Distributed conjugate gradient strategies for distributed estimation over sensor networks

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Introduction

- Distributed processing has become popular in wireless communication networks. It can collect data at each node in a given area and convey the information to the whole network in a distributed way.
- When compared with the centralized solution, the distributed solution has significant advantages.
- This kind of strategy can significantly reduce the amount of processing and the communications bandwidth requirements.
- The problem we are interested in solving: distributed estimation over sensor networks.
- We propose distributed CG algorithms with both incremental and diffusion adaptive strategies for distributed estimation over sensor networks.

System model and problem statement

- We focus on the processing of an adaptive filter for adjusting the weight vector $\boldsymbol{\omega}_o$ with coefficients $\boldsymbol{\omega}_k$.

- The desired signal of each node at time instant i:

$$d^{(i)} = \boldsymbol{\omega}_0^H \boldsymbol{x}^{(i)} + n^{(i)}, \quad i = 1, 2, \ldots, N,$$

- The output of the adaptive filter for each node:

$$y^{(i)} = \boldsymbol{\omega}^{(i)H} \boldsymbol{x}^{(i)}, \quad i = 1, 2, \ldots, N,$$

- The problem: minimize the cost function J(ω)

$$J(\boldsymbol{\omega}) = \boldsymbol{E}[||\boldsymbol{d} - \boldsymbol{X}\boldsymbol{\omega}||^2],$$

