# Low-Complexity Variable Forgetting Factor Mechanism for RLS Algorithms in Interference Mitigation Applications

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***Abstract*—In this work, we propose a low-complexity variable forgetting factor (VFF) mechanism for recursive least square (RLS) algorithms in interference mitigation applications. The proposed VFF mechanism employs a time average of the error correlation to automatically adjust the forgetting factor in order to ensure fast convergence and good tracking of the interference and the channel variations. Simulation results for a direct-sequence code-division multiple access (DS-CDMA) system are presented for nonstationary environments and show that the proposed VFF mechanism achieves superior performance to previously reported methods at a reduced complexity.**[1]

***Index Terms*- Interference mitigation, adaptive receivers, RLS algorithm, variable forgetting factor, time-varying channels.**

## I. Introduction

Wireless communication channels are dynamic by nature and present significant challenges to the design of receivers [1]. In particular, the estimation of the parameters of wireless receivers is a problem that has been around for many years and is key to the performance of wireless systems. In such time-varying environments, adaptive techniques are of fundamental importance and encounter applications in many wireless transmission schemes such as multi-input multi-output (MIMO) [2]-[4], orthogonal-frequency division multiplexing (OFDM) [5] and spread spectrum systems [6], [7], [8], [9]. Stochastic gradient (SG) and RLS algorithms [10], [11], [12] are the most common adaptive estimation techniques. Despite the linear computational complexity of SG techniques, their performance is typically poor in correlated channels and time-varying environments. For this reason, it is preferable to implement adaptive receivers with RLS algorithms due to their improved performance over SG algorithms.

Adaptive receivers are often equipped with RLS algorithms due to their fast convergence and excellent steady-state performance. In dynamic wireless environments in which users often enter and exit the system, it is impractical to compute a predetermined value for the forgetting factor [10]. Therefore, the RLS algorithm needs to be modified in order to yield satisfactory performance in time-varying environments. In this regard, one promising technique that has been proposed is to employ VFF mechanisms to adjust the forgetting factor automatically [10]-[15]. The classic VFF mechanism is the gradient-based variable forgetting factor (GVFF) algorithm which was proposed in [10]. In this work, besides the recursive expressions to adapt the receive filter, another SG recursion is used to control the forgetting factor, where the gradient with respect to the forgetting factor is obtained based on the instantaneous squared error cost function. In [13], the authors proposed a modified variable forgetting factor mechanism using the error criterion with noise variance weighting for frequency selective fading channel estimation. In [14], [15], a modified GVFF mechanism based on the gradient of the mean squared error (MSE) rather than on the gradient of the instantaneous squared error was investigated. The existing VFF mechanisms are mostly derived from the architecture of the classic GVFF algorithm [10], and their computational complexity of which is proportional to the receive filter length.

In this work, we propose a low-complexity VFF mechanism for adaptive RLS algorithms applied to linear interference suppression in DS-CDMA systems. The proposed VFF mechanism employs an updated component relating to the time average of the error correlation to automatically adjust the forgetting factor in order to ensure good tracking of the interference and the channel. We refer to the proposed VFF scheme as correlated time-averaged variable forgetting factor (CTVFF) mechanism. A convergence analysis is carried out and analytical expressions for predicting the mean squared error of the proposed adaptation technique are obtained. Interestingly, the proposed scheme is general and can be applied to any wireless system with the RLS algorithm. Simulation results are presented for time-varying environments and show that the proposed VFF mechanism obtains a superior performance to previously reported methods at a reduced complexity.

The paper is structured as follows. Section II briefly describes the DS-CDMA system model. The adaptive RLS algorithm and the existing gradient-based VFF mechanisms are reviewed in section III. The proposed CTVFF mechanism is described in section IV. A convergence analysis of the resulting algorithm and the analytical formulas to predict the steady-state MSE and the steady-state mean value of the variable forgetting factor for the proposed CTVFF mechanism are developed in section V. The simulation results are presented in section VI. Finally, section VII draws the conclusions.

## II. DS-CDMA System Model

Let us now describe a data model for the downlink of an uncoded synchronous binary phase-shift keying (BPSK) DS-CDMA system with $K$ users, $N$ chips per symbol and $L_p$ propagation paths. The delays are multiples of the chip duration and the receiver is synchronized with the main path. The $M$-dimensional received vector is given by

$$\mathbf{r}(i) = \sum_{k=1}^{K} A_k b_k(i) \mathbf{C}_k \mathbf{h}(i) + \boldsymbol{\eta}_k(i) + \mathbf{n}(i), \qquad (1)$$

where $M = N + L_p - 1$, $b_k(i) \in \{\pm 1\}$ is the $i$-th symbol for user $k$, and the amplitude associated with user $k$

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is $A_k$. The $M \times L_p$ convolution matrix $\mathbf{C}_k$ contains one-chip shifted versions of the spreading code of user $k$. The channel vector is $\mathbf{h}(i) = [h_0(i) \ldots h_{L_p-1}(i)]^T$, $\boldsymbol{\eta}_k(i)$ is the inter-symbol interference (ISI), $\mathbf{n}(i) = [n_0(i) \ldots n_{M-1}(i)]^T$ is the complex Gaussian noise vector with zero mean and $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma^2\mathbf{I}$, where $\sigma^2$ is the noise variance, $(.)^T$ and $(.)^H$ denote transpose and Hermitian transpose, respectively.

