with mean zero and $E[\mathbf{n}_m(i)\mathbf{n}_m^H(i)] = \sigma^2\mathbf{I}$, where $(.)^T$ and $(.)^H$ denote transpose and Hermitian transpose, respectively. The quantity $E[.]$ stands for expected value and the amplitude of user k is A_k. The channel vector for the users' signals transmitted from each transmit antenna n_t $(n_t = 1, 2)$ and received at the m-th receive antenna are $\mathbf{h}_m^{n_t}(i) = [h_{m,0}^{n_t}(i) \ldots h_{m,L_p-1}^{n_t}(i)]^T$ and $\boldsymbol{\eta}_m(i)$ is the intersymbol interference at the mth receive antenna. The $M \times L_p$ convolution matrix $\mathbf{C}_k^{n_t}$ contains one-chip shifted versions of the signature sequence for user k and each transmit antenna given by $\mathbf{s}_k^{n_t} = [a_k^{n_t}(1) \ldots a_k^{n_t}(N)]^T$ (the reader is referred to [8], [16], [19] for details on the structure of $\mathbf{C}_k^{n_t}$). The received data in (1) organized into a single $2M \times 1$ vector $\mathbf{y}_m(i) = [\mathbf{r}^T(2i-1) \ \mathbf{r}^T(2i)]^T$ within the ith symbol interval at the mth receive antenna is

$$\begin{aligned}\mathbf{y}_m(i) &= \sum_{k=1}^{K} A_k b_k(2i-1)\boldsymbol{\mathcal{C}}_k \boldsymbol{g}_m(i) + A_k b_k(2i)\bar{\boldsymbol{\mathcal{C}}}_k \boldsymbol{g}_m^*(i) \\ &\quad + \boldsymbol{\eta}_k(i) + \mathbf{n}(i) \\ &= \sum_{k=1}^{K} \mathbf{x}_k(i) + \bar{\mathbf{x}}_k(i) + \boldsymbol{\eta}_k(i) + \mathbf{n}(i)\end{aligned} \tag{2}$$

where

$$\boldsymbol{\mathcal{C}}_k = \begin{bmatrix} \mathbf{C}_k^1 & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_k^2 \end{bmatrix}, \bar{\boldsymbol{\mathcal{C}}}_k = \begin{bmatrix} \mathbf{0} & \mathbf{C}_k^2 \\ -\mathbf{C}_k^1 & \mathbf{0} \end{bmatrix},$$

$$\mathbf{C}_k^1 \ \text{ and } \mathbf{C}_k^2 \in \boldsymbol{\mathcal{R}}^{M \times L_p} [19] \tag{3}$$

$$\boldsymbol{g}_m(i) = \begin{bmatrix} \mathbf{h}_{k,m}^1 \\ \mathbf{h}_{k,m}^2 \end{bmatrix}, \boldsymbol{\eta}_k(i) = \begin{bmatrix} \boldsymbol{\eta}_{k,1}(2i-1) \\ \boldsymbol{\eta}_{k,2}(2i) \end{bmatrix},$$

$$\mathbf{n}(i) = \begin{bmatrix} \mathbf{n}_1(2i-1) \\ \mathbf{n}_2(2i) \end{bmatrix} \tag{4}$$

The $2M \times 1$ received vectors $\mathbf{y}_m(i)$ are linearly combined with the $2M \times 2$ parameter matrix $\mathbf{W}_{k,m}(i)$ of user k of the mth antenna at the Rx to provide the soft estimates

$$\mathbf{z}_m(i) = \mathbf{W}_{k,m}^H(i)\mathbf{y}_m(i) = [z_{k,m}(i) \ \bar{z}_{k,m}(i)]^T \tag{5}$$

By collecting the soft estimates $\mathbf{z}_m(i)$ at the Rx, the designer can also exploit the spatial diversity at the receiver as

$$\mathbf{z}(i) = \sum_{m=1}^{N_r} \boldsymbol{\alpha}_m(i)\mathbf{z}_m(i) \tag{6}$$

where $\boldsymbol{\alpha}_m(i) = \mathrm{diag}\big(\alpha_{m,1}(i), \ \alpha_{m,2}(i)\big)$ are the gains of the combiner at the receiver, which can be equal leading to Equal Gain Combining (ECG) or proportional to the channel gains as with Maximal Ratio Combining (MRC) [28].

III. Space-Time Linearly Constrained Receivers Based on the CCM Design Criterion

Consider the $2M$-dimensional received vector at the mth receiver $\mathbf{y}_m(i)$, the $2M \times 2L_p$ constraint matrices $\boldsymbol{\mathcal{C}}_k$ and $\bar{\boldsymbol{\mathcal{C}}}_k$ that were defined in (3) and the $2L_p \times 1$ space-time channel vector $\boldsymbol{g}_m(i)$ with the multipath components of the unknown channels from Tx1 and Tx2 to the mth antenna at the receiver. The space-time linearly constrained receiver design according to the CCM criterion corresponds to determining an $2M \times 2$ FIR filter matrix $\mathbf{W}_{k,m}(i) = \big[\mathbf{w}_{k,m}(i), \bar{\mathbf{w}}_{k,m}(i)\big]$ composed of two FIR filters $\mathbf{w}_{k,m}(i)$ and $\bar{\mathbf{w}}_{k,m}(i)$ with dimensions $2M \times 1$. The filters $\mathbf{w}_{k,m}(i)$ and $\bar{\mathbf{w}}_{k,m}(i)$ provide estimates of the desired symbols at the mth antenna of the receiver as given by

$$\hat{\mathbf{b}}_k(i) = \mathrm{sgn}\big(\Re\big[\mathbf{W}_{k,m}^H(i)\mathbf{y}_m(i)\big]\big) + j\ \mathrm{sgn}\ \big(\Im\big[\mathbf{W}_{k,m}^H(i)\mathbf{y}_m(i)\big]\big) \tag{7}$$

where $\mathrm{sgn}(\cdot)$ is the signum function, $\Re(.)$ selects the real component, $\Im(.)$ selects the imaginary component and $\mathbf{W}_{k,m}(i)$ is designed according to the minimization of the following constant modulus (CM) cost functions

$$J_{CM}(\mathbf{w}_{k,m}(i)) = E\big[(|\mathbf{w}_{k,m}^H(i)\mathbf{y}_m(i)|^2 - 1)^2\big] \tag{8}$$

$$J_{CM}(\bar{\mathbf{w}}_{k,m}(i)) = E\big[(|\bar{\mathbf{w}}_{k,m}^H(i)\mathbf{y}_m(i)|^2 - 1)^2\big] \tag{9}$$

subject to the set of constraints described by

$$\boldsymbol{\mathcal{C}}_k^H \mathbf{w}_{k,m}(i) = \nu\ \boldsymbol{g}_m(i), \quad \bar{\boldsymbol{\mathcal{C}}}_k^H \bar{\mathbf{w}}_{k,m}(i) = \nu\ \boldsymbol{g}_m^*(i) \tag{10}$$

where ν is a constant to ensure the convexity of (8) and (9), which is detailed in Appendix I along with the convergence properties. The proposed approach is to consider the design problems in (8) and (9) via the optimization of the two filters $\mathbf{w}_{k,m}(i)$ and $\bar{\mathbf{w}}_{k,m}(i)$ in a simultaneous fashion. The optimization of each filter aims to suppress the interference and estimate the symbols transmitted by each transmit antenna. The expressions for the filters of the space-time CCM linear receiver are derived using the method of Lagrange multipliers

