

# Blind space–time joint channel and direction of arrival estimation for DS-CDMA systems

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**Abstract:** A blind space–time joint channel and direction of arrival (DoA) estimation method is proposed for DS-CDMA systems. The new method employs a linear antenna array and requires only the signature sequence and timing of the desired user to blindly estimate the channel and the DoA parameters. Two blind algorithms that exploit antenna arrays are also proposed for enhancing the channel estimates. Simulations show the improvements for channel estimation of the method and algorithms over existing techniques and the accuracy achieved for DoA computation as compared to recently reported DoA estimators for DS-CDMA systems.

## 1 Introduction

In modern direct sequence code-division multiple access (DS-CDMA) systems, the incorporation of antenna arrays in conjunction with adaptive receivers can provide an enhanced performance for multiple access interference (MAI) and intersymbol interference (ISI) mitigation [1]. This requires the joint processing of the data received at an antenna array with elements closely spaced, which leads to the combination of multiuser or singleuser detection techniques and beamforming [2–7]. Since these signals propagate through multipath environments, the design of space–time (ST) blind detectors requires the knowledge of the spatial signature, which depends entirely upon the known signature sequence and estimation of the channel and the direction of arrival (DoA). In order to estimate the channel and the DoA parameters, the designer can resort to two different approaches: training based [8–10] or blind algorithms [11–20].

Owing to their higher transmission efficiency, blind techniques are the most promising candidates for DoA and channel estimation (CE) even though they present some drawbacks such as phase ambiguities. In this context, the classical DoA estimation methods, such as maximum likelihood (ML) [11] and ML-based [12] MUSIC [13] and ESPRIT [14], encounter limitations in a multiuser environment because of the reduced number of sensors and their high complexity requirements. Recently, some capacity improvement for DoA estimators in DS-CDMA systems was accomplished by processing despread symbol-rate signals and then applying enhanced versions of MUSIC [15] and ESPRIT [16, 17]. Indeed, blind CE methods based on subspace estimation have been reported for DS-CDMA systems in [21–24], where singular value decomposition (SVD) is required for identifying the channel. However, existing blind estimators do not exploit antenna arrays in order to improve channel estimates and to

obtain the DoA of the desired user. Furthermore, the literature on methods that combine DoA estimation techniques and exploit the spreading sequences available in DS-CDMA systems is extremely limited. In particular, there is no blind technique that jointly estimates the channel and DoA parameters for DS-CDMA signals and based on the DoA estimates further improves the channel parameter estimation. This work proposes a new joint blind channel and the DoA estimation method for DS-CDMA that can obtain accurate DoA and CE and two algorithms that exploit antenna array gain and the estimated DoA information to further enhance the channel estimates. The performance of the proposed DoA and CE techniques is investigated and analytical results on the capacity gains of the proposed methods are presented.

## 2 DS-CDMA system model

Consider the uplink of a synchronous DS-CDMA system with  $K$  users,  $N$  binary chips per symbol and  $L_p$  paths. Assuming that the channel is constant during each symbol and the receiver with a  $J$ -element linear antenna-array is synchronised with the main path, the complex envelope of the received waveforms after filtering by a chip-pulse matched filter and sampled at chip rate yields the  $JM \times 1$  observation vector corresponding to the  $i$ th signalling interval

$$\begin{aligned} \mathbf{r}(i) &= \sum_{k=1}^K A_k b_k(i) \mathcal{F}_k \mathcal{H}_k + \boldsymbol{\eta}(i) + \mathbf{n}(i) \\ &= \sum_{k=1}^K \mathbf{x}_k(i) + \boldsymbol{\eta}(i) + \mathbf{n}(i) \end{aligned} \quad (1)$$