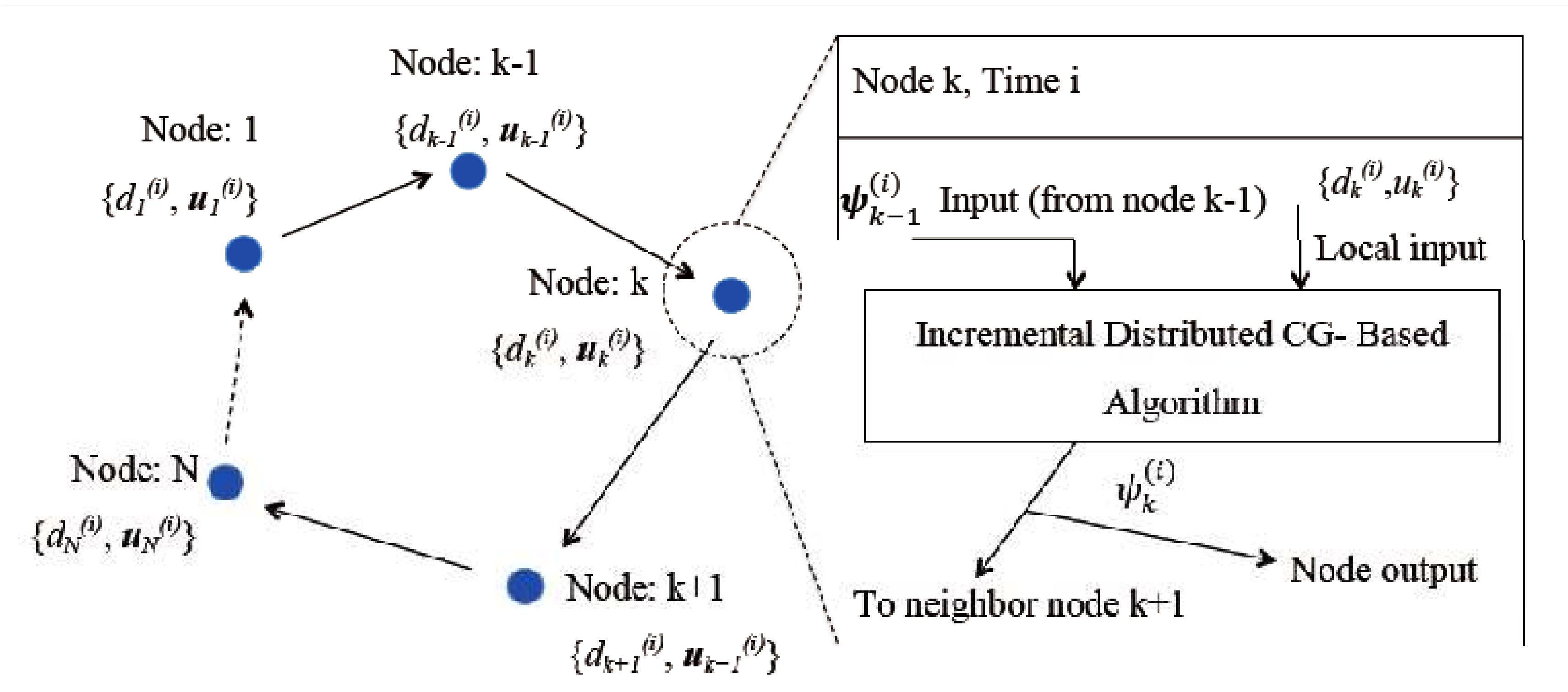
where

$$\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots \boldsymbol{x}_N], \ (N \times M)$$

$$\boldsymbol{d} = [d_1, d_2, \ldots d_N]^T, \ (N \times 1)$$

Proposed incremental distributed CG – based algorithms(1/3)

- Incremental distributed CG- based network processing:



Proposed incremental distributed CG – based algorithms(2/3)

Incremental distributed conventional CG

- It solves the following equation:

$$\boldsymbol{R}_k^{(i)}\boldsymbol{\omega}_k^{(i)} = \boldsymbol{b}_k^{(i)}$$

- In the CG- based algorithm, the iteration procedure is introduced. For the jth iteration, we choose the negative direction as:

$$\boldsymbol{g}_k^{(i)}(j) = \boldsymbol{b}_k^{(i)} - \boldsymbol{R}_k^{(i)}\boldsymbol{\omega}$$

- The CG-based weight vector is defined as:

$$\boldsymbol{\omega}_k^{(i)}(j) = \boldsymbol{\omega}_k^{(i)}(j-1) + \alpha_k^{(i)}(j)\boldsymbol{p}_k^{(i)}(j)$$

- The direction vector is defined as:

$$\boldsymbol{p}_{k+1}^{(i)} = \boldsymbol{g}_k^{(i)} + \beta_k^{(i)}\boldsymbol{p}_k^{(i)}$$

where $\beta_k^{(i)}(j)$ is calculated by the Gram Schmidt orthogonalization procedure for the conjugacy

- ’Exponentially decaying data window’ is introduced to define the correlation and cross-correlation matrices

$$\boldsymbol{R}_k^{(i)} = \lambda_f \boldsymbol{R}_{k-1}^{(i)} + \boldsymbol{x}_k^{(i)}[\boldsymbol{x}_k^{(i)}]^H$$

$$\boldsymbol{b}_k^{(i)} = \lambda_f \boldsymbol{b}_{k-1}^{(i)} + d_k^{(i)*}\boldsymbol{x}_k^{(i)}$$

Proposed incremental distributed CG – based algorithms(3/3)

Incremental distributed modified CG:

- We redefine the negative gradient vector with a recursive expression:

$$g_k^{(i)} = b_k^{(i)} - R_k^{(i)}\omega_k^{(i)} = \lambda_f g_{k-1}^{(i)} - \alpha_k^{(i)} R_k^{(i)} p_k^{(i)} + x_k^{(i)}[d_k^{(i)} - x_k^{(i)H}\omega_{k-1}^{(i)}]$$

