**Stochastic Processes**

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List 2

1. Consider the throw of 2 dice and the experiment whose result consists of the sum of the number of points in the faces turned up of the dice.

a) Define this sum as a random variable.

b) Sketch the cumulative distribution function.

c) Compute the probability of x to take on values in the interval [7 , 9].

2. Consider the lifetime of a type of lamp in hours. This lifetime can be modeled by a random variable t with probability density function given by

a) Determine and sketch the cumulative density function of the random variable t.

b) Suppose that the probability that the lifetime of the lamp exceeds 200 hours is , compute the value such that the probability that the lifetime of the lamp is less than a is 0,1.

3. Show that if x is a random variable with an exponential probability density function given by

then we have

4. A factory decided to install a voltage regulator to compensate for variations in the voltage of the local network. Consider v the variable that characterizes output voltage of the regulator. In particular, the voltage regulator works well if the temperature t degrees (in Celsius) lies in the interval [10, 40]. For this reason, when the temperature is between 10 and 40 degrees the output voltage v can be considered constant and equal to V0>0. If the temperature is not within this interval, the output voltage v can be modeled by a Gaussian random variable with parameters m = V0 and σ = V0/4.

a) Determine and sketch the probability density function of the random variable v.

b) Consider that the temperature t can be modeled by a Gaussian random variable with parameters m = 30 and σ= 5, compute the probability that output voltage is less than V0/2.

5. Consider the joint probability density function given by

a) Compute

b) Are x and y are statistically independent random variables?

c) Compute the probability that x is negative when y is positive.

d) Calculate the probability that x is greater than y.

6. An electronics factory produces two types of sensors: A e B. Consider that the lifetime of sensors A and B can be modeled by the random variables x and y, respectively. Now suppose that through experiments we have found that the joint probability density function of x and y is given by

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a) Determine the probability that sensor B fails before sensor A.

b) Are the random variables x and y statistically independent? Explain in detail.

7. The joint probability density function of 2 random variables x and y is described by

a) Determine and .

b) Are the random variables x and y statistically independent? Explain in detail.

8. A factory yields two types of devices. Type A devices occur with probability α and work for a relatively short time that is geometrically distributed with parameter r. Type B devices work much longer, occur with probability 1- α and have a lifetime that is geometrically distributed with parameter s. Let X be the lifetime of an arbitrary device.

a) Write down the conditional probabilities of X given the type of device.

b) Find the probability mass function of X.