The linear minimum MSE (MMSE) receiver design is equivalent to determining a finite impulse response (FIR) filter $\mathbf{w}_k(i)$ with $M$ coefficients that provide an estimate of the desired symbol as follows

$$z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i), \tag{2}$$

where the detected symbol is given by $\hat{b}_k(i) = \text{sign}\{\Re[\mathbf{w}_k^H(i)\mathbf{r}(i)]\}$, where the operator $\Re[.]$ retains the real part of the argument and sign$\{.\}$ is the signum function. The MMSE receive filter is given by [10]

$$\mathbf{w}_0 = \bar{\mathbf{R}}^{-1}\mathbf{p}, \tag{3}$$

where $\bar{\mathbf{R}} = E[\mathbf{r}(i)\mathbf{r}^H(i)] \approx \sum_{k'=1}^{K} A_{k'}^2 \mathbf{C}_{k'}\mathbf{h}(i)\mathbf{h}^H(i)\mathbf{C}_{k'}^H + \sigma^2\mathbf{I}$ and $\mathbf{p} = E[b_k^*(i)\mathbf{r}(i)] = A_k\mathbf{C}_k\mathbf{h}(i)$. The minimum value of the mean squared error is given by $\xi_{\min} = 1 - A_k^2\mathbf{h}^H(i)\mathbf{C}_k^H\bar{\mathbf{R}}^{-1}\mathbf{C}_k\mathbf{h}(i)$.

## III. Adaptive RLS Algorithm and Problem Statement

In this section, we first describe an RLS algorithm to estimate the parameters of the linear receiver in multipath channels. Then, the existing gradient-based VFF mechanisms are briefly reviewed.

### A. Adaptive RLS Algorithm

Let us consider the time-averaged cost function $J_{LS}(i) = \sum_{n=1}^{i} \lambda^{i-n}|b_k(n) - \mathbf{w}_k^H(i)\mathbf{r}(n)|^2$ where $\lambda$ denotes the forgetting factor. By taking the gradient of $J_{LS}(i)$ with respect to $\mathbf{w}_k^*(i)$ and setting it to zero, after further mathematical manipulations we have the adaptive RLS algorithm as follows [10]

$$\mathbf{w}_k(i) = \mathbf{w}_k(i-1) + \mathbf{k}(i)e^*(i), \tag{4}$$

where $\mathbf{k}(i) = \frac{\mathbf{R}^{-1}(i-1)\mathbf{r}(i)}{\lambda + \mathbf{r}^H(i)\mathbf{R}^{-1}(i-1)\mathbf{r}(i)}$ and $e(i) = b_k(i) - \mathbf{w}_k^H(i-1)\mathbf{r}(i)$, and the estimate $\mathbf{R}^{-1}(i)$ is updated by

$$\mathbf{R}^{-1}(i) = \lambda^{-1}\mathbf{R}^{-1}(i-1) - \lambda^{-1}\mathbf{k}(i)\mathbf{r}^H(i)\mathbf{R}^{-1}(i-1). \tag{5}$$

The adaptive algorithm is implemented by using (4)-(5) with appropriate initial values $\mathbf{R}^{-1}(0)$ and $\mathbf{w}_k(0)$. The algorithm starts its operation in the training (TR) mode, and then is switched to the decision-directed (DD) mode. The problem we are interested in solving is how to devise a cost-effective mechanism to adjust $\lambda$, which is a key factor affecting the performance of RLS-based algorithms and wireless receivers.

