Abstract—This work proposes adaptive buffer-aided distributed space-time coding schemes and algorithms with feedback for wireless networks equipped with buffer-aided relays. The proposed schemes employ a maximum likelihood receiver at the destination, and adjustable codes subject to a power constraint with an amplify-and-forward cooperative strategy at the relays. The adjustable codes are part of the proposed space-time coding schemes and the codes are sent back to relays after being updated at the destination via feedback channels. Each relay is equipped with a buffer and is capable of storing blocks of received symbols and forwarding the data to the destination if selected. Different antenna configurations and wireless channels, such as static block fading channels, are considered. The effects of using buffer-aided relays to improve the bit error rate (BER) performance are also studied. Adjustable relay selection and optimization algorithms that exploit the extra degrees of freedom of relays equipped with buffers are developed to improve the BER performance. We also analyze the pairwise error probability and diversity of the system when using the proposed schemes and algorithms in a cooperative network. Simulation results show that the proposed schemes and algorithms obtain performance gains over previously reported techniques.

Index Terms—Buffer-aided communications, cooperative communications, distributed space-time coding.

I. INTRODUCTION

COOPERATIVE relaying systems, which employ relay nodes with an arbitrary number of antennas between the source node and the destination node as a distributed antenna array, can obtain diversity gains by employing space-time coding (STC) schemes to improve the reliability of wireless links [1], [7]. In existing cooperative relaying systems, amplify-and-forward (AF), decode-and-forward (DF) or compress-and-forward (CF) [1] cooperation strategies are often employed with the help of multiple relay nodes.

The adoption of distributed space-time coding (DSTC) schemes at relay nodes in a cooperative network, providing more copies of the desired symbols at the destination node, can offer the system diversity and coding gains which enable more effective interference mitigation and enhanced performance. A recent focus of DSTC techniques lies in the design of full-diversity schemes with minimum outage probability [2]–[6]. In [2], the generalized ABBA (GABBA) STC scheme has been extended to a distributed multiple-input and multiple-output (MIMO) network with full-diversity and full-rate, while an optimal algorithm for the design of DSTC schemes that achieve the optimal diversity and multiplexing tradeoff has been derived in [3]. A quasi-orthogonal distributed space-time block coding (DSTBC) scheme for cooperative MIMO networks is presented and shown to achieve full rate and full diversity with any number of antennas in [6]. In [20], an STC scheme that multiplies a randomized matrix by the STC code matrix at the relay node before the transmission is derived and analyzed. The randomized space-time coding (RSTC) schemes can achieve the performance of a centralized STC scheme in terms of coding gain and diversity order. The intuition behind RSTC is to let each relay transmit an independent random linear combination of the columns of an STC matrix, where each node transmits signals with random gains and phases. A detailed study of randomized matrices has been reported in [20], where the criterion based on a uniform spherical randomized matrix that contains uniformly distributed elements on the surface of a complex hyper-sphere of radius $\rho$ has been shown to achieve the best BER performance.

Relay selection algorithms such as those designed in [7], [8] provide an efficient way to assist the communication between the source node and the destination node. Although the best relay node can be selected according to different optimization criteria, conventional relay selection algorithms often focus on the best relay selection (BRS) scheme [9], which selects the links with maximum instantaneous signal-to-noise ratio (SNR). The best relay forwards the information to the destination which results in an improved BER performance. Recently, cooperative schemes with more general configurations involving a source node, a destination node and multiple relays equipped with buffers has been introduced and analyzed in [10]–[18]. The main idea is to select the best link during each time slot according to different criteria, such as maximum instantaneous SNR and maximum throughput. In [10], an introduction to buffer-aided relaying networks is given, and further analysis of the throughput and diversity gain is provided in [11]. In [12] and [13], an adaptive link selection protocol with buffer-aided relays is proposed and an analysis of the network throughput and the outage probability is developed. A max-link relay selection scheme focusing on achieving full diversity gain, which selects the strongest link in each time slot is proposed in [14]. A max-max relay selection algorithm is proposed in [16] and has been extended to mimic a full-duplex relaying scheme in [15] with the help of buffer-aided relays. Recent work on relay selection strategies and power allocation algorithms has been reported in [17] and [18]. In Luo and Teh’s work an optimal relay selection algorithm is designed based on the status of the...
buffer, whereas Nomikos et al. [18] have investigated optimal power allocation and interference cancelation between relays.

Despite the early work with buffer-aided relays and its performance advantages, schemes that employ STC techniques have not been considered so far. In particular, STC and DSTC schemes encoded at the relays can provide higher diversity order and higher reliability for wireless systems. In this work, we propose adjustable buffer-aided distributed and non-distributed STC schemes, relay selection and adaptive buffer-aided relaying optimization (ABARO) algorithms for cooperative relaying systems with feedback. We examine two basic configurations of relays with STC and DSTC schemes: one in which the coding is performed independently at the relays [20], denoted multiple-antenna system (MAS) configuration, and another in which coding is performed across the relays [6], called single-antenna system (SAS) configuration.

According to the literature, STC schemes can be implemented at a single relay node with multiple antennas and DSTC schemes can be used at multiple relay nodes with a single antenna. Moreover, an adjustable STC scheme is developed in [21] which indicates that by using an adjustable coding vector at single-antenna relay nodes, a complete STC scheme can be implemented. In this work, we consider a STC scheme implemented at a multiple-antenna relay node and a DSTC scheme applied at a group of single-antenna relay nodes along with adjustable STC and DSTC schemes at both types of relays.

Compared to relays without buffers, buffer-aided relays help mitigate deep fading periods during communication between devices as the received symbols can be stored at the relays, which contributes to a significant BER performance improvement. Although the delay is a key issue for buffer-aided relays, their key advantage is to improve the error tolerance and transmission accuracy of the links in the network. Buffer-aided relay schemes can be used in networks in which the delay is not an issue and with delay tolerance.

The proposed schemes, relay selection and ABARO optimization algorithms can be structured into two parts, the first one is the relay selection part which chooses the best link with the maximum instantaneous SNR or signal-to-interference-plus-noise ratio (SINR) and checks if the state of the best relay node is available to transmit or receive, and the second part refers to the optimization of the adjustable STC schemes employed at the relay nodes. The adaptive buffer-aided relaying optimization (ABARO) algorithm is based on the maximum-likelihood (ML) criterion subject to constraints on the transmitted power at the relays for different cooperative systems. STC schemes are employed at each relay node and an ML detector is employed at the destination node in order to ensure full receive diversity. Suboptimal detectors can be also used at the destination node to reduce the detection complexity. Moreover, stochastic gradient (SG) adaptive algorithms [19] are developed in order to compute the required parameters at a reduced computational complexity. We study how the adjustable codes can be employed at buffer-aided relays combined with relay selection and how to optimize the adjustable codes by employing an ML criterion. A feedback channel is required in the proposed scheme and algorithms. All the computations are done at the destination node so that the useful information, such as relay selection information and optimized coding matrices are assumed known. We have studied the impact of feedback errors in [21], however, in this work we focus on the effects of using the proposed buffer-aided relay schemes, relay selection and optimization algorithms. The feedback is assumed to be error-free and the devices are assumed to have perfect or statistical channel state information (CSI). The proposed relay selection and optimization algorithms can be implemented with different types of STC and DSTC schemes in cooperative relaying systems with DF or AF protocols. We first study the design of adjustable STC schemes and relay selection algorithms for single-antenna systems and then extend it to multiple-antenna systems, which allow further diversity gains or multiplexing gains. The proposed algorithms and schemes are also considered with DSTC schemes. In single-antenna networks, DSTC schemes are used with an arbitrary number of relays and a group of relays is selected to implement the DSTC scheme. In multiple-antenna networks, a complete DSTC scheme can be obtained at each relay node and a superposition of multiple DSTC transmissions is received at the destination.