[25] and are given by

$$\mathbf{w}_{k,m}(i+1) = \mathbf{R}_{k,m}^{-1}(i)\Big[\mathbf{d}_{k,m}(i) - \boldsymbol{\mathcal{C}}_k(\boldsymbol{\mathcal{C}}_k^H\mathbf{R}_{k,m}^{-1}(i)\boldsymbol{\mathcal{C}}_k)^{-1} \cdot \big(\boldsymbol{\mathcal{C}}_k^H\mathbf{R}_{k,m}^{-1}(i)\mathbf{d}_{k,m}(i) - \nu\ \boldsymbol{g}_m(i)\big)\Big] \quad (11)$$

$$\bar{\mathbf{w}}_{k,m}(i+1) = \bar{\mathbf{R}}_{k,m}^{-1}(i)\Big[\bar{\mathbf{d}}_{k,m}(i) - \bar{\boldsymbol{\mathcal{C}}}_k(\bar{\boldsymbol{\mathcal{C}}}_k^H\bar{\mathbf{R}}_{k,m}^{-1}(i)\bar{\boldsymbol{\mathcal{C}}}_k)^{-1} \cdot \big(\bar{\boldsymbol{\mathcal{C}}}_k^H\bar{\mathbf{R}}_{k,m}^{-1}(i)\bar{\mathbf{d}}_{k,m}(i) - \nu\ \boldsymbol{g}_m^*(i)\big)\Big] \quad (12)$$

where $\mathbf{R}_{k,m}(i) = E[|z_{k,m}(i)|^2\mathbf{y}_m(i)\mathbf{y}_m^H(i)]$ and $\bar{\mathbf{R}}_{k,m}(i) = E[|\bar{z}_{k,m}(i)|^2\mathbf{y}_m(i)\mathbf{y}_m^H(i)]$ are correlation matrices, where $\mathbf{d}_{k,m}(i) = E[z_{k,m}^*(i)\mathbf{y}_m(i)]$ and $\bar{\mathbf{d}}_{k,m}(i) = E[\bar{z}_{k,m}^*(i)\mathbf{y}_m(i)]$ are cross-correlation vectors, which are originated from the proposed optimization problem. The expressions (11) and (12) require matrix inversions which lead to a complexity $O((2M)^3)$. It should also be remarked that (11) and (12) are functions of previous values of the filter and therefore must be iterated in order to reach a solution. Since (11) and (12) assume the knowledge of the space-time channel parameters, channel estimation is required.

IV. Space-Time Channel Estimation

In this section, we present a method that exploits the signature sequences of the desired user and the structure of STBC for blind channel estimation. Consider the received vector $\mathbf{y}_m(i)$ at the mth Rx, its associated $2M \times 2M$ covariance matrix $\mathbf{R}_m = E[\mathbf{y}_m(i)\mathbf{y}_m^H(i)]$, the space-time $2M \times 2L_p$ constraint matrices $\boldsymbol{\mathcal{C}}_k$ and $\bar{\boldsymbol{\mathcal{C}}}_k$ given in (6) and the space-time channel vector $\boldsymbol{g}_m(i)$. From (5) we have that the kth user space-time coded transmitted signals are given by

$$\mathbf{x}_k(i) = A_k b_k(2i-1)\boldsymbol{\mathcal{C}}_k\boldsymbol{g}_m(i), \quad \bar{\mathbf{x}}_k(i) = A_k b_k(2i)\bar{\boldsymbol{\mathcal{C}}}_k\boldsymbol{g}_m^*(i) \quad (13)$$

Let us perform singular value decomposition (SVD) on the space-time $JM \times JM$ covariance matrix $\mathbf{R}_m$.

$$\mathbf{R}_m = \sum_{k=1}^{K} E[\mathbf{x}_k(i)\mathbf{x}_k^H(i)] + E[\bar{\mathbf{x}}_k(i)\bar{\mathbf{x}}_k^H(i)] + E[\boldsymbol{\eta}_k(i)\boldsymbol{\eta}_k^H(i)] + \sigma^2\mathbf{I} = [\mathbf{V}_s\ \mathbf{V}_n]\begin{bmatrix} \boldsymbol{\Lambda}_s + \sigma^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma^2\mathbf{I} \end{bmatrix}[\mathbf{V}_s\ \mathbf{V}_n]^H \quad (14)$$

where $\mathbf{V}_s$ and $\mathbf{V}_n$ are the signal (that includes the ISI) and noise subspaces, respectively. Since the signal and noise subspaces are orthogonal [23], [24], we have the conditions $\mathbf{V}_n^H\mathbf{x}_k(i) = \mathbf{V}_n^H\boldsymbol{\mathcal{C}}_k\boldsymbol{g}_m(i) = \mathbf{0}$ and $\mathbf{V}_n^H\bar{\mathbf{x}}_k(i) = \mathbf{V}_n^H\bar{\boldsymbol{\mathcal{C}}}_k\boldsymbol{g}_m^*(i) = \mathbf{0}$ and hence we have $\Omega = \boldsymbol{g}_m(i)^H\boldsymbol{\mathcal{C}}_{\boldsymbol{k}}{}^H\mathbf{V}_n\mathbf{V}_n^H\boldsymbol{\mathcal{C}}_k\boldsymbol{g}_m(i) = 0$ and $\bar{\Omega} = \boldsymbol{g}_m^T(i)\bar{\boldsymbol{\mathcal{C}}}_k^H\mathbf{V}_n\mathbf{V}_n^H\bar{\boldsymbol{\mathcal{C}}}_k\boldsymbol{g}_m^*(i) = 0$. From these conditions and taking into account the conjugate symmetric properties induced by STBC [8], it suffices to consider only Ω, which allows the recovery of $\boldsymbol{g}_m(i)$ as the eigenvector corresponding to the smallest eigenvalue of the matrix $\boldsymbol{\mathcal{C}}_{\boldsymbol{k}}{}^H\mathbf{V}_n\mathbf{V}_n^H\boldsymbol{\mathcal{C}}_k$, provided $\mathbf{V}_n$ is known. To avoid the SVD on $\mathbf{R}_m$ and overcome the need for determining the noise subspace rank that is necessary to obtain $\mathbf{V}_n$, we resort to the following approach.