### B. Gradient-based Mechanisms

In order to adjust the forgetting factor automatically, the adaptive rule for the classic GVFF mechanism is derived by taking the gradient of the instantaneous cost function $|b_k(i) - \mathbf{w}_k^H(i)\mathbf{r}(i)|^2$ with respect to the variable forgetting factor $\lambda(i)$ [10], which results in

$$\lambda(i) = \Big[\lambda(i-1) + \mu\Re\big[\frac{\partial \mathbf{w}_k^H(i-1)}{\partial \lambda}\mathbf{r}(i)e^*(i)\big]\Big]_{\lambda^-}^{\lambda^+} \tag{6}$$

where $[.]_{\lambda^-}^{\lambda^+}$ denotes the truncation to the limits of the range $[\lambda^-, \lambda^+]$, $\mu$ denotes a step-size. Here, a new quantity $\frac{\partial \mathbf{w}_k(i)}{\partial \lambda}$ is introduced, the updated equation of which is given by

$$\frac{\partial \mathbf{w}_k(i)}{\partial \lambda} = \big(\mathbf{I} - \mathbf{k}(i)\mathbf{r}^H(i)\big)\frac{\partial \mathbf{w}_k(i-1)}{\partial \lambda} + \frac{\partial \mathbf{R}^{-1}(i)}{\partial \lambda}\mathbf{r}(i)e^*(i), \tag{7}$$

where $\frac{\partial \mathbf{R}^{-1}(i)}{\partial \lambda}$ is updated by $\frac{\partial \mathbf{R}^{-1}(i)}{\partial \lambda} = \lambda^{-1}(i)\big(\mathbf{I} - \mathbf{k}(i)\mathbf{r}^H(i)\big)\frac{\partial \mathbf{R}^{-1}(i-1)}{\partial \lambda}\big(\mathbf{I} - \mathbf{r}(i)\mathbf{k}^H(i)\big) + \lambda^{-1}(i)\mathbf{k}(i)\mathbf{k}^H(i) - \lambda^{-1}(i)\mathbf{R}^{-1}(i)$. The adaptive RLS receiver with the GVFF mechanism [10] is implemented by using (4)-(7) with suitable initial values. By using the error criterion with noise variance weighting and the MSE rather than the instantaneous squared error, two modified GVFF mechanisms were recently proposed in [13] and [15]. We refer to them as weighting GVFF (WGVFF) mechanism and mean squared error GVFF (MGVFF) mechanism, respectively. In the following section, we will describe the proposed low-complexity variable forgetting factor mechanism.

## IV. Proposed Variable Forgetting Factor Mechanism

In this section, we first introduce the proposed low-complexity CTVFF mechanism that adjusts the forgetting factor of the adaptive RLS algorithm used at the receiver. Then, we present the computational complexity analysis for the proposed CTVFF mechanism and the existing gradient-based mechanisms.

### A. Adaptive CTVFF Mechanism

The proposed low-complexity adaptive variable forgetting factor mechanism is given by

$$\lambda(i) = \Big[\frac{1}{1+\gamma(i)}\Big]_{\lambda^-}^{\lambda^+}, \tag{8}$$

where the component $\gamma(i)$ is updated by

$$\gamma(i) = \delta_1\gamma(i-1) + \delta_2\rho^2(i), \tag{9}$$

where $0 < \delta_1 < 1$ and $\delta_2 > 0$, and $\delta_1$ is close to 1 and $\delta_2$ is set to a small value. The quantity $\rho(i)$ denotes the time average estimate related to the correlation of $|b_k(i-1) - \mathbf{w}_k^H(i-1)\mathbf{r}(i-1)|$ and $|b_k(i) - \mathbf{w}_k^H(i)\mathbf{r}(i)|$, which is given by

$$\rho(i) = \delta_3\rho(i-1) + (1-\delta_3)|b_k(i-1) - \mathbf{w}_k^H(i-1)\mathbf{r}(i-1)| \times |b_k(i) - \mathbf{w}_k^H(i)\mathbf{r}(i)|, \tag{10}$$

where $0 < \delta_3 < 1$, and $\delta_3$ controls the time average estimate. We set $\delta_3$ to a value close to 1. The update rule of the proposed CTVFF mechanism is in accordance with the MSE criterion since it estimates the time average of the correlation between $|b_k(i-1) - \mathbf{w}_k^H(i-1)\mathbf{r}(i-1)|$ and $|b_k(i) - \mathbf{w}_k^H(i)\mathbf{r}(i)|$, which is the error between the desired signal and the filter output. At the early stage, the large error correlation estimate $\rho^2(i)$ causes the updated component $\gamma(i)$ to increase, which simultaneously reduces the forgetting factor $\lambda(i)$ and provides a faster tracking. When a small error correlation estimate $\rho^2(i)$ results in a decrease in the updated component $\gamma(i)$, the forgetting factor $\lambda(i)$ is increased to yield a smaller misadjustment.

### B. Computational Complexity

We describe the computational complexity of the proposed CTVFF and the adaptive GVFF, WGVFF and MGVFF mechanisms [10], [13], [15]. Table I shows the additional computational complexity of the algorithms for multipath channels. We estimate the number of arithmetic operations by considering the number of complex additions and multiplications required by the mechanisms. The CTVFF and MGVFF mechanisms have constant computational complexity for each received symbol while the adaptive GVFF and WGVFF techniques have additional complexity proportional to the length $M$ of the receive filter. An important advantage of the proposed adaptation rule is that it requires only a few fixed number of operations.

