

Non-Linear Precoding Techniques for Physical-Layer Security in Wireless Networks

Xiaotao Lu
Department of Electronics
University of York
Email: xtl503@york.ac.uk

Rodrigo de Lamare
CETUC/PUC-Rio
Department of Electronics, University of York
Email: rodrigo.delamare@york.ac.uk

Keke Zu
Ericsson Research, Sweden
Email: zukeke@gmail.com

Abstract—In this paper, we propose successive optimization non-linear precoding strategies for physical-layer security in wireless networks. We also investigate different precoding techniques for multi-user MIMO systems under various conditions of channel state information (CSI) between the access point and the users and the eavesdroppers. A non-linear precoding technique based on Successive Optimization Tomlinson-Harashima Precoding (SO-THP) and Simplified Generalized Matrix Inversion (S-GMI) technique is proposed along with a strategy for injecting artificial noise prior to transmission. Simulation results show that the proposed SO-THP+S-GMI precoding technique outperforms existing non-linear and linear precoding algorithms in terms of BER and secrecy rate performances.

Index Terms—Physical-layer security techniques, precoding, transmit processing.

I. INTRODUCTION

Data security in wireless systems has been dominated in the past by encryption methods like Data Encryption Standard (DES) and Advanced Encryption Standard (AES) which have played very important roles in the security aspects of data transmission. However, these encryption algorithms suffer from high complexity and latency. This has motivated the investigation of novel secrecy techniques which can be implemented in the physical layer and which are under consideration to provide secure wireless transmission.

In the 1970s, Wyner posed the Alice-Bob-Eve problem and proposed the wire-tap transmission system in [1]. The wire-tap transmission system gives a basic description on how to achieve the secrecy in the physical layer with better Gaussian channels between users than the Gaussian wire-tap channel. Later on another study reported in [2] showed that secrecy transmission is possible even when the eavesdropper has a better channel to the transmitter in a statistical sense than the receiver with confidential messages. The secrecy capacity [3] for different kinds of channels, such as the Gaussian wire-tap channel, the MIMO wire-tap channels have been studied in [3] and [4]. In some later works [5], [7], it has been found that by adding artificial noise to the system the secrecy of the transmission can be further enhanced.

A. Related Work

In recent years, precoding techniques have been widely applied to multi-user MIMO systems. Precoding techniques rely on pre-processing the transmitted data with the help

of channel state information (CSI). Different kinds of linear precoding techniques such as Zero-forcing (ZF) precoding, Minimum Mean Square Error (MMSE) precoding and Block Diagonalization (BD) precoding are introduced in [8]-[12], respectively. Some non-linear precoding techniques like Tomlinson-Harashima precoding (THP) [13], Vector Perturbation (VP) precoding [14] have also been reported. The precoding techniques can also contribute to the performance of relay systems [9]. These precoding techniques have been proven effective in improving the sum-rate and the bit-error rate (BER) performances. However, precoding strategies require CSI at the transmit side, whereas in reality CSI is available to the users of interest but is rarely available to the eavesdroppers. In most situations, a designer only has access to imperfect CSI for the users of interest. Dealing with imperfect CSI and the absence of CSI of the eavesdropper is a major challenge. Moreover, a technique that is particularly effective for improving the secrecy rate of physical-layer security is the introduction of artificial noise at the transmitter [5].

B. Contributions

In this paper we propose a novel non-linear technique based on Successive Optimization Tomlinson-Harashima Precoding (SO-THP) and Simplified Generalized Matrix Inversion (S-GMI) techniques along with a strategy of injecting artificial noise prior to transmission. We also study precoding techniques for multi-user MIMO systems under various conditions of CSI between the access point and the users and the eavesdroppers. Simulation results show that the proposed SO-THP+S-GMI precoding technique outperforms existing non-linear and linear precoding algorithms in terms of physical layer secrecy rate performances.

The remainder of this paper is organized as follows. In Section II, the system model and the performance metrics are given. The proposed SO-THP+S-GMI precoding algorithm is introduced in Section III. In Section IV, we discuss the simulation results. The conclusions are given in Section V.