where  $M = N + L_p - 1$ , the ST convolution matrix  $\mathcal{F}_k$  has dimensions  $JM \times JL_p$  and contains the matrix  $C_k$  on its main diagonal, that is,  $\mathcal{F}_k = \text{diag}(C_k, C_k, \dots, C_k)$ . The columns of the  $M \times L_p$  matrix  $C_k$  contain one-chip shifted versions by one row down of the known signature sequence  $s_k = [c_k(1) \dots c_k(N)]^T$  of user  $k$ ,  $\mathbf{n}(i) = [n_1(i) \dots n_{JM}(i)]^T$  is the complex Gaussian noise vector with  $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma^2 \mathbf{I}$ ,  $(\cdot)^T$  and  $(\cdot)^H$  denote transpose and Hermitian transpose, respectively,  $E[\cdot]$  stands for ensemble average,  $b_k(i)$  is the  $i$ th complex symbol of user  $k$ ,  $\boldsymbol{\eta}(i)$  is the ISI and the amplitude of user  $k$  is  $A_k$ . The symbols  $b_k(i)$  ( $k = 1, \dots, K$ ), pertaining to different users, are independent and have zero mean and unit variance. The  $JL_p \times 1$  ST channel vector  $\mathcal{H}_k$  is given by

$$\mathcal{H}_k = [\mathbf{h}_{k,0}^T | \mathbf{h}_{k,1}^T | \dots | \mathbf{h}_{k,J-1}^T]^T$$

$$= [(\mathbf{z}(\Theta_{k,0})\mathbf{h}_k)^T | (\mathbf{z}(\Theta_{k,1})\mathbf{h}_k)^T | \dots | (\mathbf{z}(\Theta_{k,J-1})\mathbf{h}_k)^T]^T \quad (2)$$

where the matrix  $\mathbf{z}(\Theta_{k,l}) = \text{diag}(e^{-j\Theta_{k,l}}, \dots, e^{-j\Theta_{k,l}})$  is  $L_p \times L_p$  and contains the phase shift  $\Theta_{k,m} = 2\pi(d/\lambda)\cos(\phi_{k,m})$  for user  $k$ , sensor  $l$  and the  $L_p$  paths,  $d = \lambda/2$  is the spacing between sensors,  $\lambda$  is the carrier wavelength,  $\phi_{k,m}$  is the DoA of user  $k$  and path  $m$  and  $\mathbf{h}_{k,l}(i) = [h_{k,0}^{(l)}(i) \dots h_{k,L_p-1}^{(l)}(i)]^T$  in (2) is the channel of user  $k$  at sensor  $l$ . Assuming that the spacing between elements is small enough, there is no amplitude variation between signals received at different elements and thus  $\mathbf{h}_{k,l}(i) = \mathbf{h}_k \mathbf{z}(\Theta_{k,l})$  ( $l = 0, 1, \dots, J-1$ ) or equivalently

$$h_{k,m}^{(l-1)}(i)e^{-j\Theta_{k,m}} = h_{k,m}^{(l)}(i) \quad (3)$$

### 3 Proposed channel and DoA estimation method

Let us present the theory of the proposed joint blind channel and DoA estimation method that employs an antenna-array. Consider the  $JM \times JL_p$  ST constraint matrix  $\mathcal{F}_k$  and the  $JL_p \times 1$  ST channel vector  $\mathcal{H}_k$ . From (1) we have that the  $k$ th user received signal without noise and ISI can be expressed as

$$\mathbf{x}_k = A_k b_k \mathcal{F}_k \mathcal{H}_k \quad (4)$$

Let us perform SVD on the ST  $JM \times JM$  covariance matrix  $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ . By neglecting the ISI, we have

$$\mathbf{R} = \sum_{k=1}^K E[\mathbf{x}_k \mathbf{x}_k^H] + \sigma^2 \mathbf{I}$$

$$= [\mathbf{V}_s \ \mathbf{V}_n] \begin{bmatrix} \Lambda_s + \sigma^2 \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma^2 \mathbf{I} \end{bmatrix} [\mathbf{V}_s \ \mathbf{V}_n]^H \quad (5)$$