TABLE I
ADDITIONAL COMPUTATIONAL COMPLEXITY.

| Algorithm | Number of operations per symbol | |
|---|---|---|
| | Multiplications | Additions |
| **CTVFF** | 7 | 3 |
| **GVFF [10]** | $7M^2+4M+2$ | $7M^2+M$ |
| **WGVFF [13]** | $7M^2+4M+9$ | $7M^2+M+1$ |
| **MGVFF [15]** | 29 | 18 |

## V. ANALYSES OF THE PROPOSED ALGORITHMS

In this section, we first derive the steady-state MSE expression for the adaptive RLS receiver with the proposed CTVFF mechanism in the case of time-varying channels. Then, the expression for the steady-state mean value of the CTVFF mechanism is derived.

### A. Convergence of MSE

Firstly, we give two expressions which will be used in our development [10]:

$$\mathbf{k}(i) = \mathbf{R}^{-1}(i)\mathbf{r}(i) \tag{11}$$

and

$$\lim_{i\to\infty} \mathbf{R}^{-1}(i) \approx \big(1 - E[\lambda(\infty)]\big)\bar{\mathbf{R}}^{-1}. \tag{12}$$

In a nonstationary scenario, the optimum solution takes on a time-varying form. It brings the task of tracking the minimum point of the error-performance surface, which is no longer fixed. We investigate the convergence properties of the proposed algorithm in this case. In time-varying channels, the optimum filter coefficients are considered to vary according to the model $\mathbf{w}_0(i) = \mathbf{w}_0(i-1) + \mathbf{q}(i)$, where $\mathbf{q}(i)$ denotes a random perturbation [10]. We assume that $\mathbf{q}(i)$ is an independently generated sequence with zero mean and positive definite autocorrelation matrix $E[\mathbf{q}(i)\mathbf{q}^H(i)]$. This is typical in the context of tracking analyses of adaptive filters [10].

By redefining $\boldsymbol{\epsilon}(i) = \mathbf{w}_k(i) - \mathbf{w}_0(i)$ we have

$$\begin{aligned}\boldsymbol{\epsilon}(i) &= \mathbf{w}_k(i) - \mathbf{w}_0(i)\\ &= \mathbf{w}_k(i-1) - \mathbf{w}_0(i-1) - \mathbf{q}(i) + \mathbf{k}(i)e^*(i).\end{aligned} \tag{13}$$

Let us recall (4) and assume $b_k(i) = \mathbf{w}_0^H(i-1)\mathbf{r}(i) + \xi_0(i)$, where $\xi_0(i)$ denotes the independent measurement error with zero mean and variance $\sigma_0^2$, where $\sigma_0^2 = 1 - A_k\mathbf{w}_0^H\mathbf{C}_k\mathbf{h} - A_k\mathbf{h}^H\mathbf{C}_k^H\mathbf{w}_0 + \mathbf{w}_0^H\bar{\mathbf{R}}\mathbf{w}_0$, we have

$$\boldsymbol{\epsilon}(i) = \big(\mathbf{I} - \mathbf{R}^{-1}(i)\mathbf{r}(i)\mathbf{r}^H(i)\big)\boldsymbol{\epsilon}(i-1) + \mathbf{R}^{-1}(i)\mathbf{r}(i)\xi_0^*(i) - \mathbf{q}(i). \tag{14}$$

Using (12) and direct averaging [10] we obtain

$$\boldsymbol{\epsilon}(i) \approx E[\lambda(\infty)]\boldsymbol{\epsilon}(i-1) + \big(1 - E[\lambda(\infty)]\big)\bar{\mathbf{R}}^{-1}\mathbf{r}(i)\xi_0^*(i) - \mathbf{q}(i). \tag{15}$$

Note that the vector $\mathbf{q}(i)$ is an independent zero mean vector which results in

$$\begin{aligned}\boldsymbol{\Theta}(i) &= E[\boldsymbol{\epsilon}(i)\boldsymbol{\epsilon}^H(i)]\\ &\approx E^2[\lambda(\infty)]\boldsymbol{\Theta}(i-1) + \big(1 - E[\lambda(\infty)]\big)^2\bar{\mathbf{R}}^{-1}\sigma_0^2\\ &\quad + E[\mathbf{q}(i)\mathbf{q}^H(i)],\end{aligned} \tag{16}$$

and when $i$ becomes large, the MSE in a time-varying environment is given by

$$\begin{aligned}\xi(i) &= E[|b_k(i) - \mathbf{w}_k^H(i-1)\mathbf{r}(i)|^2]\\ &= \mathbf{w}_0^H(i-1)\bar{\mathbf{R}}\mathbf{w}_0(i-1) + \mathrm{tr}[\bar{\mathbf{R}}\boldsymbol{\Theta}(i)] + 1\\ &\quad - A_k\mathbf{w}_0^H(i-1)\mathbf{C}_k\mathbf{h} - A_k\mathbf{h}^H\mathbf{C}_k^H\mathbf{w}_0(i-1).\end{aligned} \tag{17}$$