Notation: Bold uppercase letters $\mathbf{A} \in \mathbb{C}^{M \times N}$ denote matrices with size $M \times N$ and bold lowercase letters $\mathbf{a} \in \mathbb{C}^{M \times 1}$ denote column vectors with length M . Conjugate, transpose, and conjugate transpose are represented by $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ respectively; \mathbf{I}_M is the identity matrix of size $M \times M$; $\text{diag}\{\mathbf{a}\}$ denotes a diagonal matrix with the elements of the vector \mathbf{a} along its diagonal; $\mathcal{CN}(0, \sigma_n^2)$ represents *i.i.d* entries with zero mean and σ_n^2 variance.

II. SYSTEM MODEL AND PERFORMANCE METRICS

In this section we introduce the system model considered for the transmission as well as the performance metrics that are used in the evaluation of the precoders.

A. System Model

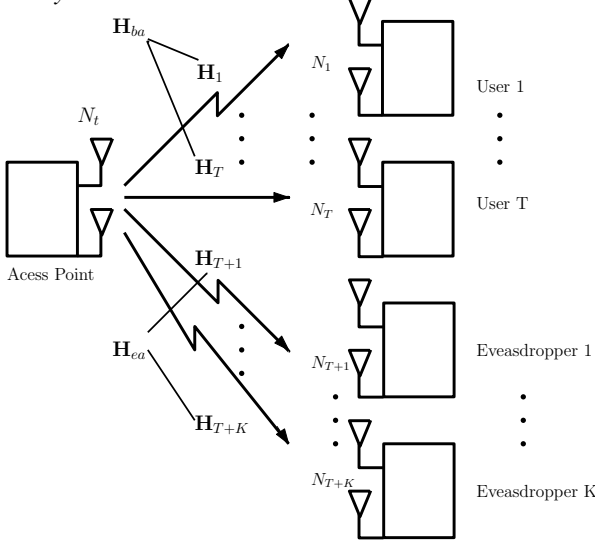


Fig. 1: System model of a MU-MIMO system with T users and K eavesdroppers

Here we consider an uncoded multi-user multiple input multiple output (MU-MIMO) broadcast channel. At the Access Point (AP) a transmitter with N_t antennas is used to transmit T users' data streams. Each user's data streams are received with N_i receive antennas. Along with the T users, K eavesdroppers each with N_k receive antennas also receive data from the transmitter. In this system we assume the eavesdroppers do not jam the transmission and each user's or eavesdropper's channel is a flat-fading MIMO channel. The quantity $\mathbf{H}_i \in \mathbb{C}^{N_i \times N_t}$ and $\mathbf{H}_k \in \mathbb{C}^{N_k \times N_t}$ corresponds to each user's or the eavesdropper's channel matrix. The Gaussian noise with *i.i.d* entries is distributed as $\mathcal{CN}(0, \sigma_B^2)$ and $\mathcal{CN}(0, \sigma_E^2)$.

In the scenarios without artificial noise, each user's or eavesdropper's received data are described by

$$\mathbf{y}_r = \beta_r^{-1} (\mathbf{H}_r \mathbf{P}_r \mathbf{s}_r + \mathbf{H}_r \sum_{j=1, j \neq r}^R \mathbf{P}_j \mathbf{s}_j + \mathbf{n}_r), \quad (1)$$

in (1) $\beta_r = \sqrt{\frac{E_r}{\|\mathbf{P}_r\|}}$ is used to guarantee that the transmit power after precoding is the same as the original transmit power E_r of user r and $\mathbf{P}_r \in \mathbb{C}^{N_t \times N_r}$ is the precoding matrix. We use the vector $\mathbf{x}_r \in \mathbb{C}^{N_t \times 1}$ to represent the signal after precoding. Without artificial noise \mathbf{x}_r is given by

$$\mathbf{x}_r = \mathbf{P}_r \mathbf{s}_r, \quad (2)$$

where $\mathbf{s}_r \in \mathbb{C}^{N_r \times 1}$ is the transmit symbols.