This paper is organized as follows. Section II introduces a cooperative two-hop relaying systems with multiple buffer-aided relays applying the AF strategy in SAS and MAS configurations, respectively. In Section III the detailed adjustable STC scheme is introduced. The proposed relay selection and code optimization algorithms are derived in Section IV and the DSTC schemes are considered in Section V. The analysis of the proposed algorithms is shown in Section VI, whereas in Section VII we provide the simulation results. Section VIII gives the conclusions of the work.

Notation: the italic, the bold lower-case and the bold upper-case letters denote scalars, vectors and matrices, respectively. The operator $\|X\|_F = \sqrt{\text{Tr}(X^H \cdot X)} = \sqrt{\text{Tr}(X \cdot X^H)}$ is the Frobenius norm. $\text{Tr}(\cdot)$ stands for the trace of a matrix, and the $N \times N$ identity matrix is written as $I_N$.

II. COOPERATIVE SYSTEM MODELS

In this section, we introduce the cooperative system models adopted to evaluate the proposed schemes and algorithms. We consider two relay configurations: SAS in which each node contains only a single antenna and MAS in which each node contains multiple antennas. The feedback scheme consists of information conveyed from the destination node to the relay nodes, which includes indices representing the buffer entries and the delays, and the parameters of the optimized coding matrices. We focus on the relay selection and adjustable code matrices optimization algorithms so that we assume that perfect or statistical CSI is available at the relays and destination nodes and perfect synchronization of all nodes. However, we remark that that CSI can be obtained in practice by using pilot sequences and cooperative channel estimation algorithms [22], [23].

A. Cooperative System Models for SAS

In this section, we consider a two-hop system, which is shown in Fig. 1 and consists of a source node, a destination...
node and \( n_r \) relays. Each node contains a single antenna. Let 
\( s[j] \) denote a block of modulated data symbols with length 
of \( M \) and covariance matrix \( E[s[j]s[j]^H] = \sigma_v^2 I_M \), where \( \sigma_v^2 \) 
denotes the signal power and \( j \) is the index of the blocks. We 
assume that the channels are static over the transmission period 
of \( s[j] \). The minimum buffer size is equal to the size of one 
block of symbols, \( M \), and the maximum buffer size is \( M J \), 
where \( J \) is the maximum number of symbol blocks. In the first 
hop, the source node sends the modulated symbol vector \( s[j] \) to 
the relay nodes and the received data are given by 
\[
\begin{align*}
    r_{sr}[j] &= \sqrt{P_s} f_{sr}[j] s[j] + n_{sr}[j], \\
    k &= 1, 2, \ldots, n_r, j = 1, 2, \ldots, J,
\end{align*}
\]
where \( f_{sr}[j] \) denotes the CSI between the source node and 
the \( k \)th relay, and \( n_{sr}[j] \) stands for the \( M \times 1 \) additive white 
Gaussian noise (AWGN) vector generated at the \( k \)th relay with 
variance \( \sigma_v^2 \). The transmit power assigned at the source node is 
denoted as \( P_s \). At the relay nodes, in order to implement an STC 
scheme the received symbols are divided into \( i = M/N \) groups, 
where \( N \) denotes the number of symbols required to encode an 
STC scheme and whose value is different according to the STC 
adopted, e.g. \( N = 2 \) for the \( 2 \times 2 \) Alamouti STBC scheme and \( N = 4 \) for the linear dispersion code (LDC) scheme in [24].

The transmission in the second hop is expressed as follows: 
\[
\begin{align*}
    r_{rd}[i] &= \sqrt{P_r} g_{rd}[i] c_{rand}[i] + n_{rd}[i], \\
    k &= 1, 2, \ldots, n_r, i = 1, 2, \ldots, M/N,
\end{align*}
\]
where \( g_{rd}[i] \) denotes the \( i \)th \( T \times 1 \) received symbol vec-
tor. The \( T \times 1 \) adjustable STC scheme is denoted by \( c_{rand}[i] \), 
and \( g_{rd}[i] \) denotes the CSI between the \( k \)th relay and 
the destination node. The transmit power assigned at the 
relay node is denoted as \( P_r \). The vector \( n_{rd}[i] \) stands for the 
AWGN vector generated at the destination node with variance 
\( \sigma_v^2 \). It is worth mentioning that during the transmission period 
of each group the channel is static. The details of adjustable 
STC encoding and decoding procedures are given in the next 
section.

### B. Cooperative System Models for MAS

In this section, we extend the single-antenna system model 
to a two-hop multiple-antenna system that is shown in Fig. 2 
Each node contains \( N \geq 2 \) antennas. Let \( s[j] \) denote a modu-
lated data symbol vector with length \( M \), which is a block of 
symbols in a packet. The data symbol vector \( s[j] \) can be sent 
from the source to the relays within one time slot since mul-
iple antennas are employed. We assume that the channels are 
static over the transmission period of \( s[j] \) and, for simplicity, 
we assume that \( M = N \) and the minimum buffer size is equal 
to \( M \). In the first hop, the source node sends \( s[j] \) to the relay 
nodes and the received data are described by 
\[
\begin{align*}
    r_{sr}[j] &= \sqrt{P_s} F_{sr}[j] s[j] + n_{sr}[j], \\
    k &= 1, 2, \ldots, n_r, j = 1, 2, \ldots, J,
\end{align*}
\]
where \( F_{sr}[j] \) denotes the \( N \times N \) CSI matrix between the 
source node and the \( k \)th relay, and \( n_{sr}[j] \) stands for the \( N \times 1 \) 
AWGN vector generated at the \( k \)th relay with variance \( \sigma_v^2 \). At 
each relay node, an adjustable code vector is randomly gener-
ated before the forwarding procedure and the received data are 
expressed as: 
\[
\begin{align*}
    R_{rd}[i] &= \sqrt{P_r} G_{rd}[i] c_{rand}[i] + N_{rd}[i], \\
    k &= 1, 2, \ldots, n_r, i = 1, 2, \ldots, M/N,
\end{align*}
\]
where \( c_{rand}[i] \) denotes the \( N \times T \) standard STC scheme with \( T \) 
being the number of codewords and \( V[j] = \text{diag}(v[j]) \) stands 
for the \( N \times N \) diagonal adjustable code matrix whose elements 
are from the adjustable vector \( v = [v_1, v_2, \ldots, v_N] \). The \( N \times T \) 
adjustable code matrix is denoted by \( C_{rand}[j] \). An equivalent 
representation of the received data is given by the received
vector $r_{RD}[j]$, which replaces the received symbol matrix $R_{RD}[j]$ in (4) and is written as
\[
r_{RD}[j] = \sqrt{\frac{P_R P_S}{N}} V_{eq}[j] H[j] s[j] + \sqrt{\frac{P_R}{N}} V_{eq}[j] G_{RD}[j] n_{sr,j}[j] + n_{RD}[j] \\
= \sqrt{\frac{P_R P_S}{N}} V_{eq}[j] H[j] s[j] + n[j].
\]

where $V_{eq}[j] = I_{T \times T} \otimes V[j]$ denotes the $TN \times TN$ block diagonal equivalent adjustable code matrix and $\otimes$ is the Kronecker product, and $H[j]$ stands for the equivalent channel matrix which is the combination of $F_{SR,j}[j]$ and $G_{RD,j}[j]$. The $TN \times 1$ vector $n[j]$ contains the equivalent noise vector at the destination node, which can be modeled as AWGN with zero mean and covariance matrix $\left(\sigma_d^2 + \|V_{eq}[j] G_{RD}[j]\|^2 \sigma_v^2\right) I_{NT}$.