By using (16), when $i$ becomes large, we have

$$\mathrm{tr}[\bar{\mathbf{R}}\boldsymbol{\Theta}(i)] \approx \Big(\frac{1 - E[\lambda(\infty)]}{1 + E[\lambda(\infty)]}\Big)\sigma_0^2 M + \frac{\mathrm{tr}\big[\bar{\mathbf{R}}E[\mathbf{q}(i)\mathbf{q}^H(i)]\big]}{1 - E^2[\lambda(\infty)]}. \tag{18}$$

The MSE for a situation in which the adaptive receiver is tracking a channel can be computed by the following expression

$$\xi(\infty) \approx \xi_{\min} + \Big(\frac{1 - E[\lambda(\infty)]}{1 + E[\lambda(\infty)]}\Big)\sigma_0^2 M + \frac{\mathrm{tr}\big[\bar{\mathbf{R}}E[\mathbf{q}(i)\mathbf{q}^H(i)]\big]}{1 - E^2[\lambda(\infty)]}. \tag{19}$$

Note that we need to compute the quantities $E[\lambda(\infty)]$ and $E[\mathbf{q}(i)\mathbf{q}^H(i)]$ to calculate the above expression.

### B. Steady-State Mean Value of the CTVFF Mechanism

In order to derive the expression for the steady-state mean value of the variable forgetting factor for the CTVFF mechanism, we first show the convergence and derive the expression for the steady-state statistical property of the updated component $\gamma(i)$. By recalling (10), the estimate of $\rho(i)$ can be alternatively written as

$$\begin{aligned}\rho(i) &= (1-\delta_3)\sum_{l=0}^{i-1}\delta_3^l|b_k(i-l-1) - \mathbf{w}_k^H(i-l-1)\mathbf{r}(i-l-1)|\\ &\quad \times |b_k(i-l) - \mathbf{w}_k^H(i-l)\mathbf{r}(i-l)|\end{aligned} \tag{20}$$

and by squaring (20) we have

$$\begin{aligned}\rho^2(i) &= (1-\delta_3)^2\sum_{l=0}^{i-1}\sum_{j=0}^{i-1}\delta_3^l\delta_3^j|b_k(i-l-1)\\ &\quad - \mathbf{w}_k^H(i-l-1)\mathbf{r}(i-l-1)|\\ &\quad \times |b_k(i-j-1) - \mathbf{w}_k^H(i-j-1)\mathbf{r}(i-j-1)|\\ &\quad \times |b_k(i-l) - \mathbf{w}_k^H(i-l)\mathbf{r}(i-l)|\\ &\quad \times |b_k(i-j) - \mathbf{w}_k^H(i-j)\mathbf{r}(i-j)|.\end{aligned} \tag{21}$$

When $i \to \infty$, we assume that $|b_k(i-l)-\mathbf{w}_k^H(i-l)\mathbf{r}(i-l)|$ and $|b_k(i-j)-\mathbf{w}_k^H(i-j)\mathbf{r}(i-j)|$ are uncorrelated, thus, we have $E[|b_k(i-l)-\mathbf{w}_k^H(i-l)\mathbf{r}(i-l)||b_k(i-j)-\mathbf{w}_k^H(i-j)\mathbf{r}(i-j)|] = E[|b_k(i-l)-\mathbf{w}_k^H(i-l)\mathbf{r}(i-l)|]E[|b_k(i-j)-\mathbf{w}_k^H(i-j)\mathbf{r}(i-j)|] \approx 0$, where $\forall j \neq l$. Hence, the expectation of $\rho^2(i)$ is given by

$$E[\rho^2(i)] = (1-\delta_3)^2 \sum_{l=0}^{i-1} \delta_3^{2l} E[|b_k(i-l-1) - \mathbf{w}_k^H(i-l-1)\mathbf{r}(i-l-1)|^2] \times E[|b_k(i-l) - \mathbf{w}_k^H(i-l)\mathbf{r}(i-l)|^2]. \quad (22)$$