If the artificial noise is applied before the transmission, following the description in [7], the signal after precoding is given by

$$\mathbf{x}_r = \mathbf{P}_r \mathbf{s}_r + \mathbf{P}'_r \mathbf{s}'_r, \quad (3)$$

where $\mathbf{P}_r \in \mathbb{C}^{N_t \times N_r}$, $\mathbf{P}'_r \in \mathbb{C}^{N_t \times m}$ are the precoding matrices. The vector $\mathbf{s}_r \in \mathbb{C}^{N_r \times 1}$ represents the information to be transmitted. The vector $\mathbf{s}'_r \in \mathbb{C}^{m \times 1}$ is the jamming signal. Suppose $0 < \rho < 1$ is the fraction of the power devoted to the information signal and $\mathbf{P}'_r{}^H \mathbf{P}'_r = \mathbf{I}$. The artificial noise then should satisfy

$$E\{\mathbf{s}'_r \mathbf{s}'_r{}^H\} = \mathbf{Q}'_s, \quad (4)$$

$$\text{Tr}(\mathbf{Q}'_s) = (1 - \rho)E_s, \quad (5)$$

In (4) and (5) $\mathbf{Q}'_s \in \mathbb{C}^{m \times m}$ is the covariance matrix associated with the jamming signal \mathbf{s}'_r . The covariance matrix of the transmit signal $\mathbf{Q}_s \in \mathbb{C}^{N_r \times N_r}$ satisfies

$$\text{Tr}(\mathbf{Q}_s) = \rho E_s, \quad (6)$$

and proper artificial noise power ratio ρ will result in an increase of the secrecy rate. Based on the construction of the artificial noise, there will be a different effect on the secrecy rate performance. In [22], it shows a generalized artificial noise with more flexible covariance matrix to improve the secrecy rate. Besides the construction introduced in [7], the study in [20] reports a system with a helping interferer to improve the achievable secrecy rate.

B. Performance Metrics

From [1]-[4], we have the definition of the secrecy capacity as the maximum transmission rate Alice can achieve during the transmission without any information detected by the eavesdropper. From [4] without artificial noise, we have the secrecy capacity

$$\begin{aligned} C_{sec} &= \max_{\mathbf{Q}_s \geq 0, \text{Tr}(\mathbf{Q}_s) \leq E_s} I(X_s; Y_i) - I(X_s; Y_k) \\ &= \max_{\mathbf{Q}_s \geq 0, \text{Tr}(\mathbf{Q}_s) \leq E_s} \log(\det(\mathbf{I} + \mathbf{H}_{ba} \mathbf{Q}_s \mathbf{H}_{ba}^H / \sigma_B^2)) \\ &\quad - \log(\det(\mathbf{I} + \mathbf{H}_{ea} \mathbf{Q}_s \mathbf{H}_{ea}^H / \sigma_E^2)), \end{aligned} \quad (7)$$

In (7) $I(X_s; Y_i)$ and $I(X_s; Y_k)$ represent the mutual information between the transmitter to the user as well as the eavesdropper. X_s, Y_i, Y_k are the counterparts of different signal symbols. \mathbf{Q}_s is the covariance matrix associated with the signal after precoding \mathbf{x}_r . The secrecy capacity achieved in this situation with prior knowledge of the users' channel \mathbf{H}_{ba} as well as the eavesdroppers' channel \mathbf{H}_{ea} can be used as a benchmark for further study. In reality the channel between the transmitter and the eavesdroppers is not perfectly known. This situation known as the imperfect CSI is discussed in [5]. However, for the imperfect CSI \mathbf{H}_{ea} we suppose the distribution of the eavesdroppers' channel is known to the transmitter and users.

Another performance metric is the BER estimated by the different links. Ideally, we would like the users to experience a reliable communication and the eavesdropper to have a very high BER (virtually no reliability when communicating). In this paper, different precoding techniques are supposed to suppress the interferences between different users thus leading to a better BER performance.