Lemma: Consider the SVD on $\mathbf{R}_m$ as in (14), then we have:

$$\lim_{p\rightarrow\infty}(\mathbf{R}_m/\sigma^2)^{-p} = \mathbf{V}_n\mathbf{V}_n^H \quad (15)$$

Proof: Using the decomposition in (14) and since $\mathbf{I} + \boldsymbol{\Lambda}_s/\sigma^2$ is a diagonal matrix with elements strictly greater than unity, by induction we have as $p \rightarrow \infty$ that $(\mathbf{R}_m/\sigma^2)^{-p} = \mathbf{V}_n\mathbf{V}_n^H$.

To blindly estimate the space-time channel of user k at the mth antenna of the receiver we propose the optimization:

$$\hat{\boldsymbol{g}}_m(i) = \arg\min_{\boldsymbol{g}_m(i)} \quad \boldsymbol{g}_m^H(i)\boldsymbol{\mathcal{C}}_{\boldsymbol{k}}{}^H\hat{\mathbf{R}}_m^{-p}(i)\boldsymbol{\mathcal{C}}_{\boldsymbol{k}}\boldsymbol{g}_m(i) \quad (16)$$

subject to $||\boldsymbol{g}_m(i)|| = 1$, where p is an integer, $\hat{\mathbf{R}}_m(i)$ is an estimate of the covariance matrix $\mathbf{R}_m(i)$ and whose solution is the eigenvector corresponding to the minimum eigenvalue of the $JL_p \times JL_p$ matrix $\boldsymbol{\mathcal{C}}_{\boldsymbol{k}}{}^H\hat{\mathbf{R}}_m^{-p}(i)\boldsymbol{\mathcal{C}}_{\boldsymbol{k}}$ that can be obtained using SVD. The performance of the estimator can be improved by increasing p even though our studies reveal that it suffices to use powers up to $p = 2$ to obtain a good estimate of $\mathbf{V}_n\mathbf{V}_n^H$. For the space-time block coded CCM receiver design, we employ the matrix $\mathbf{R}_{k,m}(i)$ instead of $\mathbf{R}_m$ to avoid the estimation of both $\mathbf{R}_m$ and $\mathbf{R}_{k,m}(i)$, and an equivalence of these matrices is established in Appendix II. An analysis of the capacity of the system and the necessary and sufficient conditions for the method to work is included in Appendix III.

V. Blind Adaptive RLS Algorithms for Receiver and Channel Parameter Estimation

In this section we present RLS algorithms for estimating the parameters of the space-time receiver and channel as described in Sections III and IV, respectively.

A. RLS Algorithm for CCM Receiver Parameter Estimation

Considering the expressions obtained for $\mathbf{w}_{k,m}(i)$ and $\bar{\mathbf{w}}_{k,m}(i)$ in (11) and (12), replacing $E[\cdot]$ with time averages, we can develop an RLS algorithm through the recursive estimation of the matrices $\mathbf{R}_{k,m}^{-1}$, $\mathbf{R}_{k,m}^{-1}$, $\boldsymbol{\Gamma}_{k,m}^{-1}(i) = (\boldsymbol{\mathcal{C}}_k^H\mathbf{R}_{k,m}^{-1}(i)\boldsymbol{\mathcal{C}}_k)^{-1}$ and $\bar{\boldsymbol{\Gamma}}_{k,m}^{-1}(i) = (\bar{\boldsymbol{\mathcal{C}}}_k^H\bar{\mathbf{R}}_{k,m}^{-1}(i)\bar{\boldsymbol{\mathcal{C}}}_k)^{-1}$ using the matrix inversion lemma (MIL) and Kalman RLS recursions [25]. The space-time CCM linear receiver estimates are obtained with

$$\hat{\mathbf{w}}_{k,m}(i+1) = \hat{\mathbf{R}}_{k,m}^{-1}(i)\Big[\hat{\mathbf{d}}_{k,m}(i) - \boldsymbol{\mathcal{C}}_k\boldsymbol{\Gamma}_{k,m}^{-1}(i) \cdot \Big(\boldsymbol{\mathcal{C}}_k^H\hat{\mathbf{R}}_{k,m}^{-1}(i)\hat{\mathbf{d}}_{k,m}(i) - \nu\ \hat{\boldsymbol{g}}_m(i)\Big)\Big] \quad (17)$$

$$\hat{\bar{\mathbf{w}}}_{k,m}(i+1) = \hat{\bar{\mathbf{R}}}_{k,m}^{-1}(i)\Big[\hat{\bar{\mathbf{d}}}_{k,m}(i) - \bar{\boldsymbol{\mathcal{C}}}_k\bar{\boldsymbol{\Gamma}}_{k,m}^{-1}(i) \cdot \Big(\bar{\boldsymbol{\mathcal{C}}}_k^H\hat{\bar{\mathbf{R}}}_{k,m}^{-1}(i)\hat{\bar{\mathbf{d}}}_{k,m}(i) - \nu\ \hat{\boldsymbol{g}}_m^*(i)\Big)\Big] \quad (18)$$

where

$$\hat{\mathbf{d}}_{k,m}(i) = \alpha\hat{\mathbf{d}}_{k,m}(i-1) + (1-\alpha)z_{k,m}^*(i)\mathbf{y}_m(i) \quad (19)$$

$$\hat{\bar{\mathbf{d}}}_{k,m}(i) = \alpha\hat{\bar{\mathbf{d}}}_{k,m}(i-1) + (1-\alpha)\bar{z}_{k,m}^*(i)\mathbf{y}_m(i) \quad (20)$$

correspond to estimates of $\mathbf{d}_{k,m}(i)$ and $\bar{\mathbf{d}}_{k,m}(i)$,respectively. In terms of computational complexity, the space-time CCM-RLS algorithm requires $O((2M)^2)$ to suppress MAI and ISI against $O((2M)^3)$ required by (11) and (12).

B. RLS Algorithm for Space-Time Channel Estimation

We develop an RLS algorithm for the estimation of the space-time channel $\boldsymbol{g}_m(i)$ at the mth receive antenna. The proposed RLS algorithm avoids the SVD and the matrix inversion required in (19) via the MIL and a variation of the power method used in numerical analysis [26]. Following this approach, we first compute the inverse of the matrices $\mathbf{R}_{k,m}^{-1}$ and $\bar{\mathbf{R}}_{k,m}^{-1}$ with the MIL, as part of the space-time receiver design. Then, we construct the matrices $\boldsymbol{\Gamma}_{k,m}(i) = \boldsymbol{\mathcal{C}}_k^H \hat{\mathbf{R}}_{k,m}^{-1}(i)\boldsymbol{\mathcal{C}}_k$ and $\bar{\boldsymbol{\Gamma}}_{k,m}(i) = \bar{\boldsymbol{\mathcal{C}}}_k^H \hat{\bar{\mathbf{R}}}_{k,m}^{-1}(i)\bar{\boldsymbol{\mathcal{C}}}_k$. At this point, the SVD on the $2L_p \times 2L_p$ matrices $\boldsymbol{\Gamma}_{k,m}(i)$ and $\bar{\boldsymbol{\Gamma}}_{k,m}(i)$ that requires $O(L_p^3)$ is avoided and replaced by a single matrix-vector multiplication, resulting in the reduction of the corresponding computational complexity on one order of magnitude and no performance loss. To estimate the channel and avoid the SVD on $\boldsymbol{\Gamma}_{k,m}(i)$ and $\bar{\boldsymbol{\Gamma}}_{k,m}(i)$, we employ the variant of the power method introduced in [24]