III. ADJUSTABLE SPACE-TIME CODING SCHEME

In this section, we detail the adjustable STC schemes in the SAS and MAS configurations. The encoding procedure of the adjustable coding schemes as compared to standard STC and DSTC schemes is different in the SAS and the MAS configuration, and we describe them in the following.

A. Adjustable Space-Time Coding Scheme for SAS

Here, we develop the procedure of adjustable STC for the SAS configuration. In [20] and [21], adjustable codes are employed to allow relays with a single antenna to transmit STC schemes. In the second hop, the whole packet will be forwarded to the destination node. Due to the consideration of the performance of an $N \times T$ STC scheme, the received packet is divided into $i = M/N$ groups and each group contains $N$ symbols. These $N$ symbols will be encoded by an STC generation matrix and then forwarded to the destination. For example, suppose that a packet contains $M = 100$ symbols and the $2 \times 2$ Alamouti space-time block coding (STBC) scheme is used at the relay nodes. We first split $r_{SR}$ into 50 groups, encode the symbols in the first group by the Alamouti STBC scheme and then multiply a $1 \times 2$ randomized vector $v$. The original $2 \times 2$ orthogonal Alamouti STBC scheme $C$ results in the following code:
\[
c_{rand} = vC = \left[ v_1 v_2 \right] \left[ r_{SR,1} -r_{SR,2} \right] \\
= \left[ v_1 r_{SR,1} v_2 r_{SR,2} \right] \left[ r_{SR,1} -r_{SR,2} \right].
\]

where $r_{SR,1}$ and $r_{SR,2}$ are symbols in the first group, and the $1 \times 2$ vector $v$ denotes the randomized vector whose elements are generated randomly according to different criteria described in [20]. As shown in (6), the $2 \times 2$ STBC matrix changes to a $1 \times 2$ STBC vector which can be transmitted by a relay node with a single antenna in 2 time slots. Different STC schemes such as the LDC scheme in [24] can be easily adapted to the randomized vector encoding in (6). Therefore, the transmission of the randomized STC schemes can be described as:
\[
r = \sqrt{P_T} h_{rand} + n = \sqrt{P_T} h v C + n,
\]

where $h$ denotes the channel coefficient which is assumed to be constant within the transmission time slots, and $n$ stands for the noise vector. The decoding methods of the randomized STC schemes are the same as that of the original STC schemes. At the destination, instead of the estimation of the channel coefficients, the resulting composite parameter vector $vh$ is estimated. As a result, the transmission of a randomized STC vector is similar to the transmission of a deterministic STC scheme over an effective channel. Taking the randomized Alamouti scheme as an example, the linear ML decoding for the information symbols $s_1$ and $s_2$ is given by
\[
\hat{s}_1 = h_{rand}^* v_1 + h_{rand} v_2, \\
\hat{s}_2 = h_{rand}^* v_1 + h_{rand} v_2,
\]

where $h_{rand,1}$ and $h_{rand,2}$ are the randomized channel coefficients in $vh$. Different decoding methods can be employed in this context. In [21], optimization algorithms to compute the randomized code vector $v$ are proposed in order to obtain a performance improvement.

Since the adjustable STC scheme is employed at the relay node, the received vector $r_{RD}[i]$ in (2) can be rewritten as:
\[
r_{RD}[i] = \sqrt{\frac{P_R P_S}{N}} V_{eq}[i] h[i] s[i] + \sqrt{\frac{P_R}{N}} V_{eq}[i] G_{RD}[i] n_{sr,i}[i] + n_{RD}[i] \\
= \sqrt{\frac{P_R P_S}{N}} V_{eq}[i] h[i] s[i] + n[i],
\]

where $V_{eq}[i]$ denotes the $T \times N$ block diagonal equivalent adjustable code matrix, and $h[i] = f_{SR,i}[i] g_{RD,i}[i]$ stands for the equivalent channel. The vector $n[i]$ contains the equivalent noise vector at the destination node, which can be modeled as AWGN with zero mean and covariance matrix $\left(\sigma_d^2 + \|V_{eq}[i] G_{RD,i}\|^2 \sigma_v^2\right) I_{NT}$.

B. Adjustable Space-Time Coding Scheme for MAS

In this section, the details of the adjustable STC encoding procedure in the MAS configuration are given. As mentioned in the previous section, we assume $M = N$ so that in the MAS configuration we do not need to divide the received symbols into different groups to implement the adjustable STC scheme. Take the $2 \times 2$ Alamouti STBC scheme as an example, the adjustable STC scheme is encoded as:
\[
C_{rand} = vC = \left[ v_1 v_2 \right] \left[ r_{SR,1} -r_{SR,2} \right] \\
= \left[ v_1 r_{SR,1} v_2 r_{SR,2} \right] \left[ r_{SR,1} -r_{SR,2} \right].
\]

where $r_{SR,1}$ and $r_{SR,2}$ are the first symbols in the separate groups, and the $2 \times 2$ matrix $V$ denotes the randomized matrix whose elements at the main diagonal are generated randomly according to different criteria described in [20]. The transmission of the randomized STC schemes is described in (4) and the decoding is given in (8).
IV. ADAPTIVE BUFFER-AIDED STC AND RELAY
OPTIMIZATION ALGORITHMS

In this section, the proposed ABARO algorithm in SAS is
derived in detail. The optimization in MAS follows a similar
procedure with different channel vectors so that we will skip
the derivation. The main idea of the proposed algorithm is to
choose the best relay node which contains the highest instan-
taneous SNR for transmission and reception in order to achieve
full diversity order and higher coding gain as compared to stan-
ard STC and DSTC designs. The relay nodes are assumed
to contain buffers to store the received data and forward the
data to the destination over the best available channels. In addi-
tion, the best relay node is always chosen in order to enhance
the detection performance at the destination. As a result, with
buffer-aided relays the proposed ABARO algorithm will result
in improved performance.

Before each transmission, the instantaneous SNR (SNR\textsubscript{ins})
of the SR and RD links are calculated at the destination and
conveyed with the help of signaling and feedback channels [15].
The expressions for the instantaneous SNR of the SR and RD
links are respectively given by

\[
\text{SNR}_{\text{SR}}[i] = \frac{\|f_{SR}[i]\|^2_f}{\sigma_f^2}, \quad \text{SNR}_{\text{RD}}[i] = \frac{\|V_{eq}[i]g_{RD}[i]\|^2_f}{\sigma_d^2},
\]

and the best link is chosen according to

\[
\text{SNR}_{\text{opt}}[i] = \arg \max_{b, k} \text{SNR}_{\text{ins}, b, k}[i], \quad k, b = 1, 2, \ldots, n_r, \quad i = 1, 2, \ldots, M/N.
\]

where \(b\) denotes the occupied number of packets in the buffer.
After the best relay is determined, the transmission described
in (1) and (2) is implemented. The SNR\textsubscript{ins} is calculated first
and then the destination chooses a suitable relay which has
enough room in the buffer for the incoming data. For example,
if the \(k\)th SR link is chosen but the buffer at the \(k\)th relay node
is full, the destination node will skip this node and check the
state of the buffer which has the second best link. In this case
the optimal relay with maximum instantaneous SNR and mini-
imum buffer occupation at a certain SNR level will be chosen
for transmission.