Note that $0 < \delta_3 < 1$, by using $\lim_{i\to\infty} E[|b_k(i) - \mathbf{w}_k^H(i)\mathbf{r}(i)|^2] \approx \xi_{min}$, we obtain $E[\rho^2(i)] = \frac{(1-\delta_3)\xi_{min}^2}{(1+\delta_3)}$. Note that $0 < \delta_1 < 1$, based on (9) we obtain the steady-state mean for $\gamma(i)$

$$E[\gamma(\infty)] = \frac{\delta_2(1-\delta_3)\xi_{min}^2}{(1-\delta_1)(1+\delta_3)}. \quad (23)$$

From (9) and (10), we can see that the quantity $\gamma(i)$ is a small value, and $\gamma(i)$ varies slowly around its mean value. Thus, when $i \to \infty$, by using (8) and (23) we assume that the steady-state mean value of the variable forgetting factor for the CTVFF mechanism is given as

$$E[\lambda(\infty)] \approx \frac{1}{1+E[\gamma(\infty)]} = \frac{(1-\delta_1)(1+\delta_3)}{(1-\delta_1)(1+\delta_3)+\delta_2(1-\delta_3)\xi_{min}^2}. \quad (24)$$

The above value can be substituted in (19) to predict the MSE.

## VI. Simulations

In this section, we first adopt a simulation approach and conduct several experiments in order to verify the effectiveness of the proposed CTVFF mechanism in RLS algorithms applied to interference suppression problems with DS-CDMA systems. Then, we verify the effectiveness of the proposed analytical expressions in (19) and (24) to predict the performance of adaptive linear receivers in several situations. The DS-CDMA system employs random sequences as the spreading codes, and the spreading gain is $N = 16$. The sequence of channel coefficients for each path is given by $h_f(i) = p_f\alpha_f(i) (f = 0, 1, 2)$, where $\alpha_f(i)$ is computed according to the Jakes' model [1]. All channels have a profile with three paths whose powers are $p_0$ = 0dB, $p_1 = -6$dB and $p_2 = -10$dB, respectively. All channels are normalized so that $\sum_{f=1}^{L_p} p_f^2 = 1$.

We evaluate the performance of a linear receiver using the proposed CTVFF mechanism with the RLS algorithm and compare it with the classic GVFF mechanism with the RLS algorithm [10], the MGVFF mechanism with the RLS algorithm [15], the WGVFF mechanism with the RLS algorithm [13], the RLS algorithm with the fixed forgetting factor mechanism and the SG algorithm. We optimized the parameters of the adaptive CTVFF scheme with $\delta_1 = 0.99$, $\delta_2 = 4\times10^{-4}$, $\delta_3 = 0.99$, $\lambda^- = 0.98$, $\lambda^+ = 0.99998$, and the initial values are $\gamma(0) = 0$ and $\rho(0) = 0$. The parameters for the adaptive GVFF scheme are $\lambda(0) = 0.998$, $\mu = 10^{-4}$, $\lambda^- = 0.995$ and $\lambda^+ = 0.99998$, and the initial values are $\frac{\partial \mathbf{R}_k^{-1}(0)}{\partial\lambda} = \mathbf{I}$ and $\frac{\partial \mathbf{w}_k(0)}{\partial\lambda} = \mathbf{0}$. The parameters of the MGVFF and WGVFF mechanisms are set as [15], [13]. We set $\mathbf{R}_k^{-1}(0) = \mathbf{I}$ and $\mathbf{w}_k(0) = 0.01\times\mathbf{1}$, where $\mathbf{1}$ denotes an all-one vector, the fixed forgetting factor mechanism employs $\lambda = 0.998$ and the step-size for the SG receiver is 0.0006. We remark that the parameters of the adaptive GVFF mechanism, the fixed forgetting factor mechanism and the SG algorithm are tuned to optimize the performance. All simulations are averaged over 1000 runs. We set the power of the desired user $|A_k|^2 = 1$.

In the first experiment, we choose the received signal to interference plus noise ratio (SINR) as the performance index to evaluate the convergence performance in nonstationary scenarios. The results shown in Fig. 1 illustrate that the performance in terms of SINR of the analyzed algorithms in a nonstationary scenario. The system starts with four users including one high-power level interferer with 3 dB above the desired user. At 1000 symbols, four interferers including two users operating at 3 dB above and two users operating at 6 dB above the desired user's power level enter the system. The normalized Doppler frequency is $f_dT = 5\times10^{-5}$. From Fig. 1, we can see that the proposed CTVFF mechanism with the RLS algorithm achieves the best performance, followed by the WGVFF mechanism and the GVFF mechanism. The MGVFF mechanism does not work well in a nonstationary scenario with multipath fading channels. The algorithms process 250 symbols in the TR mode and are then switched to DD mode. The signal-to-noise ratio (SNR) is 15dB.