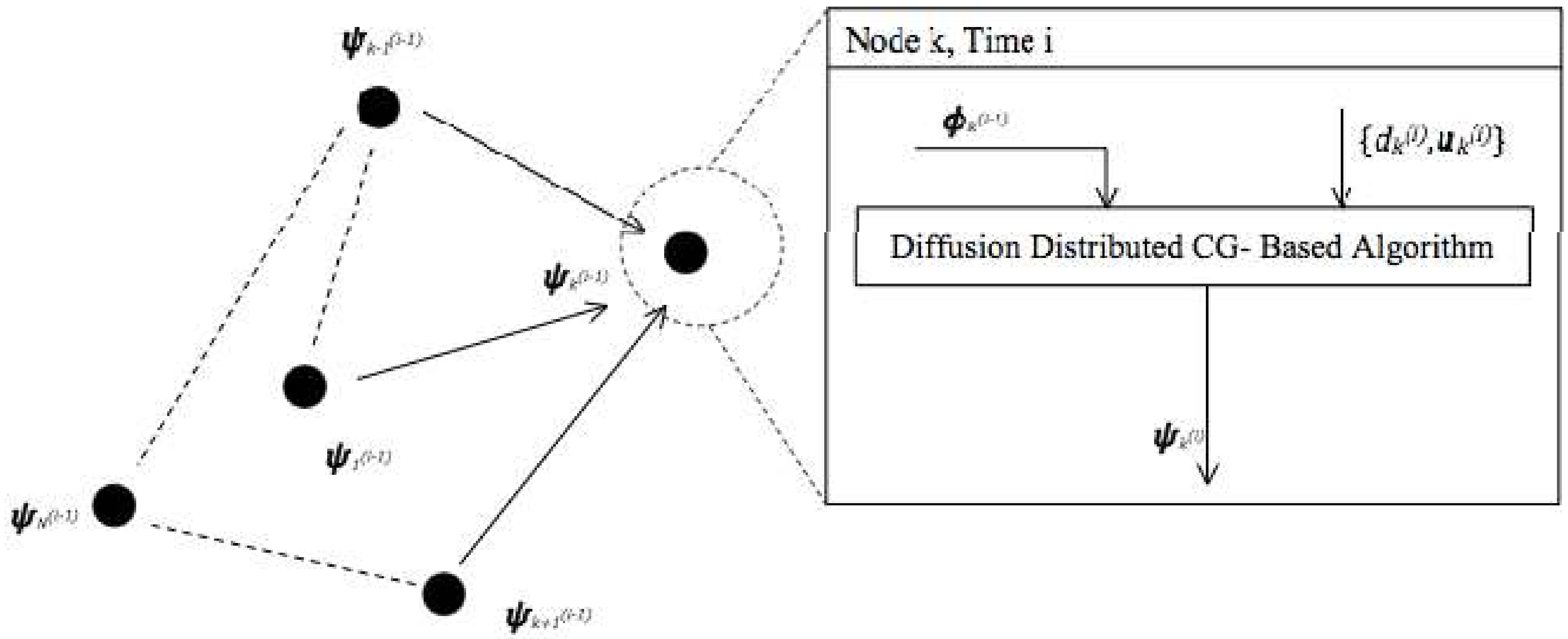
- The direction vector is defined as:

$$p_{k+1}^{(i)} = g_k^{(i)} + \beta_k^{(i)} p_k^{(i)}$$

where β_k is computed to avoid the residue produced by using the Polak-Ribiere approach

Proposed diffusion distributed CG – based algorithms(1/2)

- Diffusion distributed CG- based network processing



Proposed diffusion distributed CG – based algorithms(2/2)

- Local estimates are combined at node k as:

$$\phi_k^{(i-1)} = \sum_{l \in N_{k,i-1}} c_{kl} \psi_l^{(i-1)}$$

where C_{kl} should be satisfied:

$$\sum_l c_{kl} = 1, l \in N_{k,i-1} \forall k$$

- The combiner C is defined through the Metropolis rule:

$$\begin{cases} c_{kl} = \frac{1}{max(n_k, n_l)}, & \text{if } k \neq l \text{ are linked} \\ c_{kl} = 0, & \text{for } k \text{ and } l \text{ not linked} \\ c_{kk} = 1 - \sum_{l \in N_k / k} c_{kl}, & \text{for } k = l \end{cases}$$