After the detection of the first group of the received symbol
vector at the destination node, the adjustable code \(v\) will be optim-
ized. The constrained ML optimization problem that involves
the detection of the transmitted symbols and the computation
of the adjustable code matrix at the destination is written as

\[
\hat{\mathbf{s}}[i], \hat{V}_{eq}[i] = \arg \min_{\mathbf{s}, \mathbf{V}} \|\mathbf{r}[i] - \sqrt{P_R P_S V_{eq}[i]} h_i \hat{\mathbf{s}}[i]\|^2_f,
\]

\[\text{s.t.} \quad \text{Tr}(V_{eq}[i] V_{eq}^H[i]) \leq P_V, \quad i = 1, 2, \ldots, M/N, \quad (13)\]

where \(\mathbf{r}[i]\) is the received symbol vector in the \(i\)th group and
\(\hat{\mathbf{s}}[i]\) denotes the detected symbol vector in the \(i\)th group. For ex-
ample, if the number of antennas \(N = 4\) and the number
of symbols stored at the buffer is \(M = 8\), we have \(M/N = 2\)
groups of symbols to implement the adjustable STC scheme.

According to the properties of the adjustable code vector, the
computation of \(\hat{\mathbf{s}}[i]\) is the same as the decoding procedure of
the original STC schemes. In order to obtain the optimal code
vector \(\mathbf{v}[i]\), the cost function in (13) should be minimized with
respect to the equivalent code matrix \(V_{eq}[i]\) subject to a con-
straint on the transmitted power. The Lagrangian expression of
the optimization problem in (13) is given by

\[
L = \|\mathbf{r}[i] - \sqrt{P_R P_S V_{eq}[i]} h_i \hat{\mathbf{s}}[i]\|^2_f + \lambda (\text{Tr}(V_{eq}[i] V_{eq}^H[i]) - P_V).
\]

It is worth mentioning that the power constraint expressed in
(13) is ignored during the optimization of the adjustable code
and in order to enforce the power constraint, we introduce a
normalization procedure after the optimization which reduces
the computational complexity. A stochastic gradient algorithm is
used to solve the optimization algorithm in (14) with lower
computational complexity as compared to least-squares algo-
rithms which require the inversion of matrices. By taking the
instantaneous gradient of \(L\), discarding the power constraint
and equating it to zero, we obtain

\[
\nabla L = -\sqrt{P_R P_S} (r[i] - \sqrt{P_R P_S} V_{eq}[i] h_i \hat{\mathbf{s}}[i]) \hat{\mathbf{s}}^H[i] h_i^H.
\]

and the ABARO algorithm for the proposed scheme can be
expressed as follows

\[
V_{eq}[i + 1] = V_{eq}[i] - \mu \sqrt{P_R P_S} (r[i] - \sqrt{P_R P_S} V_{eq}[i] h_i \hat{\mathbf{s}}[i]) \hat{\mathbf{s}}^H[i] h_i^H,
\]

where \(\mu\) is the step size. After the update of the equivalent
coding matrix \(V_{eq}\) in SAS, we can recover the original coding
vector \(v[i]\) from the entries of the main diagonal of \(V_{eq}\). A
normalization of the original code vector \(v[i]\) that circumvents
the power constraint in (13) is given by

\[
v[i + 1] = v[i + 1] \frac{P_V}{\sqrt{\text{Tr}(\hat{\mathbf{s}}[i] \hat{\mathbf{s}}^H[i])}}.
\]

Similarly, the ABARO algorithm in the MAS configuration
is implemented step-by-step as shown in (11) to (17). A
summary of the ABARO algorithm in the MAS configuration
is shown in Table I.

V. BEST RELAY SELECTION WITH DSTC SCHEMES

In this section, we assume that the relays contain buffers and
employ DSTC schemes in the second hop for the SAS and MAS
configurations. In particular, we also present the design of a best
group relay selection algorithm for performance enhancement.
The details of the deployment of DSTC schemes in the MAS
configuration is similar to that in the SAS scheme. Therefore,
we will not repeat it to avoid redundancy. The main difference
between the relay selection algorithm for DSTC schemes as
compared to that for STC schemes is due to the fact that for
DSTC schemes a group of relays is selected. Specifically for
DSTC schemes, the source node broadcasts data to all the relays
and a DF protocol is employed at the relays. After the detec-
tion, the proposed group relay selection algorithm is employed.
TABLE I
SUMMARY OF THE ADAPTIVE BUFFER-AIDED RELAYING OPTIMIZATION ALGORITHM FOR MAS CONFIGURATION

<table>
<thead>
<tr>
<th>Initialization:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty the buffer at the relays,</td>
</tr>
<tr>
<td>for ( j = 1, 2, \ldots )</td>
</tr>
<tr>
<td>if ( j = 1 )</td>
</tr>
<tr>
<td>compute: ( \text{SNR}_{SR_k}[j] = \frac{</td>
</tr>
<tr>
<td>compare: ( \text{SNR}<em>{SR_k}[j] = \arg \min</em>{b} \text{SNR}_{\text{ink}, b}[j], \quad k = 1, 2, \ldots, n_r, \quad b = 1, 2, \ldots, B, )</td>
</tr>
<tr>
<td>( r_{SR_k}[j] = \sqrt{\frac{P_S}{N}} F_{SR_k}[j] s[j] + n_{SR_k}[j]. )</td>
</tr>
<tr>
<td>else</td>
</tr>
<tr>
<td>compute: ( \text{SNR}_{SR_k}[j] = \frac{</td>
</tr>
<tr>
<td>( \text{SNR}_{RD}[j] = \frac{</td>
</tr>
<tr>
<td>compare: ( \text{SNR}<em>{opt}[j] = \max { \text{SNR}</em>{SR_k}[j], \text{SNR}_{RD}[j] } )</td>
</tr>
<tr>
<td>if ( \text{SNR}<em>{max}[j] = \text{SNR}</em>{SR_k}[j] ) &amp; Relay ( k ) is not full</td>
</tr>
<tr>
<td>( r_{SR_k}[j] = \sqrt{\frac{P_S}{N}} F_{SR_k}[j] s[j] + n_{SR_k}[j]. )</td>
</tr>
<tr>
<td>else ( \text{SNR}<em>{max}[j] = \max { \text{SNR}</em>{RD}[j] } ) &amp; Relay ( k ) is not empty</td>
</tr>
<tr>
<td>( r_{RD}[j] = \sqrt{\frac{P_S}{N}} V_{eq}[j] H[j] s[j] + n[j], )</td>
</tr>
<tr>
<td>ML detection:</td>
</tr>
<tr>
<td>( \hat{s}[j] = \arg \min_{s[j]} | r_{RD}[j] - \sqrt{\frac{P_S}{N}} V_{eq}[j] H[j] \hat{s}[j] |^2. )</td>
</tr>
<tr>
<td>Adjustable Matrix Optimization:</td>
</tr>
<tr>
<td>( V_{eq}[j + 1] = V_{eq}[j] - \mu \sqrt{\frac{P_S}{N}} (r_{RD}[j] - \sqrt{\frac{P_S}{N}} V_{eq}[j] H[j] \hat{s}[j]) (H[j] \hat{s}[j])^H. )</td>
</tr>
<tr>
<td>Normalization:</td>
</tr>
<tr>
<td>( V[j + 1] = V[j + 1] \frac{P_S}{|V[j + 1]|^2}. )</td>
</tr>
<tr>
<td>if ( \text{SNR}_{SR_k} ) is full &amp; Relay ( k ) is full</td>
</tr>
<tr>
<td>skip this Relay,</td>
</tr>
<tr>
<td>else ( \text{SNR}_{RD} ) is full &amp; Relay ( k ) is empty</td>
</tr>
<tr>
<td>skip this Relay,</td>
</tr>
<tr>
<td>...repeat...</td>
</tr>
</tbody>
</table>