Thirdly, the algorithm computational complexity should also take into consideration. With the increasing of the number of users, the high computational complexity algorithm will suffer from a long time delay. Low computational complexity will contribute to reduce the latency of the system.

III. PROPOSED PRECODING ALGORITHMS

In this section, we detail the proposed precoding algorithms.

A. SO-THP Precoding

According to [21], we assume the modified data symbol vector $\mathbf{u}_r = \mathbf{s}_r + \mathbf{z}_r$, where $\mathbf{u}_r \in \mathbf{C}^{N_r \times 1}$ and $\mathbf{z}_r \in \mathbf{C}^{N_r \times 1}$. With the feedback matrix we can then obtain the output of the modulo operation \mathbf{v}_r

$$\mathbf{v}_r = (\mathbf{I} + \mathbf{B})_r^{-1} \mathbf{u}_r \quad (8)$$

where $\mathbf{v}_r \in \mathbf{C}^{N_r \times 1}$. When we have the feedforward matrix $\mathbf{F}_r \in \mathbf{C}^{N_t \times N_r}$, the transmit signal can be obtained as $\mathbf{x}_r = \mathbf{F}_r \mathbf{v}_r$, where $\mathbf{x}_r \in \mathbf{C}^{N_t \times 1}$. The received signal $\mathbf{y}_{r-pro} \in \mathbf{C}^{N_t \times 1}$ then can be written in the following form:

$$\mathbf{y}_{r-pro} = \mathbf{H}_r \mathbf{x}_r + \mathbf{n}_r = \mathbf{H}_r \mathbf{F}_r (\mathbf{I} + \mathbf{B})_r^{-1} \mathbf{u}_r + \mathbf{n}_r \quad (9)$$

In [15], the SO-THP algorithm is illustrated as a combination of a successive optimization technique and Tomlinson-Harashima precoding. The reordered feedforward and feedback matrices are expressed as

$$\mathbf{F} = (\mathbf{F}_1 \cdots \mathbf{F}_R) \quad (10)$$

$$\mathbf{B} = \text{lower triangular} (\mathbf{DHF} \bullet \text{diag}([\mathbf{DHF}]_{ii}^{-1})) \quad (11)$$

where the complex unitary matrix $\mathbf{D} = \begin{pmatrix} \mathbf{D}_1 & & \\ & \ddots & \\ & & \mathbf{D}_T \end{pmatrix}$.

B. S-GMI Precoding

The concept of generalized matrix inversion precoding has been introduced in [16], where two approaches called Generalized Zero-Forcing Channel Inversion (GZI) and Generalized MMSE Channel Inversion (GMI) are illustrated. The GMI scheme uses the QR decomposition to decompose the MMSE channel inversion $\bar{\mathbf{H}}_r$ as expressed by

$$\bar{\mathbf{H}}_r = (\mathbf{H}_r^H \mathbf{H}_r + \alpha \mathbf{I})^{-1} \mathbf{H}_r^H, \quad (12)$$

$$\bar{\mathbf{H}}_r = \bar{\mathbf{Q}}_r \bar{\mathbf{R}}_r \quad \text{for } r = 1, \dots, R, \quad (13)$$

where $\bar{\mathbf{H}}_r \in \mathbf{C}^{N_t \times N_r}$, $\bar{\mathbf{Q}}_r \in \mathbf{C}^{N_t \times N_t}$, $\bar{\mathbf{R}}_r \in \mathbf{C}^{N_r \times N_r}$ and GMI takes the noise into account. It overcomes the noise enhancement in the BD algorithm. Meanwhile extra interference may be introduced in the following steps. To solve this problem, a transmit combining matrix \mathbf{T}_r is applied to $\bar{\mathbf{Q}}_r$. Under the total transmit power constraint we use the minimum total MSE criterion to achieve the transmit combining matrix