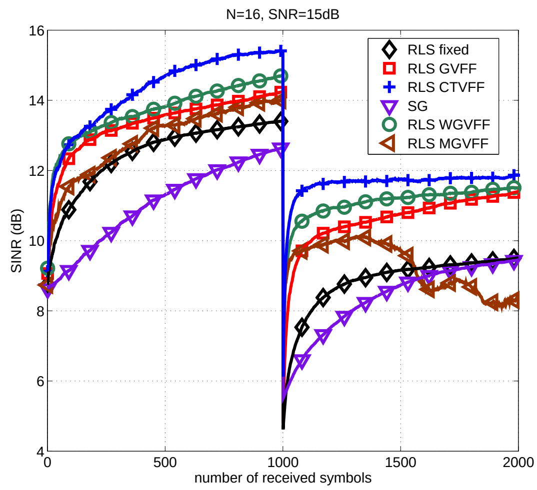


Fig. 1. SINR performance in nonstationary environment of multipath fading channels. SNR= 15dB. $f_dT = 0.00005$.

Fig. 2 (a) and (b) illustrate the BER performance of the desired user versus SNR and number of users $K$, where we set $f_dT = 1\times10^{-4}$. The users have the same power level. We can see that the best performance is achieved by the RLS algorithm with the CTVFF mechanism, followed by the RLS algorithm with the WGVFF mechanism, the RLS with the GVFF mechanism, the RLS with the fixed forgetting factor mechanism, the adaptive SG receiver and the conventional Rake receiver. In particular, the adaptive RLS receiver with the CTVFF mechanism can save up to over 5dB and support up to three more users in comparison with the RLS with the

GVFF mechanism, at the BER level of $10^{-2}$. The algorithms process 250 symbols in TR and 1500 symbols in DD.

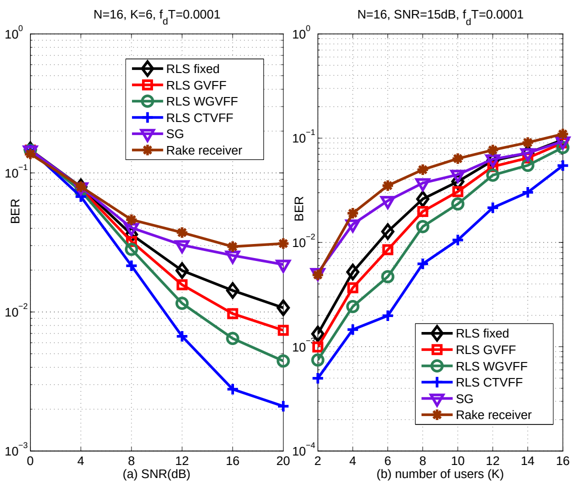


Fig. 2. BER performance versus (a) SNR and (b) the number of users (K) in multipath fading channels. $f_dT = 0.0001$.

Then, we examine the convergence and tracking analyses of the proposed CTVFF mechanism with the RLS. The steady-state MSE between the desired and the estimated symbol obtained through simulation is compared with the steady-state MSE computed via the expressions derived in Section V. We verify the analytical results (19) and (24) to predict the steady-state MSE. In this simulation, we assume that four users having the same power level operate in the system. All experiments are averaged over 1000 runs. We employ $\delta_1 = 0.99$, $\delta_2 = 0.35 \times 10^{-2}$ and $\delta_3 = 0.995$ for the case of invariant channels, and employ $\delta_1 = 0.99$, $\delta_2 = 0.4 \times 10^{-3}$ and $\delta_3 = 0.99$ for the case of time-varying fading channels, where $f_dT = 1 \times 10^{-5}$. 250 symbols are used in TR. The quantity of $E[\mathbf{q}(i)\mathbf{q}^H(i)]$ is estimated by using the average over the 10000 independent experiments, namely, $\left(\sum_{i=1}^{N_e} \mathbf{q}(i)\mathbf{q}^H(i)\right)/N_e$, where $N_e = 10000$ and $\mathbf{q}(i) = \mathbf{w}_0(i) - \mathbf{w}_0(i-1)$. By comparing the curves in Fig. 3, it can be seen that as the number of received symbols increases and the simulated MSE values converge to the analytical results, showing the usefulness of our analyses and assumptions.

## VII. Conclusion

In this paper, we proposed a low-complexity variable forgetting factor mechanism for estimating the parameters of adaptive CDMA linear receiver that operate with RLS algorithms. We compared the computational complexity of the new algorithm with the existing gradient-based methods and further investigated the convergence analysis of the proposed CTVFF scheme. We also derived expressions to predict the steady-state MSE of the adaptive RLS algorithm with the CTVFF mechanism. The simulation results verify the analytical results and show that the proposed scheme significantly outperforms existing algorithms and supports systems with higher loads. We remark that our proposed algorithm can also be extended to take into account and other types of applications.