A. DSTBC Schemes

In this subsection, we detail the DSTBC scheme used in this study. In the SAS configuration, a single antenna is used in each node and the DF protocol is employed at the relay nodes. In the first hop, the source node broadcasts information symbol vector \( s \) to the relay node which is given by

\[
\begin{align*}
\text{Initialization:} & \quad \text{Empty the buffer at the relays,} \\
\text{for } j = 1, 2, \ldots & \\
\text{if } j = 1 & \text{compute: } \text{SNR}_{SR_k}[j] = \frac{|F_{SR_k}[j]|^2}{\sigma_n^2}, \quad k = 1, 2, \ldots, n_r, \\
\text{compare: } \text{SNR}_{SR_k}[j] & = \arg \min_{b} \text{SNR}_{\text{ink}, b}[j], \quad k = 1, 2, \ldots, n_r, \quad b = 1, 2, \ldots, B, \\
\text{else} & \text{compute: } \text{SNR}_{SR_k}[j] = \frac{|F_{SR_k}[j]|^2}{\sigma_n^2}, \quad k = 1, 2, \ldots, n_r, \\
\text{compare: } \text{SNR}_{opt}[j] & = \max \{ \text{SNR}_{SR_k}[j], \text{SNR}_{RD}[j] \}, \quad k = 1, 2, \ldots, n_r, \\
\text{if } \text{SNR}_{max}[j] & = \text{SNR}_{SR_k}[j] \text{ & Relay}_k \text{ is not full} \\
\text{else } \text{SNR}_{max}[j] & = \max \{ \text{SNR}_{RD}[j] \} \text{ & Relay}_k \text{ is not empty} \\
\text{ML detection:} & \hat{s}[j] = \arg \min_{s[j]} \| r_{RD}[j] - \sqrt{\frac{P_S}{N}} V_{eq}[j] H[j] \hat{s}[j] \|^2, \\
\text{Adjustable Matrix Optimization:} & V_{eq}[j + 1] = V_{eq}[j] - \mu \sqrt{\frac{P_S}{N}} (r_{RD}[j] - \sqrt{\frac{P_S}{N}} V_{eq}[j] H[j] \hat{s}[j]) (H[j] \hat{s}[j])^H, \\
\text{Normalization:} & V[j + 1] = V[j + 1] \frac{P_S}{\|V[j + 1]\|^2}, \\
\text{if } \text{SNR}_{SR_k} \text{ is full & Relay}_k \text{ is full} & \text{skip this Relay,} \\
\text{else } \text{SNR}_{RD} \text{ is full & Relay}_k \text{ is empty} & \text{skip this Relay,} \\
\ldots \text{repeat...} \\
\end{align*}
\]

\[
\begin{align*}
r_{SR_k}[j] & = \sqrt{\frac{P_S}{N}} F_{SR_k}[j] s[j] + n_{SR_k}[j], \quad k = 1, 2, \ldots, n_r, \\
& \text{where } s[j] \text{ is a block of symbols with length of } M, \quad f_{SR_k}[j] \text{ denotes the CSI and } n_{SR_k}[j] \text{ stands for the } M \times 1 \text{ AWGN. The} \\
& \text{transmission power assigned at the source node is denoted as } P_S. \text{ After the detection at the } k\text{th node, } \hat{s}_k \text{ can be obtained. The} \\
& \text{relays are then divided into } m = N_{\text{DSTC}}/n_r \text{ groups to implement the DSTC scheme, where } N_{\text{DSTC}} \text{ denotes the number of} \\
& \text{antennas to form the DSTC scheme. It should be noted that} \\
& \text{synchronization at the symbol level and of the carrier phase is} \\
& \text{assumed in this work. If one considers the distributed Alamouti} \\
& \text{STBC as an example, the encoding procedure is detailed in} \\
& \text{Table II, where } s = [s_1, s_2] \text{ denotes the estimated symbols.}
\end{align*}
\]
at relay 1, and $s = [s_1^2, s_2^2]$ denotes the symbols estimated at relay 2. Note that it is assumed that the best relays will be chosen in the second hop and synchronization is perfect so after the relays forward the DSTC schemes to the destination, a composite signal comprising DSTC transmissions from multiple relays is received. The signal received in the second hop is described by

$$r_{RD_m}[j] = \sum_{m=1}^{N_{DSTC}/n} \sqrt{\frac{P_R}{N_{DSTC}}} g_{RD_m}[j] C_m[j] + n_{RD_m}[j],$$

where $r_{RD_m}[j]$ denotes the $T \times 1$ received symbol vector, and $g_{RD_m}[j]$ denotes the $m$th channel coefficients vector. The parameter $M$ denotes the number of symbols stored in the buffers, $m$ denotes the number of relay groups to implement the DSTC scheme and $j$ denotes the DSTC scheme index.

### B. Best Relay Selection With DSTC in SAS

In this subsection, we describe the best relay selection algorithm used in conjunction with the DSTC scheme in the SAS configuration. In particular, the best relay selection algorithm is based on the techniques reported in [9] and [27], however, the approach presented here is modified for DSTC schemes and buffer-aided relay systems. In the first hop, the $M \times 1$ modulated signal vector $s[j]$ is broadcast to the relays during $M$ time slots and the $M \times 1$ received symbol vector $s_{SR}[j]$ is given by

$$s_{SR}[j] = \sqrt{P} f_{SR}[j] s[j] + n[j], \quad k = 1, 2, \ldots, n_r, \quad j = 1, 2, \ldots, J,$$

where $f_{SR}[j]$ denotes the complex scalar channel gain between the $k$th relay and the destination, and the AWGN noise vector $n[j]$ is generated at the $k$th relay node with variance equal to $\sigma_n^2$. The relays are equipped with buffers to store the received symbol vectors and the optimal relays are chosen according to the approach reported in [28] in order to implement the DSTC scheme among the relays. Specifically, all the relays will be divided into $m = N_{DSTC}/n_r$ groups and the best relay group with the highest SINR will be chosen to forward the received symbols. The opportunistic relay selection algorithm is given by

$$\text{SINR}_k[j] = \arg\max_{g_{RD_k}[j]} \frac{\|g_{RD_k}[j]\|^2_F}{\sum_{m=1, m \neq k}^K \|g_{RD_m}[j]\|^2_F + \sigma_d^2},$$

where $g_{RD_k}[j]$ denotes the $1 \times N_{DSTC}$ channel vector between the chosen relays and the destination to implement the DSTC scheme and $K = C_n^{N_{DSTC}}$ denotes all possible relay group combinations. The noise variance is given by $\sigma_d^2$. After the relay group selection, the optimal relay group transmits the DSTC signals to the destination node and the received data at the destination is described by

$$r_{RD_m}[j] = \sqrt{\frac{P_R}{N_{DSTC}}} g_{RD_1}[j] C_m[j] + n_{RD_m}[j].$$

where $C_m[j]$ denotes the DSTC scheme encoded among the chosen relays. The DSTC decoding process is similar to that of the original STC scheme. It is worth mentioning that the adjustable coding schemes can be introduced in DSTC schemes and the optimization of the adjustable code vector will result in a performance improvement. The summary of the ABARO algorithm for DSTC schemes in the SAS configuration is shown in Table III.