$$\hat{\boldsymbol{g}}_m(i) = (\mathbf{I} - \gamma(i)\boldsymbol{\Gamma}_{k,m}(i))\hat{\boldsymbol{g}}_m(i-1) \quad (21)$$

where $\gamma(i) = 1/tr[\boldsymbol{\Gamma}_{k,m}(i)]$ and we make $\hat{\boldsymbol{g}}_m(i) \leftarrow \hat{\boldsymbol{g}}_m(i)/||\hat{\boldsymbol{g}}_m(i)||$ to normalize the channel. It is worth pointing out that due to certain conjugate symmetric properties induced by STBC, it is possible to exploit the data record size for estimation purposes by using both $\boldsymbol{\Gamma}_{k,m}(i)$ and $\bar{\boldsymbol{\Gamma}}_{k,m}(i)$ and thus the proposed RLS algorithm computes

$$\hat{\boldsymbol{g}}_m(i) = \Big(\mathbf{I} - \vartheta(i)(\boldsymbol{\Gamma}_{k,m}(i) + \bar{\boldsymbol{\Gamma}}_{k,m}(i))\Big)\hat{\boldsymbol{g}}_m(i-1) \quad (22)$$

where $\vartheta(i) = 1/tr[\boldsymbol{\Gamma}_{k,m}(i) + \bar{\boldsymbol{\Gamma}}_{k,m}(i)]$ and the normalization procedure remains the same. The algorithm in (22) is adopted since it has a faster convergence than (21) due to the use of more data samples and spreading codes.

C. Computational Complexity

We illustrate the computational complexity of the proposed algorithms and compare them with existing RLS algorithms and subspace techniques, as shown in Table I. The proposed space-time CCM-RLS algorithms have a complexity which is quadratic with N_tM, i.e. the number of transmit antennas N_t and proportional to the processing gain plus the channel order ($M = N + L_p - 1$). The complexities of the trained RLS algorithm and the blind CMV-RLS [8] are also quadratic with N_tM, whereas the complexity of the Subspace algorithm of Reynolds *et al.* [9] is higher due to the subspace computation. The complexity of the RLS channel estimation algorithm in (21) is $2(N_tL_p)^2 + N_tL_p$ as it requires the modified power method on the sum of the $N_tL_p \times N_tL_p$ matrices $\boldsymbol{\Gamma}_{k,m}(i)$ and $\bar{\boldsymbol{\Gamma}}_{k,m}(i)$. In contrast, the subspace algorithm of [9] requires an SVD or the use of the power method on the $N_tM \times N_tM$ matrix $\mathbf{R}_m$. It turns out that since $M >> L_p$ in practice, the proposed algorithm is substantially simpler than the one in [9].

TABLE I
COMPUTATIONAL COMPLEXITY OF RLS ESTIMATION ALGORITHMS PER SYMBOL.

Algorithm	Multiplications
STBC-Trained	$6(N_tM)^2 + 2N_tM + 2$
STBC-CCM	$5(N_tM)^2 + 3(N_tL_p)^2 + 3N_t^2L_pM$ $+4N_tM + 4N_tL_p + 2$
STBC-CMV	$4(N_tM)^2 + 2N_t^2L_pM + 2(N_tL_p)^2$ $+3N_tM + 4N_tL_p + 2$
STBC-Subspace	$(N_tM)^2 + N_t^2L_pM + 3N_tM$ $160N_tMK + 12N_tM + 30K^2 + 14K$

VI. SIMULATIONS

In this section we evaluate the bit error rate (BER) performance of the proposed blind space-time linear receivers based on the CCM design (STBC-CCM). We also assess the proposed space-time channel estimation method (STBC-CCM-CE) in terms of mean squared error (MSE) performance and their corresponding RLS-type adaptive algorithms. We compare the proposed algorithms with some previously reported techniques, namely, the constrained minimum variance (CMV) with a single antenna [16] and with STBC [8] and the subspace receiver of Wang and Poor without [27] and with STBC [9]. The DS-CDMA system employs randomly generated spreading sequences of length $N = 32$, one or two transmit antennas with the Alamouti STBC [3] and one or two receive antennas with MRC. The downlink channels assume that $L_p = 6$ (upper bound). We use three-path channels with powers $p_{l,m}^{1,2}$ given by 0, -3 and -6 dB, where in each run and for each transmit antenna and each receive antenna, the second path delay (τ_2) is given by a discrete uniform random variable (d. u. r. v.) between 1 and 4 chips and the third path delay is taken from a d. u. r. v. between 1 and $5 - \tau_2$ chips. The sequence of channel coefficients for each transmit antenna $n_t = 1, 2$ and each receive antenna $m = 1, 2$ is $h_{l,m}^{n_t}(i) = p_{l,m}^{n_t}\alpha_{l,m}^{n_t}(i)$ ($l = 0, 1, 2, \ \ldots$), where $\alpha_{l,m}^{n_t}(i)$, is obtained with Clarke's model [28]. The phase ambiguity of the blind space-time channel estimation method in [24] is eliminated in our simulations using the phase of $\hat{\boldsymbol{g}}_m(0)$ as a reference to remove the ambiguity and for fading channels we assume ideal phase tracking and express the results in terms of the normalized Doppler frequency f_dT (cycles/symbol). Alternatively, differential modulation can be used to account for the phase rotations as in [9] or the semi-blind approach of [8] adopted.

We evaluate the BER convergence performance of the proposed RLS algorithms for both receiver and channel parameter estimation in a scenario where the system has initially 10 users, the power distribution among the interferers follows a log-normal distribution with associated standard deviation of 3 dB. After 1500 symbols, 6 additional users enter the system and the power distribution among interferers is loosen to 6 dB. The results shown in Fig. 2 indicate that the proposed STBC-CCM receiver design achieves the best performance among the analyzed techniques.