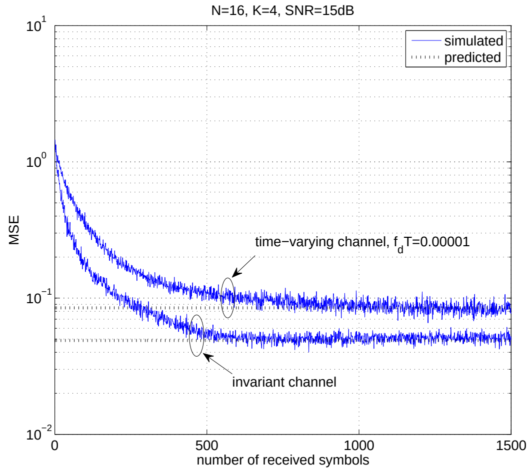


Fig. 3. Analytical MSE versus simulated performance for convergence analysis of the proposed CTVFF scheme.

## References


[1] T. S. Rappaport, *Wireless Communications*, Prentice-Hall, Englewood Cliffs, NJ, 1996.

[2] A. Rontogiannis, V. Kekatos, and K. Berberidis," A Square-Root Adaptive V-BLAST Algorithm for Fast Time-Varying MIMO Channels," *IEEE Signal Processing Letters*, Vol. 13, No. 5, pp. 265-268, May 2006.

[3] J. H. Choi, H. Y. Yu, Y. H. Lee, Adaptive MIMO decision feedback equalization for receivers with time-varying channels, *IEEE Trans. Signal Proc.*, 2005, 53, no. 11, pp. 4295-4303.

[4] R. C. de Lamare, and R. Sampaio-Neto, "Adaptive Reduced-Rank Equalization Algorithms Based on Alternating Optimization Design Techniques for MIMO Systems," *IEEE Transactions on Vehicular Technology*, vol.60, no.6, pp.2482-2494, July 2011.

[5] G. L. Stuber, J. R. Barry, S. W. MacLaughlin, Y. Li, M. A. Ingram and T. G. Pratt "Broadband MIMO-OFDM Wireless Communications", Proceedings of the IEEE, vol. 92, no. 2, Feb 2004.

[6] R. C. de Lamare, R. Sampaio-Neto, and A. Hjorungnes, "Joint Iterative Interference Cancellation and Parameter Estimation for CDMA Systems," *IEEE Communications Letters*, vol.11, no. 12, pp. 916-918, Dec. 2007.

[7] Y. Cai and R. C. de Lamare,"Adaptive Space-Time Decision Feedback Detectors with Multiple Feedback Cancellation", *IEEE Trans. on Veh. Techn.*, vol. 58, no. 8, October 2009, pp. 4129 - 4140.

[8] Y. Cai, R. C. de Lamare and R. Fa, "Switched Interleaving Techniques with Limited Feedback for Interference Mitigation in DS-CDMA Systems," *IEEE Trans. Commun.*, vol.59, no. 7, pp. 1946-1956, Jul. 2011.

[9] R. C. de Lamare and R. Sampaio-Neto,"Minimum Mean-Squared Error Iterative Successive Parallel Arbitrated Decision Feedback Detectors for DS-CDMA Systems," *IEEE Trans. Commun.*, vol.56, no. 5, pp. 778-789, May 2008.

[10] S. Haykin, "Adaptive Filter Theory," 4th ed. Englewood Cliffs, NJ: Prentice-Hall, 2002.

[11] R. C. de Lamare and R. Sampaio-Neto, "Adaptive Reduced-Rank Processing Based on Joint and Iterative Interpolation, Decimation, and Filtering," *IEEE Transactions on Signal Processing*, vol. 57, no. 7, July 2009, pp. 2503 - 2514.

[12] R. C. de Lamare and R. Sampaio-Neto, "Reduced-Rank Space-Time Adaptive Interference Suppression With Joint Iterative Least Squares Algorithms for Spread-Spectrum Systems," *IEEE Transactions on Vehicular Technology*, vol.59, no.3, March 2010, pp.1217-1228.

[13] S. Song, J. Lim, S. Baek, and K. Sung,"Variable Forgetting Factor Linear Least Squares Algorithm for Frequency Selective Fading Channel Estimation," *IEEE Trans. Vehicular Tech.*, vol. 51, no. 3, pp. 613-616, May 2002.

[14] C. F. So and S. H. Leung,"Variable forgetting factor RLS algorithm based on dynamic equation of gradient of mean square error," *Electronics Letters*, vol. 37, no. 3, pp. 202-203, Feb. 2001.

[15] S. H. Leung and C. F. So,"Gradient-Based Variable Forgetting Factor RLS Algorithm in Time-Varying Environments," *IEEE Trans. Signal Proc.*, vol. 53, no. 8, pp. 3141-3150, Aug. 2005.