### C. Best Relay Selection With DSTC in MAS

The best relay selection algorithm described in the previous section is now extended to the MAS configuration in this subsection. The main difference between the best relay selection for SAS and MAS is the use of multiple antennas at each node. Moreover, the relays equipped with multiple antennas will obtain a complete STC scheme and only one best relay node will be chosen according to the best relay selection algorithm. Assuming $M = N$, each node equips $N \geq 2$ antennas and in the first hop, the $M \times 1$ modulated signal vector $s[j]$ is broadcast to the relays within $T$ time slots and the $M \times 1$ received symbol matrix $r_{SR}[j]$ is given by

$$r_{SR}[j] = \sqrt{\frac{P}{N}} F_{SR}[j] s[j] + n[j], \quad k = 1, 2, \ldots, n_r, \quad j = 1, 2, \ldots, J,$$

where $F_{SR}[j]$ denotes the channel coefficient matrix between the $k$th relay and the destination, and the AWGN noise vector $n[j]$ is generated at the $k$th relay node with variance $\sigma_n^2$. The $N \times 1$ received symbol vector is stored at the relays and the optimal relay will be chosen according to [28]. The opportunistic relay selection algorithm for the DSTC scheme and the MAS configuration is given by

$$\text{SINR}_k[j] = \arg\max_{G_{RD_k}[j]} \frac{\|G_{RD_k}[j]\|^2_F}{\sigma_d^2}, \quad k = 1, 2, \ldots, n_r, \quad j = 1, 2, \ldots, N,$$

where $G_{RD_k}[j]$ denotes the $N \times N$ channel matrix between the $k$th relay and the destination. After the best relay with the maximum SINR is chosen, the data is encoded by the DSTC scheme. The DSTC encoded and transmitted data in the second hop is received at the destination as described by


where $M[j]$ denotes the $N \times T$ DSTC encoded data, $R[j]$ denotes the $N \times T$ received data matrix, and $N[j]$ is the AWGN matrix with variance $\sigma_d^2$. 

<table>
<thead>
<tr>
<th>Relay</th>
<th>1st Time Slot</th>
<th>2nd Time Slot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relay 1</td>
<td>$s_1^{(1)}$</td>
<td>$-s_2^{(1)*}$</td>
</tr>
<tr>
<td>Relay 2</td>
<td>$s_2^{(2)}$</td>
<td>$-s_2^{(2)*}$</td>
</tr>
</tbody>
</table>
TABLE III
SUMMARY OF THE ADAPTIVE BUFFER-AIDED RELAYING OPTIMIZATION ALGORITHM FOR DSTC SCHEMES IN SAS

<table>
<thead>
<tr>
<th>Initialization:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty the buffer at the relays,</td>
</tr>
<tr>
<td>for ( j = 1, 2, \ldots )</td>
</tr>
<tr>
<td>if ( j = 1 )</td>
</tr>
<tr>
<td>( r_{SR_1}(j) = \sqrt{F_S f_{SR_1}(j)} s(j) + n(j) ),</td>
</tr>
<tr>
<td>else</td>
</tr>
<tr>
<td>compute: ( SNR_{SR_1}[j] = \sum_{k=1}^{n_r} \frac{</td>
</tr>
<tr>
<td>( SNR_{RD}[j] = \sum_{k=1}^{n_r} \frac{</td>
</tr>
<tr>
<td>compare: ( SNR_{opt}(j) = \arg \max {SNR_{SR_1}[j], SNR_{RD}[j]} ),</td>
</tr>
<tr>
<td>if ( SNR_{max}(j) = SNR_{SR_1}(j) ) &amp; All the Relays are not full</td>
</tr>
<tr>
<td>( r_{SR_1}(j) = \sqrt{F_S f_{SR_1}(j)} s(j) + n_{SR_1}(j) ),</td>
</tr>
<tr>
<td>else ( SNR_{max}(j) = SNR_{RD}[j] ) &amp; All the Relays are not empty</td>
</tr>
<tr>
<td>( SNR_R(j) = \arg \max {SNR_{RD}[j]} \frac{\sum_{k=1}^{n_r}</td>
</tr>
<tr>
<td>( r_{RD}(j) = \sqrt{F_R f_{RD}(j)} s(j) + n_{RD}(j) ),</td>
</tr>
<tr>
<td>else ( SNR_{SR_1}(j) ) is max &amp; Relay ( k ) is full</td>
</tr>
<tr>
<td>skip this Relay,</td>
</tr>
<tr>
<td>else ( SNR_{RD}(j) ) is max &amp; Relay ( k ) is empty</td>
</tr>
<tr>
<td>skip this Relay,</td>
</tr>
<tr>
<td>...repeat...</td>
</tr>
<tr>
<td>end</td>
</tr>
<tr>
<td>end</td>
</tr>
</tbody>
</table>

VI. ANALYSIS

In this section, we assess the computational complexity of the proposed algorithms, derive the pairwise error probability (PEP) of cooperative systems that employ adaptive STC and DSTC schemes and analyze delay aspects caused by buffers. The expression of the PEP upper bound is adopted due to its relevance to assess STC and DSTC schemes. We also study the effects of the use of buffers and adjustable codes at the relays, and derive analytical expressions for their impact on the PEP. As mentioned in Section II, the adjustable codes are considered in the derivation as it affects the performance by reducing the upper bound of the PEP. Similarly, the buffers store the data and forward it by selecting the best available associated channel for transmission so that the performance improvement is quantified in our analysis. The PEP upper bound of the traditional STC schemes in [25] is used for comparison purposes. The main difference between the PEP upper bound in [25] and that derived in this section lies in the increase of the eigenvalues of the adjustable codes and channels which leads to higher coding gains. The derived upper bound holds for systems with different sizes and an arbitrary number of relay nodes.

A. Computational Complexity Analysis

According to the description of the proposed algorithms in Section IV and V, the SG algorithms reduces the computational complexity by avoiding the channel inversion as compared to the existing algorithms. The computational complexity of the proposed SG adjustable matrix optimization in the SAS and MAS configurations is \((3 + T)N\) and \((3 + T)N^2\), respectively. The main difference between the proposed algorithms in the SAS and MAS configurations is the number of antennas. For example, the computational complexity of SNR in SR and RD links in SAS configuration is \(2N(1 + T)\) according to (11), while the computational complexity of SNR in SR and RD links in the MAS configuration is \(2N^2(1 + T)\). In addition, if a higher-level modulation scheme is employed, larger relay networks and more antennas are used at the relay node, the STC and DSTC schemes and the relay selection algorithm as well as the coding vector optimization algorithm become more complex. For example, if a 4-antenna relay node is employed, the number of multiplications will be increased from 10 when using a 2-antenna relay node to 28, and if 4 single-antenna relay nodes are employed to implement a DSTC scheme the number of multiplications will be increased from 20 to 112.
B. Pairwise Error Probability