We assess the channel estimation (CE) RLS algorithms with single transmit antennas of [27], [24], with STBC of Reynolds *et al* [9] and the proposed space-time RLS channel estimator with STBC given in (21) in terms of MSE between the actual and the estimated channels using the same dynamic scenario of the first experiment. The results, shown in Fig 3, reveal that the proposed space-time channel estimator outperforms

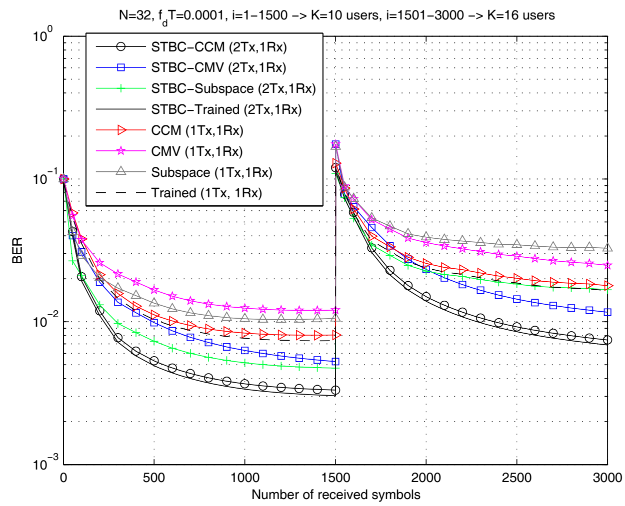

Fig. 2. BER performance versus number of received symbols in a scenario where users enter the system and receivers operate at $SNR = E_b/N_0 = 15$ dB for the desired user.

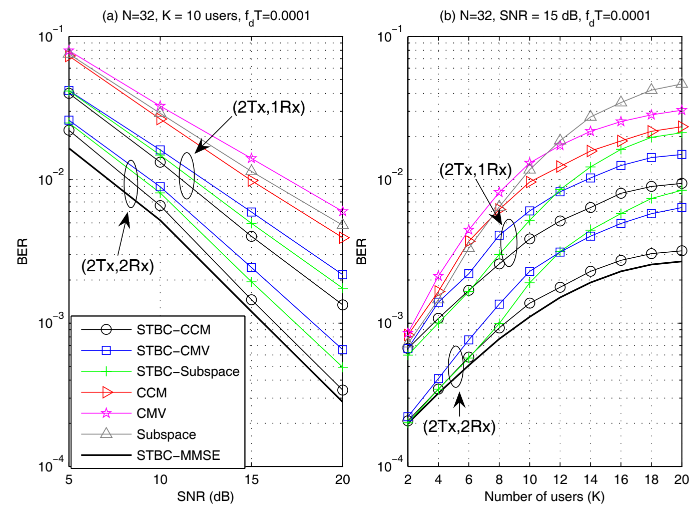

Fig. 4. BER performance versus (a) SNR with $K = 10$ users and (b) number of users (K) at $SNR = E_b/N_0 = 15$ dB .

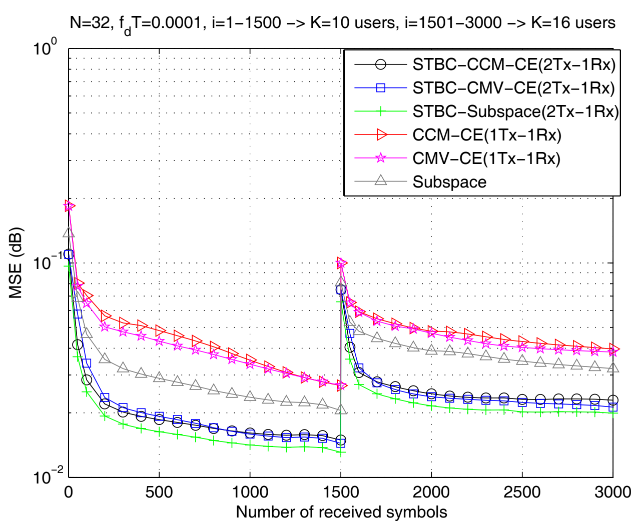

Fig. 3. MSE performance of channel estimation versus number of received symbols in a dynamic scenario where receivers operate at $SNR = E_b/N_0 = 15$ dB for the desired user.

the single-antenna channel estimator because it exploits the information transmitted by 2 antennas.

The BER performance versus SNR and number of users is shown in Fig. 4. We consider data packets of $P = 1500$ symbols, 2 transmit antennas, 1 and 2 receive antennas. We measured the BER after 200 independent transmissions. A comparison with previously reported blind techniques with configurations of 2 transmit antennas and 1 and 2 receive antennas ((2Tx,1Rx) and (2Tx,2Rx)) is shown in Fig. 4. The curves illustrate that the schemes with multiple antennas at the transmitter outperform those with single-antennas and the capacity of the system is also increased. With a (2Tx,2Rx) configuration, the diversity is further exploited and the proposed STBC-CCM achieves a performance close to the MMSE (also with (2Tx,2Rx)) which assumes the knowledge of the channel and the noise variance.

VII. Conclusions

We presented blind adaptive space-time block-coded linear receivers for DS-CDMA systems in multipath channels. A CCM design criterion based on constrained optimization was considered and RLS algorithms for parameter estimation were developed. We also derived a blind space-time channel estimation scheme along with an efficient RLS algorithm. The necessary and sufficient conditions for the channel identifiability of the proposed method were established. Simulations for a downlink scenario have shown the proposed techniques outperform previously reported schemes.

Appendix

We present an analysis of the proposed space-time CCM algorithms and examine their convergence properties and conditions. Let us consider the cost function expressed as (we will drop the time index (i) for simplicity)

$$\mathcal{J}_{CM} = E\big[(|\mathbf{w}_{k,m}^H \mathbf{y}_m|^2 - 1\big] = E\big[|z_{k,m}|^4 - 2E[|z_{k,m}|^2\big] + 1 \quad (23)$$

Let us recall (2) and define $\mathbf{x} = \sum_{k=1}^{K} \mathbf{x}_k = \sum_{k=1}^{K} A_k b_k(2i-1)\mathbf{p}_{k,m}$. The received data can be expressed by $\mathbf{y}_m = \mathbf{x} + \bar{\mathbf{x}} + \boldsymbol{\eta} + \mathbf{n}$. Since the symbols b_k are independent and identically distributed (i.i.d.) random variables with zero mean and unit variance, and b_k and $\mathbf{n}$ are statistically independent, we have $\mathbf{R} = \mathbf{Q} + \mathbf{T} + \sigma^2\mathbf{I}$, where $\mathbf{Q} = E[\mathbf{x}\mathbf{x}^H]$, and $\mathbf{T} = E[\boldsymbol{\eta}\boldsymbol{\eta}^H]$. Let us consider user 1 and antenna 1 as the desired ones, let $\mathbf{w}_1 = \mathbf{w}$ and define $u_k = A_k \mathbf{p}_k^H \mathbf{w}$ and $\mathbf{u} = \mathbf{A}^T\mathbf{P}^H\mathbf{w} = [u_1 \ldots u_K]^T$, where $\mathbf{A} = \mathrm{diag}(A_1 \ldots A_K)$, $\mathbf{P} = [\mathbf{p}_1, \ldots, \mathbf{p}_K]$, and $\mathbf{b} = \big[\mathbf{b}_1(2i-1) \ldots \mathbf{b}_K(2i-1)\big]$. Using the constraints $\boldsymbol{\mathcal{C}}_1\mathbf{w} = \nu\boldsymbol{g}$ and $\boldsymbol{\mathcal{C}}_1\bar{\mathbf{w}} = \nu\boldsymbol{g}^*$, we have for the desired user the conditions $u_1 = A_1\mathbf{p}_1^H\mathbf{w} = A_1\boldsymbol{g}^H$ and $\boldsymbol{\mathcal{C}}_1^H\mathbf{w} = \nu A_1\boldsymbol{g}^H\hat{\boldsymbol{g}}$. Note that the conditions resemble one another and only differ by a conjugate term, i.e. $\bar{u}_1 = u_1^*$. In the absence of noise and neglecting ISI, the (user 1) cost function is (24) where $F = u_1 u_1^* = |u_1|^2 = \nu^2 A_1^2 |\boldsymbol{g}^H\hat{\boldsymbol{g}}|$. Since $\bar{u}_1 = u_1^*$ and $|u_1|^2 = |\bar{u}_1|^2$. The cost functions expressed