- The unbiased estimates for node k are calculated as:

$$\psi_k^{(i)}(j) = \phi_k^{(i-1)}(j) + \alpha_k^{(i)}(j) p_k^{(i)}(j)$$

Analysis of the algorithms: incremental distributed CG

- The computational complexity is used to analyse the proposed incremental distributed CG algorithms

Algorithm	Additions	Multiplications
IDCCG	m^2+m $+J(m^2+6m-4)$	$2m^2+2m$ $J(m^2+7m+3)$
IDMCG	$2m^2+10m-4$	$3m^2+12m+3$
IDLMS	$4m-1$	$3m+1$
IDRLS[3]	$4m^2+12m+1$	$4m^2+12m-1$

Analysis of the algorithms: diffusion distributed CG

- The computational complexity is used to analyse the proposed diffusion distributed CG algorithms

Algorithm	Additions	Multiplications
DDCCG	$m^2 + m$ $+J(m^2 + 6m$ $+Lm - 4)$	$2m^2 + 2m$ $+J(m^2 + 7m$ $+Lm + 3)$
DDMCG	$2m^2 + 10m - 4$ $+Lm$	$3m^2 + 12m + 3$ $+Lm$
DDLMS	$4m - 1 + Lm$	$3m + 1 + Lm$
DDRLS	$4m^2 + 16m + 1 + Lm$	$4m^2 + 12m - 1 + Lm$

Simulations (1/4)

- There are 20 nodes in the network
- The number of taps of the adaptive filter is 10
- The number of repetitions is 1000
- The variance for the input signal and the noise are 1 and 0.001, respectively.
- The noise samples are modeled as complex Gaussian noise.

Simulations (2/4)