Consider an $N \times N$ STC scheme at the relay node with $T$ codewords. The codeword $C^1$ is transmitted and decoded as another codeword $C^j$ at the destination node, where $i = 1, 2, \ldots, T$. According to [25], the probability of error for this code can be upper bounded by the sum of all the probabilities of incorrect decoding, which is given by

$$P_e \leq \sum_{i=2}^{T} P(C^1 \rightarrow C^i). \quad (26)$$

Assuming that the codeword $C^2$ is decoded at the destination node and that we know the channel information perfectly, we can derive the conditional PEP of the STC encoded with the adjustable code matrix $V$ as [26]

$$P(C^1 \rightarrow C^2|V) = Q\left(\sqrt{\frac{NT}{2}}|VG_{RD}(C^1 - C^2)|_F\right), \quad (27)$$

where $G_{RD}$ stands for the channel coefficients matrix. Let $U^H \Lambda_C U$ be the eigenvalue decomposition of $(C^1 - C^2)H(C^1 - C^2)$, where $U$ is a unitary matrix with the eigenvectors and $\Lambda_C$ is a diagonal matrix which contains all the eigenvalues of the difference between two different codewords $C^1$ and $C^2$. Let $Y^H \Lambda_Y Y$ be the eigenvalue decomposition of $(G_{RD}U)^H \Lambda_C G_{RD}U$, where $Y$ is a unitary matrix that contains the eigenvectors and $\Lambda_Y$ is a diagonal matrix with the eigenvalues arranged in decreasing order. The eigenvalue decomposition of $(YY^H)^2YY^H$ is denoted by $W^H \Lambda_W W$, where $W$ is a unitary matrix that contains the eigenvectors and $\Lambda_W$ is a diagonal matrix with the eigenvalues. Therefore, the conditional PEP can be written as

$$P(C^1 \rightarrow C^2|V) = \sum_{n=1}^{NT} \prod_{m=1}^{N} \left(1 - \frac{1}{2} e^{-\frac{\lambda_{Y_n} \lambda_{C_s}}{\lambda_{V_n}}}|\xi_{n,m}|^2\right), \quad (28)$$

where $\lambda_{V_n}$ and $\lambda_{Y_n}$ are the $(n,m)$th element in $V$ and $Y$, respectively, and $\lambda_{C_s}$ are the $n$th eigenvalues in $\Lambda_{V_n}$, $\Lambda_{C_g}$ and $\Lambda_{G_p}$, respectively.

It is important to note that the value of $\lambda_Y$ and $\lambda_{G_p}$ are positive and real because $(G_{RD}U)^H G_{RD}U$ and $(YY^H)^2YY^H$ are Hermitian symmetric matrices. According to [25], an appropriate upper bound assumption of the $Q$ function is $Q(x) \leq \frac{x}{\sqrt{2}}$, thus the upper bound of the PEP for an adaptive STC scheme is given by

$$P_{ev} \leq \frac{1}{2} \exp\left(-\frac{\sqrt{\frac{NT}{4}}}{\sqrt{\frac{N}{2}}}\sum_{m=1}^{N} \lambda_{Y_n} \lambda_{C_s} |\xi_{n,m}|^2\right) \leq \frac{1}{\prod_{n=1}^{NT}(1 + \frac{\sqrt{\frac{NT}{4}}}{\lambda_{Y_n} \lambda_{C_s}})}, \quad (29)$$

By comparing (31) and (32), employing an adjustable code matrix for an STC scheme at the relay node introduces $\lambda_{V_n}$ in the PEP upper bound. The adjustable code matrices are chosen according to the criterion introduced in [20] and the Hermitian matrix $V_n^H V_n$ is positive semi-definite. With the aid of numerical tools, we have found that $\lambda_{V_n}$ is diagonal with one eigenvalue less than 1 and others much greater than 1. We define the coding gain factor $\eta$ which denotes the quotient of the traditional STC PEP and the adjustable STC PEP as described by

$$\eta \triangleq \frac{P_e}{P_{ev}} = \prod_{n=1}^{N} \lambda_{V_n}^{NT} \gg 1. \quad (33)$$

As a result, by using the adjustable code matrices at the relays contributes to a decrease of the BER performance. The effect of employing and optimizing the adjustable code matrix corresponds to introducing coding gain into the STC schemes. The power constraint enforced by (17) introduces no additional power and energy during the optimization. As a result, employing the adjustable code matrices in the MAS and the SAS configurations can provide a decrease in the BER upper bound since the value in the denominator increases without additional transmit power.
D. Effect of Buffer-Aided Relays

In this subsection, the effect of using buffers at the relays is mathematically analyzed. The expression of the PEP upper bound is adopted again in this subsection. The traditional STC scheme is employed in this subsection in order to highlight the performance improvement by using buffers at the relays.

Let \( U^H \Lambda_c U \) be the eigenvalue decomposition of \( (C^1 - C^2)^H (C^1 - C^2) \) and \( Y^H \Lambda_{G_{R,B}} Y \) be the eigenvalue decomposition of \( (G_{R,B}C)U^H G_{R,B} U \), the PEP upper bound of a traditional STC scheme in buffer-aided relays is given by

\[
P_{e_{opt}} \leq E \left[ \frac{1}{2} \exp \left( - \frac{\gamma NT}{4} \sum_{n=1}^{\lambda_{G_{opt}}} \lambda_{C_n} |\xi_{n,m}|^2 \right) \right]
\]

where \( \lambda_{C_n} \) denotes the eigenvalues of the traditional STC scheme and \( \lambda_{G_{opt}} \) denotes the eigenvalue of the channel components. The PEP performance of a traditional STC scheme without buffer-aided relays is given by

\[
P_e \leq \frac{1}{\prod_{n=1}^{\lambda_{G_n}} (1 + \frac{\gamma_{opt} \lambda_{C_n}}{\gamma_{G_n}})^{\lambda_{G_n}}} \approx \left( \frac{\gamma}{\lambda_{G_n}} \right)^{-\lambda_{G_n} NT} \prod_{n=1}^{\lambda_{G_n}} \lambda_{C_n}^{-\lambda_{G_n} NT}
\]

where \( \lambda_{C_n} \) denotes the eigenvalues of the traditional STC scheme and \( \lambda_{G_n} \) denotes the eigenvalue of the channels in the second hop. By comparing (34) and (35), the only difference is the product of the channel eigenvalues. To show the advantage of employing buffer-aided relays, we need to prove that

\[
P_{e_{opt}} < P_e.
\]

Through (38), we have shown that \( P_{e_{opt}} < P_e \) which indicates the BER performance of a system that employs buffer-aided relays is improved as compared to that of a system using relays without buffers. Despite the result in (38), we have not obtained formulas relating \( P_{e_{opt}} \) as a function of the buffer size \( M J \). This is an interesting subject for future work.

E. Delay Aspects

The use of buffer-aided relays improves the performance of wireless links at the expense of a higher delay in the system. In this subsection, we analyze the average delay of the proposed scheme, which is based on the work reported in [29].

We assume that the source always has data to transmit, the delay is mostly caused by the buffer at the relays and relay selection has been performed with the algorithms described in the previous sections. Let \( T_{SAS}[i] \) and \( T_{MAS}[i] \) denote the delay of the packet of \( M \) symbols transmitted by the source and the queue length at time \( i \) for SAS schemes, respectively, and \( T_{MAS}[j] \) and \( T_{MAS}[j] \) denote the delay of the packet of \( M \) symbols transmitted by the source and the queue length at time \( j \) for DSTC schemes, respectively.