\mathbf{T}_r . In [16] the transmit combining matrix \mathbf{T}_r is given by $\mathbf{T}_r = \beta \bar{\mathbf{T}}_r$ as described by

$$\bar{\mathbf{T}}_r = \left(\bar{\mathbf{Q}}_r^H \sum_{j=1}^R \mathbf{H}_j^H \mathbf{H}_j \bar{\mathbf{Q}}_r + \alpha \mathbf{I} \right)^{-1} \bar{\mathbf{Q}}_r^H \mathbf{H}_r^H \mathbf{H}_r \bar{\mathbf{Q}}_r, \quad (14)$$

where $\mathbf{T}_r, \bar{\mathbf{T}}_r \in \mathbf{C}^{N_t \times N_t}$. Once we have $\bar{\mathbf{Q}}_r$ and \mathbf{T}_r , the following approach is

$$\mathbf{H}_r \bar{\mathbf{Q}}_r \mathbf{T}_r = \bar{\mathbf{U}}_r \bar{\mathbf{\Sigma}}_r \bar{\mathbf{V}}_r^H, \quad (15)$$

In [17], it has been shown that the transmit combining matrix \mathbf{T}_r is not necessary. Therefore, a simplified GMI (S-GMI) is developed in [17] as an improvement of the original RBD precoding in [8]. This is known as S-GMI. Therefore, the decomposition in (15), the precoder and the receive filter are

$$\mathbf{H}_r \bar{\mathbf{Q}}_r = \tilde{\mathbf{U}}_r \tilde{\mathbf{\Sigma}}_r \tilde{\mathbf{V}}_r^H, \quad (16)$$

$$\mathbf{P}_{S-GMI} = [\bar{\mathbf{Q}}_1 \tilde{\mathbf{V}}_1 \quad \bar{\mathbf{Q}}_2 \tilde{\mathbf{V}}_2 \quad \cdots \quad \bar{\mathbf{Q}}_R \tilde{\mathbf{V}}_R], \quad (17)$$

$$\mathbf{M}_{S-GMI} = \text{diag}\{\tilde{\mathbf{U}}_1^H \quad \tilde{\mathbf{U}}_2^H \quad \cdots \quad \tilde{\mathbf{U}}_R^H\}, \quad (18)$$

where $\mathbf{P}_{S-GMI} \in \mathbf{C}^{N_t \times N_t}$, $\mathbf{M}_{S-GMI} \in \mathbf{C}^{N_t \times N_t}$. In the SO-THP+S-GMI algorithm, the feedforward matrix (10) is obtained by reordering the S-GMI precoding matrix (17).

IV. SIMULATION RESULTS

A system with $N_t = 4$ transmit antennas and $T = 2$ users as well as $K = 2$ eavesdroppers is considered. All the transmitted symbols are modulated with QPSK. In the simulations, we assume that the channel for each user is uncorrelated and is generated following a complex Gaussian model with zero mean and variance equal to one.

A. Computational complexity

In assessing the computational complexity, the number of FLOPS (Floating-Point Operations Per Second) is used as a measurement of the cost of an algorithm. In [18], the FLOPS for real QR, SVD and complex QR algorithms are given. According to [19] for an $m \times n$ complex matrix \mathbf{A} , the FLOPS of the SVD of the complex matrix \mathbf{A} are equivalent to its $2m \times 2n$ extended real matrix. The number of FLOPS of the multiplication, the QR decomposition, the matrix inversion as well as SVD with different matrices obtained have been reported in [18].

In Fig. 2 it is shown that although the BD precoding algorithm has the lowest complexity, its BER and secrecy rate performances are not satisfactory. Among the SO-THP type algorithms the SO-THP+S-GMI algorithm shows an advantage over the standard SO-THP algorithm in terms of complexity.