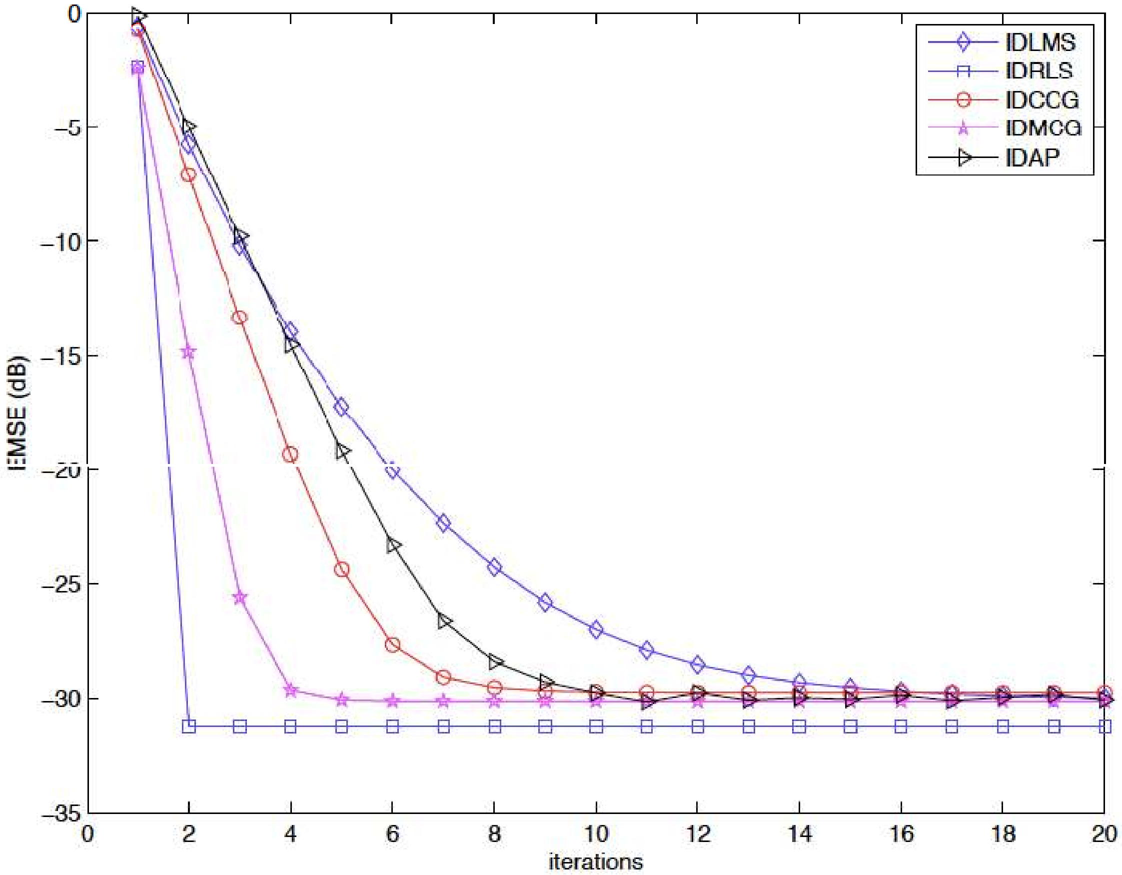


Fig. 3. Output EMSE against the number of iterations for Incremental Strategy with α_{IDLMS}=0.005, λ=0.2, $\lambda_{f-IDCCG}$=0.3,$\lambda_{f-IDMCG}$=0.25, $\eta_{f-IDCCG}$=$\eta_{f-IDMCG}$=0.15, j=5, α_{IDAP}=0.06, K=2

Simulations (3/4)

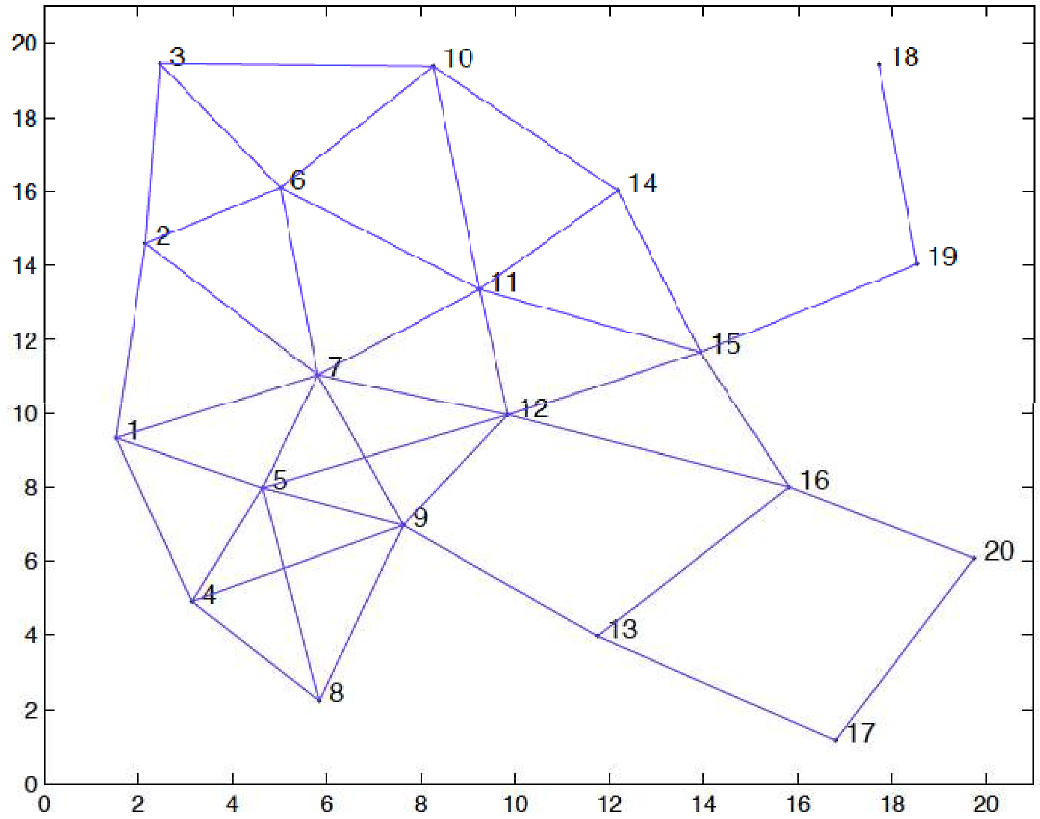


Fig. 4. Network Structure

Simulations (4/4)