According to Little’s law [30], the average delays \( T_{SAS} = E[T_{SAS}[i]] \) and \( T_{MAS} = E[T_{MAS}[j]] \) due to the time the packets are stored in the relay buffer are given by

\[
T_{SAS} = \frac{Q_{SAS}}{R} \text{ time slots,} \quad (39)
\]

\[
T_{MAS} = \frac{Q_{MAS}}{R} \text{ time slots,} \quad (40)
\]

where \( Q_{SAS} = E[Q_{SAS}[i]] \) and \( Q_{MAS} = E[Q_{MAS}[j]] \) are the average queue lengths at the buffer for the SAS and MAS configurations, respectively, and \( R \) is the average arrival rate into the queue, which is assumed fixed.

For simplicity and without loss of generality, we assume the source node transmits one packet of \( M \) symbols at each time slot, i.e., \( R = 1 \) packets/\( J \) symbols/\( J \) slots. We also assume for simplicity that the error probability for the source/relay link \( P_{SR} \) and the relay/destination link \( P_{RD} \) is the same, i.e., \( P = P_{SR} = P_{RD} \).

For a buffer of size \( J \) packets, the average queue length can be expressed as

\[
Q_{SAS} = \sum_{i=0}^{J} i P_{G_i} = J P_{G_i}, \quad (41)
\]

\[
Q_{MAS} = \sum_{j=0}^{J} j P_{G_j} = J P_{G_j}, \quad (42)
\]

where the probability of the buffer states, \( P_{G_i} \) and \( P_{G_j} \), are given in [29] and \( P_{G_j} = P_{full} \) (probability of full buffer) and \( P_{G_i} = P_{empty} \) (probability of empty buffer).

The average arrival rate in the buffer-aided relay is given by

\[
R = (1 - P_{G_j}) P + P_{G_0} P. \quad (43)
\]
Using the above equation, we obtain

\[ T_{\text{SAS}} = \frac{Q_{\text{SAS}}}{R} = \frac{P_{GJ}}{1 - P_{GJ}} P + P_{G0} P \]

\[ = J \text{ packets/slot} = JM \text{ symbols/slot}, \] (44)

\[ T_{\text{MAS}} = \frac{Q_{\text{MAS}}}{R} = \frac{P_{GJ}}{1 - P_{GJ}} P + P_{G0} P \]

\[ = JN \text{ packets/slot} = JMN \text{ symbols/slot}, \] (45)

where \( P_{G0} = P_{GJ} \) which means \( P_{\text{empty}} = P_{\text{full}} \). This analysis shows that the MAS configuration leads to an average delay which is \( N \) times greater than that of the SAS configuration.

VII. SIMULATION

The simulation results are provided in this section to assess the proposed scheme and algorithms in the SAS and the MAS configurations. In this work, we consider the AF protocol with the standard Alamouti STBC scheme and randomized Alamouti (R-Alamouti) scheme in [20]. The BPSK modulation is employed and each link between the nodes is characterized by static block fading with AWGN. The period during which the channel is static is equal to one symbol transmission period in Figs. 4, 5 and 6, whereas in Figs. 3 and 7 such period is equal to one packet size. The packet size is \( M = 100 \) symbols and the number of packets is \( J = 200 \). The effects of different buffer sizes are also evaluated. Different STC schemes can be employed with a simple modification as well as the proposed relay selection and the ABARO algorithms can be incorporated.

We employ \( n_r = 1, 2 \) relay nodes and \( N = 1, 2 \) antennas at each node, and we set the symbol power \( \sigma_s^2 \) to 1.

The upper bounds of the D-Alamouti, the proposed ABARO algorithm and the buffer-aided relays in the SAS configurations are shown in Fig. 3. The theoretical PEP result of a standard SAS configuration, which does not employ STC schemes or buffer-aided relays, is shown as the curve contains the largest decoding errors. By comparing the first two BER curves in Fig. 3 we can conclude that by employing buffers at relays, the decoding error upper bound is decreased. In this case, the effect of using buffers at the relays contributes to reducing the PEP performance dramatically. If the STC scheme is employed at the relays, an increase of diversity order is observed in Fig. 3 By comparing the lower BER curves in Fig. 3, we can see that by employing the ABARO algorithm which optimizes the adjustable matrices after each transmission contributes to a lower error probability upper bound. As shown in the previous section, by employing adjustable code matrices and the proposed ABARO algorithm, an improvement of the coding gain is obtained which improves performance.

The proposed ABARO algorithm with the Alamouti scheme and an ML receiver in the SAS configuration is evaluated with a single-relay system in Figs. 4 and 5. Different buffer sizes are considered at the relay node. A static channel is employed during the simulation and the corresponding period in which the channel is static corresponds to one symbol. The BER results of
associated with the proposed ABARO algorithm is the same as that of using the RSTC scheme at the relay node. Compared to the MMRS algorithm derived in [16] with the same buffer size, the ABARO algorithm achieves a 1 dB to 2 dB improvement.

The proposed ABARO algorithm with the Alamouti scheme and an ML receiver is evaluated in a MAS configuration with two relays in Fig. 7. It is shown in the figure that the buffer-aided relay selection systems achieve 3 dB to 5 dB gains compared to the previously reported relay systems. When the BSR algorithm is considered at the relay node, an improvement of diversity order is shown in Fig. 7 which leads to significantly improved BER performance. According to the simulation results in Fig. 7, a 1 dB gain can be achieved by using the RSTC scheme at the relays as compared to the network using the standard STC scheme at the relay node. When the proposed ABARO algorithm is employed at the relays, a 2 dB saving for the same BER performance as compared to the standard STC encoded system can be observed. The diversity order of using the proposed ABARO algorithm is the same as that of using the RSTC scheme at the relay node.

The impact of imperfect CSI at the destination node is considered for different schemes as shown in Fig. 7. In particular, we verify that a 2 dB loss in SNR for the same BER performance is obtained for BRS with Alamouti and R-Alamouti schemes due to the imperfect CSI employed at the destination node. Moreover, as we introduce errors in the channel parameters in (13)–(16), the accuracy of the code vectors obtained with the ABARO algorithm is affected. However, according to the simulation result, a 1 dB loss in SNR for the same BER is observed in Fig. 7 due to the channel errors. The proposed ABARO algorithm is able to maintain the BER performance gain in the presence of imperfect CSI at the destination node.

In Fig. 8, we show the average delay for buffers of finite size for different values of \( J \), where we compare simulation and analytical results. We assume the links are i.i.d. In particular, we observe that as the buffer size increases, the average delay with finite buffer size linearly increases and that the average delay of SAS is twice lower than that of MAS for a system with Alamouti codes. This is expected because the MAS configuration requires \( N \) times longer to encode the data at the relays. We also verify that the simulation and analytical results are in good agreement.

**VIII. Conclusion**

We have proposed a buffer-aided space-time coding scheme, relay selection and the adaptive buffer-aided relaying optimization (ABARO) algorithms for cooperative systems with limited feedback using an ML receiver at the destination node to achieve a better BER performance. Simulation results have illustrated the advantage of using the adjustable STC and DSTC schemes in the buffer-aided cooperative systems compared to the best relay selection algorithms. In addition, the proposed ABARO algorithm can achieve a better performance in terms of lower BER at the destination node as compared to prior art. The ABARO algorithm can be used with different STC schemes and can also be extended to cooperative systems with any number of antennas.
REFERENCES


