Sensor Array Signal Processing – 2016W

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List of exercises – 2

1. Explain with your own words the concept of beamforming and how it can be used to mitigate interference and enhance the reception of a desired signal.

2. Consider a linearly constrained minimum variance (LCMV) design for a beamformer employed at the output of an arbitrary sensor array with N elements that must process signals from D sources according to the model given by

$$\boldsymbol{x}[i] = \sum_{d=1}^{D} s_d[i]\boldsymbol{a}(\theta_d) + \boldsymbol{n}[i],$$

where $\boldsymbol{x}[i]$ is an N x 1 received vector, $s_d[i]$ is the dth signal and $\boldsymbol{a}(\theta_d)$ is its steering vector. The noise vector $\boldsymbol{n}[i]$ represents the measurement noise which is modeled as a complex Gaussian random variable with zero mean and variance σ^2.

The LCMV design amounts to solving the following optimization problem:

$$\boldsymbol{w_o} = \arg\ \min\ \boldsymbol{w}^H\boldsymbol{R}\boldsymbol{w}, \quad \text{subject to } \boldsymbol{C^H}\boldsymbol{w} = \boldsymbol{g},$$

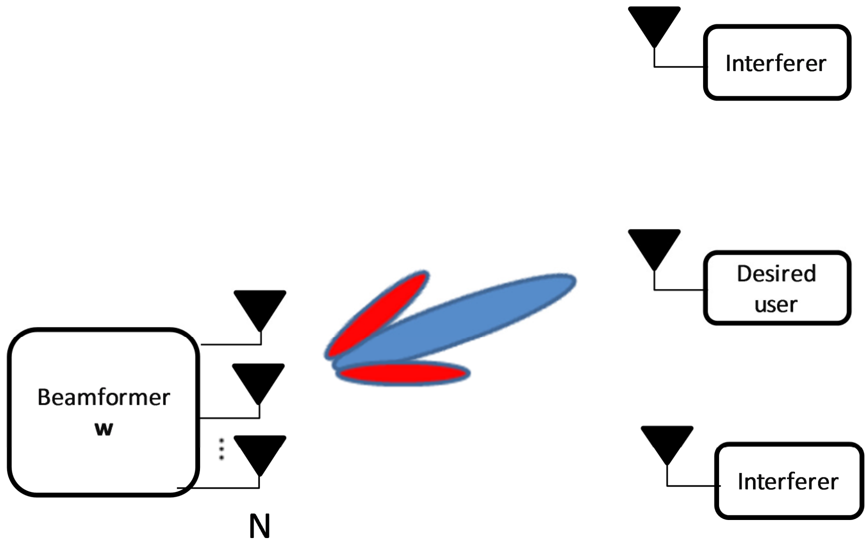
where $\boldsymbol{C} = [\ \boldsymbol{a}(\theta_1)\ \boldsymbol{a}(\theta_2) \dots\ \boldsymbol{a}(\theta_D)] \in \mathbb{C}^{N\times D}$.

a) Compute the expression for the optimal LCMV beamformer.

b) What are the advantages and disadvantages of the LCMV design as compared to the MVDR design.

3. Consider a sensor array system with N sensors and MVDR beamforming (see the Matlab code on the website). Compare the SINR performance of a uniform linear array (ULA) and a uniform circular array (UCA).

4. Consider a beamforming problem in which a uniform linear array (ULA) is used and an MVDR beamforming algorithm is employed.



The signal model is given by

$$\boldsymbol{x}[i] = s_d[i]\boldsymbol{a}(\theta_d) + \sum_{k=1,k\neq d}^{K} s_k[i]\boldsymbol{a}(\theta_k) + \boldsymbol{n}[i]$$

where $\boldsymbol{x}[i]$ is an N x 1 received vector, $s_d[i]$ is the desired signal and $\boldsymbol{a}(\theta_d)$ is the steering vector of the desired signal, $s_k[i]$ are the interfering signals that are generated by real Gaussian random variables and $\boldsymbol{a}(\theta_k)$ are the steering vectors for the k=1,2, ...,K signals impinging on the array. The noise vector $\boldsymbol{n}[i]$ represents the measurement noise which is modelled as a complex Gaussian random variable with zero mean and variance σ^2. The system employs an MVDR beamformer to suppress the interference as shown in the Matlab programme on the website of the course.

Write Matlab recursions to study and simulate the following:

a) Model mismatch errors in the steering vector of the desired signal using complex Gaussian random variables with a variance $\sigma_e^2 = 0.1\sigma^2$ and design a robust MVDR beamformer with diagonal loading. Show SINR x snapshots, SINR x SNR and beampattern plots.

b) Develop LMS and RLS versions of the MVDR beamformer and compare them using SINR x snapshots, SINR x SNR and beampattern plots.