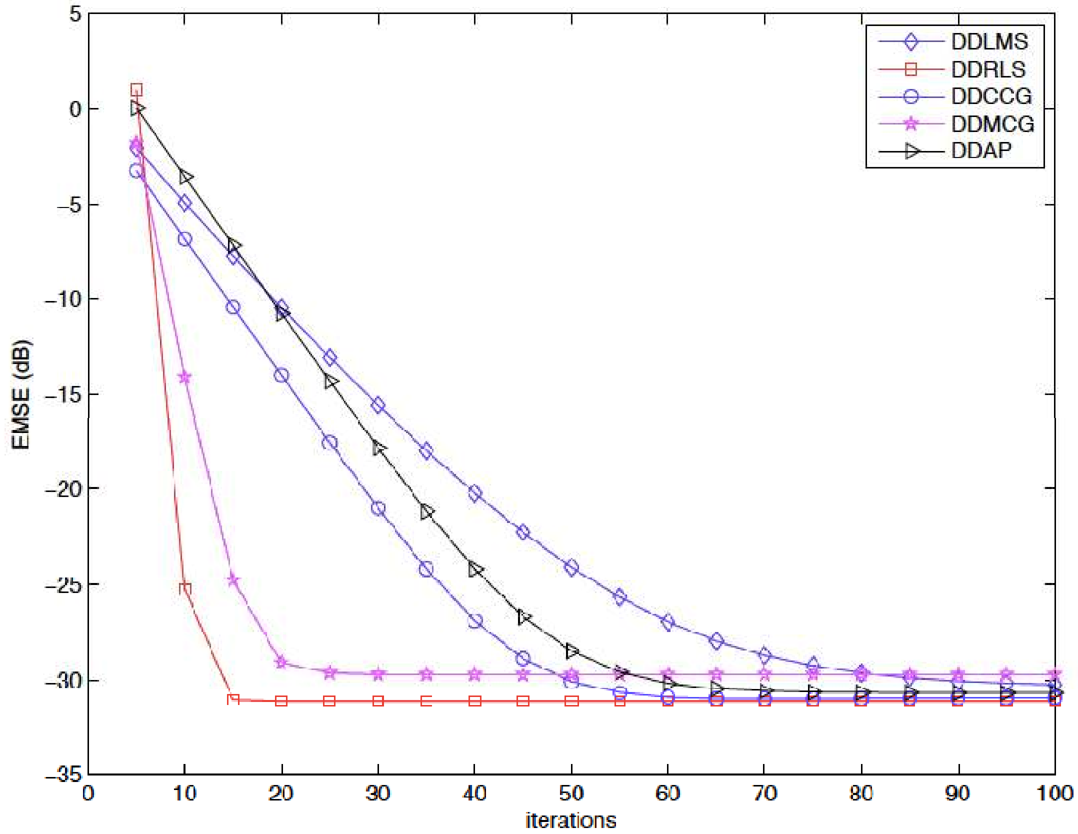


Fig. 5. Output EMSE against the number of iterations for Diffusion Strategy with α_{DDLMS}=0.0075, λ=0.998, $\lambda_{f-DDCCG}$=0.25, $\eta_{f-DDCCG}$=0.25, j=5, $\lambda_{f-DDMCG}$=0.46, $\eta_{f-DDMCG}$=0.45, α_{DDAP}=0.075, K=2

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Conclusions

- We have developed distributed CG algorithms for incremental and diffusion type distributed estimation over sensor networks.

- The CG- based strategies avoid the matrix inversion and numerical instability of RLS algorithms and have a faster convergence than LMS and AP algorithms.

- Simulation results have shown that the proposed IDMCG and DDMCG algorithms have a better performance than the LMS and AP algorithm, and a close performance to the RLS algorithm.

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