$$\begin{aligned}\mathcal{J}_{CM}(\mathbf{w}) &= E\big[(\mathbf{u}^H\mathbf{b}\mathbf{b}^H\mathbf{u})^2\big] - 2E\big[(\mathbf{u}\mathbf{b}\mathbf{b}^H\mathbf{u})\big] + 1\\ &= 8\big(F + \sum_{k=2}^{K}|u_k|^2\big)^2 - 4F^2\sum_{k=2}^{K}|u_k|^4 - 4F - 4\sum_{k=2}^{K}|u_k|^2 + 1\end{aligned} \tag{24}$$

as above are equivalent. Thus, it suffices to examine the properties of only one of them. Let us consider the constraint $\mathcal{C}_1\mathbf{w} = \nu\hat{\boldsymbol{g}}$ and rewrite the cost function $\mathcal{J}_{CM}(\mathbf{w})$ as

$$\begin{aligned}\mathcal{J}_{CM}(\mathbf{w}) = \tilde{\mathcal{J}}_{CM}(\mathbf{u}) = 8(F + \mathbf{u}'^H\mathbf{u}')^2 - 4F^2 - 4\sum_{k=2}^{K}|u_k|^2\\ - 4F - 4(\mathbf{u}^{H\,\prime}\mathbf{u}')^2 + 1\end{aligned} \tag{25}$$

where $\mathbf{u}' = \big[u_2 \dots u_K\big]^T = \mathcal{G}\mathbf{w}$, $\mathcal{G} = \mathbf{A}'^H\mathbf{P}'^H$, $\mathbf{P}' = \big[\mathbf{p}_2 \dots \mathbf{p}_K\big]$ and $\mathbf{A} = \text{diag}\big(A_2 \dots A_K\big)$. To evaluate the convexity of $\tilde{\mathcal{J}}_{CM}(\mathbf{u}')$, we compute its Hessian ($\mathbf{H}$) using the differentiation rule $\mathbf{H} = \frac{\partial\tilde{\mathcal{J}}_{CM}(\mathbf{u}')}{\partial\mathbf{u}'^H}\frac{\partial(\tilde{\mathcal{J}}_{CM}(\mathbf{u}'))}{\partial\mathbf{u}'}$ which yields:

$$\mathbf{H} = \big[16(F - \frac{1}{4})\mathbf{I} + 16\mathbf{u}'^H\mathbf{u}'\mathbf{I} + 16\mathbf{u}\mathbf{u}'^H - 16\text{diag}\big(|u_2|^2 \dots |u_k|^2\big) \tag{26}$$

Specifically, $\mathbf{H}$ is positive definite if $\mathbf{a}^H\mathbf{H}\mathbf{a} > 0$ for all nonzero $\mathbf{a} \in \mathbf{C}^{K-1\times K-1}$. The second, third and fourth terms of $\mathbf{H}$ are positive definite matrices, whereas the first term provides the following condition for convexity

$$\nu A_1^2|\boldsymbol{g}^H\hat{\boldsymbol{g}}| \geq \frac{1}{4} \tag{27}$$

Since $\mathbf{u}' = \mathcal{G}\mathbf{w}$ is a linear function of $\mathbf{w}$ then $\tilde{\mathcal{J}}_{CM}(\mathbf{u}')$ being a convex function of $\mathbf{u}'$ implies that $\mathcal{J}_{CM}(\mathbf{w}) = \tilde{\mathcal{J}}_{CM}(\mathcal{G}\mathbf{w})$ and $\bar{\mathcal{J}}_{CM}(\bar{\mathbf{w}}) = \tilde{\mathcal{J}}_{CM}(\bar{\mathcal{G}}\bar{\mathbf{w}})$ are convex function of $\mathbf{w}$ and $\bar{\mathbf{w}}$, respectively, where $\bar{\mathcal{G}} = \mathbf{A}'^H\bar{\mathbf{P}}'^H$ and $\bar{\mathbf{P}}' = \big[\bar{\mathbf{p}}_2 \dots \bar{\mathbf{p}}_K\big]$. As the extrema of the cost functions can be considered for small noise variance σ^2 a slight perturbation of the noise free case, the cost functions will also be convex for small σ^2 when $\nu A_1^2|\boldsymbol{g}^H\hat{\boldsymbol{g}}| \geq \frac{1}{4}$. If we assume ideal channel estimation, i.e. $|\boldsymbol{g}^H\hat{\boldsymbol{g}}| = 1$, and $\nu = 1$, the condition will collapse to $|A_1|^2 \geq \frac{1}{4}$, which corroborates previous results with the constant modulus algorithms [18]. In the case of larger values of σ^2, the designer should adjust ν in order to enforce the convexity of the cost functions in (8) and (9).

We discuss here the the suitability of the matrix $\mathbf{R}_{k,m}$, that arises from the space-time CCM design method, for use in the space-time channel estimator. From the analysis in Appendix I for the linear receiver, we have for an ideal and asymptotic case that (we consider receive antenna 1 and user 1 for simplicity) $u_k = (A_1\mathbf{p}_k^H)\mathbf{w}_1 \approx 0$, for for $k = 2, \dots, K$. Then, we have that $\mathbf{w}_1^H\mathbf{y} \approx A_1b_1(2i-1)\mathbf{w}_1^H\mathbf{p}_1 + \mathbf{w}_1^H\mathbf{n}$, and $|\mathbf{w}_1^H\mathbf{y}|^2 \approx A_1^2|\mathbf{w}_1^H\mathbf{p}_1|^2 + A_1b_1(2i-1)(\mathbf{w}_1^H\mathbf{p}_1)\mathbf{n}^H\mathbf{w}_1 + A_1b_1^*(2i-1)(\mathbf{p}_1^H\mathbf{w}_1)\mathbf{w}_1^H\mathbf{n} + \mathbf{w}_1^H\mathbf{n}\mathbf{n}^H\mathbf{w}_1$. Therefore, we have for the desired user :

$$\mathbf{R}_1 = E[|\mathbf{w}_1^H\mathbf{y}|^2\mathbf{y}\mathbf{y}^H] \cong \beta\mathbf{R} + \tilde{\mathbf{N}} \tag{28}$$

where $\mathbf{R} = \mathbf{Q} + \sigma^2\mathbf{I}$, $\mathbf{Q} = E[\mathbf{x}\mathbf{x}^H] = \sum_{k=1}^{K}|A_k|^2\mathbf{p}_k\mathbf{p}_k^H$, the scalar factor is $\beta = A_1^2\big(|\mathbf{w}_1^H\mathbf{p}_1|^2 + \sigma^2\big)$ and the noise-like term is $\tilde{\mathbf{N}} = A_1^2\sigma^2\big[\big(\mathbf{w}_1^H\mathbf{p}_1(\mathbf{w}_1\mathbf{p}_1^H) + \sigma(\mathbf{p}_1^H\mathbf{w}_1)(\mathbf{p}_1\mathbf{w}_1^H) + \big(\big[\text{diag}(|\mathbf{w}_1|^2 \dots |\mathbf{w}_{2M}|^2) + \mathbf{w}\mathbf{w}^H\big] - \mathbf{w}^H\mathbf{w}\mathbf{I}\big)\big]$.