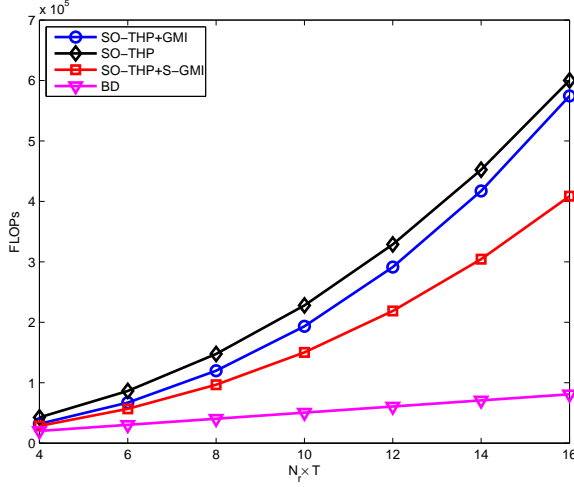


Fig. 2: Computational complexity in FLOPs for MU-MIMO systems

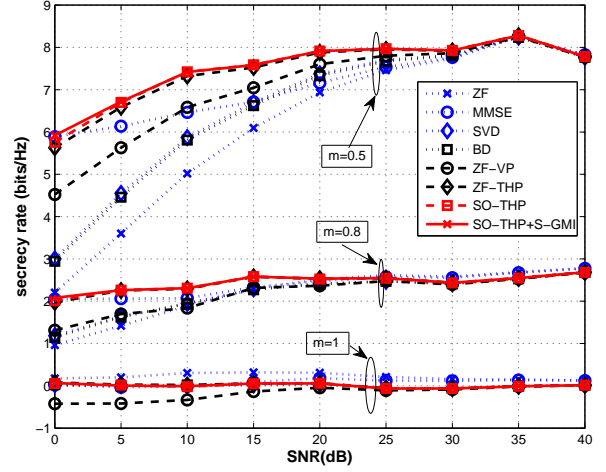


Fig. 4: Secrecy rate performance with precoding techniques in $4 \times 4 \times 4$ MU-MIMO broadcast channel

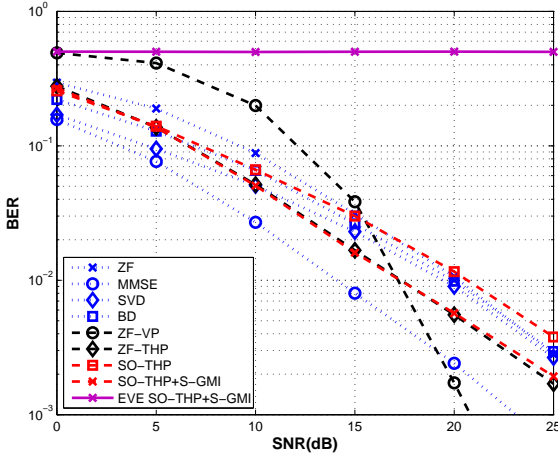


Fig. 3: BER performance with precoding techniques in $4 \times 4 \times 4$ MU-MIMO broadcast channel, $m = 0.5$

B. Perfect Channel State Information

Fig. 3 shows the BER performance for different precoding techniques. The proposed SO-THP+S-GMI precoding requires less than 5 dB than the original SO-THP precoding in terms of SNR to achieve 10^{-2} bit error rate. According to Fig. 4, the secrecy rate will level out according to the ratio between the main channel and the wire-tap channel m . From Fig. 4, we can also see that among all the investigated precoding algorithms, the proposed SO-THP+S-GMI precoding can achieve a certain secrecy rate with lower SNR.