The conditions presented show that $\mathbf{R}_k$ for a general user k can be approximated by $\mathbf{R}$ multiplied by a scalar factor β plus a noise-like term $\tilde{\mathbf{N}}$, that for sufficient signal-to-noise ratio (SNR) values has an insignificant contribution. The same analysis applies to $\bar{\mathbf{R}} = E\big[|\bar{\mathbf{w}}_1^H\mathbf{y}|^2\mathbf{y}\mathbf{y}^H\big]$, which is given by

$$\bar{\mathbf{R}}_1 \cong \bar{\beta}\mathbf{R} + \tilde{\bar{\mathbf{N}}} \tag{29}$$

where $\bar{\beta} = A_1^2\big(|\bar{\mathbf{w}}_1^H\bar{\mathbf{p}}_1|^2 + \sigma^2\big)$ and $\tilde{\bar{\mathbf{N}}} = A_1^2\sigma^2\big[\big(\bar{\mathbf{w}}_1^H\bar{\mathbf{p}}_1(\bar{\mathbf{w}}_1\bar{\mathbf{p}}_1^H) + \sigma(\bar{\mathbf{p}}_1^H\bar{\mathbf{w}}_1)(\bar{\mathbf{p}}_1\bar{\mathbf{w}}_1^H) + \big(\big[\text{diag}(|\bar{\mathbf{w}}_1|^2 \dots |\bar{\mathbf{w}}_{2M}|^2) + \bar{\mathbf{w}}\bar{\mathbf{w}}^H\big] - \bar{\mathbf{w}}^H\mathbf{w}\mathbf{I}\big)\big]$. An interesting interpretation of this behavior is the fact that when the symbol estimates $z_k = \mathbf{w}_k^H\mathbf{y}$ and $\bar{z}_k = \bar{\mathbf{w}}_k^H\mathbf{y}$ are reliable and the cost functions in (8) and (9) are sufficiently small, then $|z_k|^2$ and $|\bar{z}_k|^2$ have small variations around unity, yielding the approximation $E[|z_k|^2\mathbf{y}\mathbf{y}^H] = E[\mathbf{y}\mathbf{y}^H] + E[(|z_k^2 - 1)\mathbf{y}\mathbf{y}^H] \cong E[\mathbf{y}\mathbf{y}^H] = \mathbf{R}$ and $E[|\bar{z}_k|^2\mathbf{y}\mathbf{y}^H] = E[\mathbf{y}\mathbf{y}^H] + E[(|\bar{z}_k^2 - 1)\mathbf{y}\mathbf{y}^H] \cong E[\mathbf{y}\mathbf{y}^H] = \mathbf{R}$. Therefore, we can employ $\mathbf{R}_k$ in lieu of $\mathbf{R}$, since the properties of $\mathbf{R}$ studied for the proposed space-time channel estimation method hold for $\mathbf{R}_k$.

We develop an expression of the capacity of the space-time system and discuss necessary and sufficient conditions for the identifiability of space-time channels and the consistency of the estimates. Let q_s and q_n denote the signal and the noise subspace ranks, respectively. The matrix $\mathbf{V}_n^H\mathcal{C}_{\boldsymbol{k}}$ of dimensions $r_n \times JL_p$ will be used for our analysis. If the noise subspace $\mathbf{V}_n$ is the exact subspace then, due to $\mathbf{V}_n^H\mathcal{C}_{\boldsymbol{k}}\boldsymbol{g}_{\boldsymbol{k}} = \mathbf{0}$, we can verify that the column rank of $\mathbf{V}_n^H\mathcal{C}_{\boldsymbol{k}}$ can at most be $JL_p - 1$. In order to have a unique solution (times a phase ambiguity) the column rank of $\mathbf{V}_n^H\mathcal{C}_{\boldsymbol{k}}$ must be exactly equal to $N_TL_p - 1$. Since a column rank of a square matrix is equal to its row rank, a necessary condition for a row rank equal to $N_TL_p - 1$ is to have at least $N_TL_p - 1$ rows, i.e. $q_n \geq N_TL_p - 1$. Since $q_s + q_n = N_TM$ this yields

$$q_s \leq N_TM - N_TL_p + 1 \tag{30}$$

Consider now the signal subspace rank q_s and assume symbol-by-symbol estimation in a synchronous downlink system. The number of columns of $\mathbf{V}_s$ (which is an orthonormal basis for $\mathbf{x}_k$) is q_s and is composed by the effective spatial signatures of all K users transmitted by the N_T antennas, that corresponds to a matrix with size $N_TM \times K$, and ISI. The ISI corresponds to a matrix with dimensions $N_T(L_p - 1) \times K$ since for our transmit diversity configuration we have N_T independent multipath channels with a maximum of L_p paths each. Assuming that $K < N_TN$, we have that if $K + 2\min\{N_T(L_p - 1), K\} \leq N_TM - N_TL_p + 1 = N_T(N-1) + 1$ then the necessary condition on q_s is satisfied and an upper bound for the maximum load of the system with N_T transmit antennas is

$$K \leq N_T\Big[N - \frac{(N_T - 1)}{N_T} - 2\min\Big\{\Big(\frac{N}{3} - \frac{(N_T - 1)}{3N_T}\Big), L_p - 1\Big\}\Big] \tag{31}$$

The above result indicates an increase in the system capacity as compared to the result $K \leq N - 2\min\{N/3, L_p - 1\}$ for a single antenna system reported in [24]. In order to increase the capacity, a designer should choose $N_T \geq 2$.