C. Imperfect Channel State Information

In Fig. 5, the secrecy rate performance is evaluated in the imperfect CSI scenario, which shows a degradation at low SNR. Furthermore, the secrecy rate requires a very high SNR

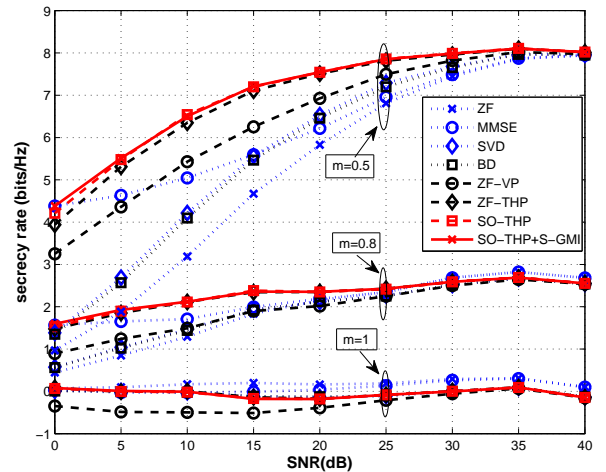


Fig. 5: Secrecy rate with precoding techniques $4 \times 4 \times 4$ MU-MIMO broadcast channel with imperfect CSI

to converge to a high secrecy rate. The proposed SO-THP+S-GMI precoder has the best secrecy rate performance among all the studied precoding techniques.

D. Imperfect Channel State Information With Artificial Noise

In Fig. 6 and Fig. 7, we assume that artificial noise is generated with extra free antennas. Although the artificial noise is added, the total transmit power E_s will remain the same. In the simulation 0.4 of the transmit power E_s is used to generate the artificial noise. In Figure 6, with higher SNR, we can have better secrecy rate performance. Even when the eavesdropper has better statistical channel, the secrecy rate keeps almost the same according to Figure 7.

V. CONCLUSION

Precoding techniques used in MU-MIMO broadcast systems can achieve good BER and secrecy-rate performances.

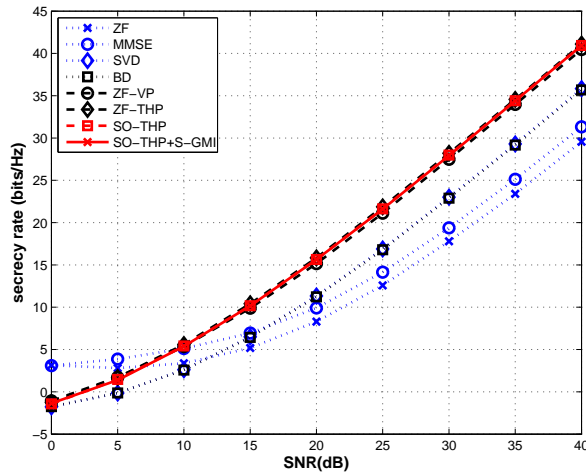


Fig. 6: Secrecy rate with precoding techniques $4 \times 4 \times 4$ MU-MIMO broadcast channel with imperfect CSI adding artificial noise, $m = 0.5$

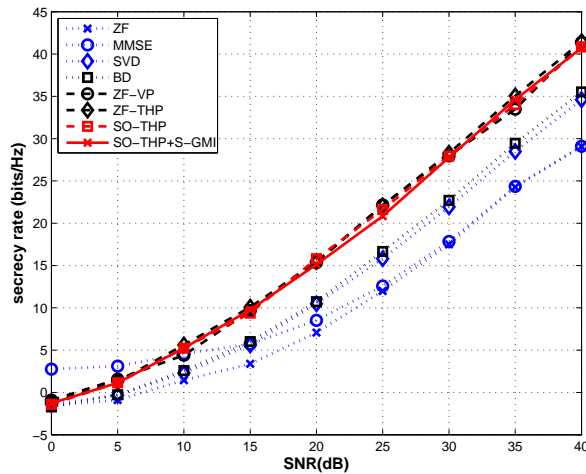


Fig. 7: Secrecy rate with precoding techniques $4 \times 4 \times 4$ MU-MIMO broadcast channel with imperfect CSI adding artificial noise, $m = 2$

The proposed SO-THP+S-GMI algorithm requires the lowest computational complexity among non-linear techniques and outperforms most linear and non-linear techniques in BER performance. Moreover, SO-THP+S-GMI can achieve a high secrecy rate, which means the security of the transmission with SO-THP+S-GMI is superior to existing techniques. When artificial noise is added, the performance is further improved.

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