Now, we are interested in establishing the sufficient conditions for the identifiability of $\boldsymbol{g_k}$. We will follow the approach reported in [23]. Let us first consider the effective space-time signatures $\boldsymbol{\mathcal{C}_k g_k}$ and $\boldsymbol{\bar{\mathcal{C}}_k g_k^*}$, and rewrite them as

$$\boldsymbol{\mathcal{C}_k g_k} = \mathbf{v}_k = \boldsymbol{\mathcal{X}}\mathbf{e}_k/A_k, \quad \boldsymbol{\bar{\mathcal{C}}_k g_k^*} = \bar{\mathbf{v}}_k = \boldsymbol{\bar{\mathcal{X}}}\mathbf{e}_k/A_k, \quad (32)$$

where $\mathbf{e}_k = [\underbrace{0, \ \ldots, \ 0}_{k-1}, \ 1, \ \underbrace{0, \ \ldots, \ 0}_{2M-k}]^T$. From the above we have $\dim\{\text{range}(\boldsymbol{\mathcal{C}_k}) \cap \text{range}(\boldsymbol{\mathcal{X}})\} = 1, \ \dim\{\text{range}(\boldsymbol{\bar{\mathcal{C}}_k}) \cap \text{range}(\boldsymbol{\bar{\mathcal{X}}})\} = 1$ where $\dim\{\cdot\}$ stands for the dimension of a subspace and $\boldsymbol{\mathcal{X}} = \sum_{k=1}^{K} E[\mathbf{x}_k(i)\mathbf{x}_k^H(i)]$ and $\boldsymbol{\bar{\mathcal{X}}} = \sum_{k=1}^{K} E[\bar{\mathbf{x}}_k(i)\bar{\mathbf{x}}_k^H(i)]$. The following theorem establishes the identifiability result for the proposed method.

Theorem: Let $\boldsymbol{\mathcal{C}_k}$, $\boldsymbol{\mathcal{X}}$, $\boldsymbol{\bar{\mathcal{C}}_k}$ and $\boldsymbol{\bar{\mathcal{X}}}$ be full-column rank matrices and α be an arbitrary scalar. Then the equations

$$\mathbf{V}_n^H \boldsymbol{\mathcal{C}_k \tilde{g}} = \mathbf{0}, \quad \mathbf{V}_n^H \boldsymbol{\bar{\mathcal{C}}_k \tilde{g}^*} = \mathbf{0}, \quad (33)$$

have a nontrivial solution other than $\alpha \boldsymbol{g_k}$ if and only if the following condition holds:

Condition: There exists the vectors $\boldsymbol{\tilde{g}} \neq \alpha \boldsymbol{g_k}$, $\mathbf{q}_k$ and $\bar{\mathbf{q}}_k$ such that $\boldsymbol{\mathcal{C}_k \tilde{g}} = \boldsymbol{\mathcal{X}}\mathbf{q}_k, \quad \boldsymbol{\bar{\mathcal{C}}_k \tilde{g}} = \boldsymbol{\bar{\mathcal{X}}}\bar{\mathbf{q}}_k$ which are equivalent to $\dim\{\text{range}(\boldsymbol{\mathcal{C}_k}) \cap \text{range}(\boldsymbol{\mathcal{X}})\} = 1, \quad \dim\{\text{range}(\boldsymbol{\bar{\mathcal{C}}_k}) \cap \text{range}(\boldsymbol{\bar{\mathcal{X}}})\} = 1$ respectively.

*P*roof: The equations in (33) can be combined together and written as follows $\mathbf{V}_n^H \boldsymbol{\mathcal{C}_k \tilde{g}} + \mathbf{V}_n^H \boldsymbol{\bar{\mathcal{C}}_k \tilde{g}^*} = \mathbf{0}$. Let the latter condition be satisfied with some $\boldsymbol{\tilde{g}} \neq \alpha \boldsymbol{g_k}$. As the matrix $[\boldsymbol{\mathcal{X}} \ \boldsymbol{\bar{\mathcal{X}}}]$ is full-column rank, the relation $\mathbf{V}_n^H \boldsymbol{\mathcal{C}_k \tilde{g}} + \mathbf{V}_n^H \boldsymbol{\bar{\mathcal{C}}_k \tilde{g}^*} = \mathbf{0}$ directly yields $[(\boldsymbol{\mathcal{C}_k \tilde{g}})^T \ (\boldsymbol{\bar{\mathcal{C}}_k \tilde{g}^*})^T] \in \text{range}(\boldsymbol{\mathcal{X}} \ \boldsymbol{\bar{\mathcal{X}}})$. Hence, from the above we have $\boldsymbol{\mathcal{C}_k \tilde{g}} = \boldsymbol{\mathcal{X}}\mathbf{q}_k, \quad \boldsymbol{\bar{\mathcal{C}}_k \tilde{g}} = \boldsymbol{\bar{\mathcal{X}}}\bar{\mathbf{q}}_k$. The relations $\dim\{\text{range}(\boldsymbol{\mathcal{C}_k}) \cap \text{range}(\boldsymbol{\mathcal{X}})\} = 1$ and $\dim\{\text{range}(\boldsymbol{\bar{\mathcal{C}}_k}) \cap \text{range}(\boldsymbol{\bar{\mathcal{X}}})\} = 1$ follows from $\boldsymbol{\mathcal{C}_k \tilde{g}} = \boldsymbol{\mathcal{X}}\mathbf{q}_k, \quad \boldsymbol{\bar{\mathcal{C}}_k \tilde{g}} = \boldsymbol{\bar{\mathcal{X}}}\bar{\mathbf{q}}_k$ along with (32) and the fact that the vectors $\boldsymbol{g_k}$ and $\boldsymbol{\tilde{g}}$ are linearly independent. This proves the condition in $\boldsymbol{\mathcal{C}_k \tilde{g}} = \boldsymbol{\mathcal{X}}\mathbf{q}_k, \quad \boldsymbol{\bar{\mathcal{C}}_k \tilde{g}} = \boldsymbol{\bar{\mathcal{X}}}\bar{\mathbf{q}}_k$. Now, to prove the sufficiency part, let us assume that the condition $\boldsymbol{\mathcal{C}_k \tilde{g}} = \boldsymbol{\mathcal{X}}\mathbf{q}_k, \quad \boldsymbol{\bar{\mathcal{C}}_k \tilde{g}} = \boldsymbol{\bar{\mathcal{X}}}\bar{\mathbf{q}}_k$ holds. From the latter, we have $\boldsymbol{\mathcal{C}_k \tilde{g}} = \boldsymbol{\mathcal{X}}\mathbf{q}_k$ and $\boldsymbol{\bar{\mathcal{C}}_k \tilde{g}} = \boldsymbol{\bar{\mathcal{X}}}\bar{\mathbf{q}}_k$ and $[(\boldsymbol{\mathcal{C}_k \tilde{g}})^T \ (\boldsymbol{\bar{\mathcal{C}}_k \tilde{g}^*})^T] \in \text{range}(\boldsymbol{\mathcal{X}} \ \boldsymbol{\bar{\mathcal{X}}})$. It follows that $\boldsymbol{\tilde{g}} \neq \alpha \boldsymbol{g_k}$ is a solution to $\mathbf{V}_n^H \boldsymbol{\mathcal{C}_k \tilde{g}} + \mathbf{V}_n^H \boldsymbol{\bar{\mathcal{C}}_k \tilde{g}^*} = \mathbf{0}$. This completes the proof.

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