where  $\mathbf{V}_s$  and  $\mathbf{V}_n$  are the signal and noise subspaces, respectively. Since these subspaces are orthogonal, we have the condition  $\mathbf{V}_n^H \mathbf{x}_k = \mathbf{V}_n^H \mathcal{F}_k \mathcal{H}_k = \mathbf{0}$  and hence  $\mathcal{H}_k^H \mathcal{F}_k^H \mathbf{V}_n \mathbf{V}_n^H \mathcal{F}_k \mathcal{H}_k = 0$  that allows the recovery of  $\mathcal{H}_k$  as the eigenvector corresponding to the smallest eigenvalue of the matrix  $\mathcal{F}_k^H \mathbf{V}_n \mathbf{V}_n^H \mathcal{F}_k$ , provided  $\mathbf{V}_n$  is known. In this regard,  $\mathcal{H}_k$  belongs to the null space of  $\mathcal{F}_k^H \mathbf{V}_n \mathbf{V}_n^H \mathcal{F}_k$  and is a linear combination of all eigenvectors corresponding to eigenvalue zero. To avoid the SVD on  $\mathbf{R}$  and overcome the need for determining the noise subspace rank that is necessary to obtain  $\mathbf{V}_n$ , we resort to the following approach.

*Lemma:* Consider the SVD on  $\mathbf{R}$  as in (5), then we have

$$\lim_{p \rightarrow \infty} (\mathbf{R}/\sigma^2)^{-p} = \mathbf{V}_n \mathbf{V}_n^H \quad (6)$$

*Proof:* Using the decomposition in (5) and since  $\mathbf{I} + \Lambda_s/\sigma^2$  is a diagonal matrix with elements strictly greater than unity, we have the following limit as  $p \rightarrow \infty$

$$\left(\frac{\mathbf{R}}{\sigma^2}\right)^{-p} = [\mathbf{V}_s \ \mathbf{V}_n] \begin{bmatrix} (\mathbf{I} + \Lambda_s/\sigma^2)^{-p} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} [\mathbf{V}_s \ \mathbf{V}_n]^H$$

$$\rightarrow [\mathbf{V}_s \ \mathbf{V}_n] \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} [\mathbf{V}_s \ \mathbf{V}_n]^H = \mathbf{V}_n \mathbf{V}_n^H \quad (7)$$

To blindly estimate the ST channel of user  $k$  we propose the following optimisation

$$\hat{\mathcal{H}}_k(i) = \arg \min_{\mathcal{H}} \mathcal{H}^H \mathcal{F}_k^H \mathbf{R}^{-p}(i) \mathcal{F}_k \mathcal{H} \quad (8)$$

subject to  $\|\hat{\mathcal{H}}_k(i)\| = 1$ , where  $p$  is an integer, and whose solution is the eigenvector corresponding to the minimum eigenvalue of the  $JL_p \times JL_p$  matrix  $\mathcal{F}_k^H \mathbf{R}^{-p}(i) \mathcal{F}_k$  that can be obtained using SVD. The performance of the estimator can be improved by increasing  $p$ , as in [23, 24], although, for DS-CDMA ST signals, our studies reveal that it suffices to use powers up to  $p = 2$  to obtain a good estimate of  $\mathbf{V}_n \mathbf{V}_n^H$ . To estimate the phase shift  $\Theta_{k,m}$  we propose the following approach

$$\hat{\Theta}_{k,m}(i) = \arg \left( \frac{1}{J-1} \sum_{l=1}^{J-1} \frac{\hat{h}_{k,m}^{(l)*}(i) \hat{h}_{k,m}^{(l-1)}(i)}{|\hat{h}_{k,m}^{(l)}(i)|^2} \right), \quad J \geq 2 \quad (9)$$

Once an estimate of the phase shift is obtained, the DoA estimate of the desired user and path is given by

$$\hat{\phi}_{k,m}(i) = \cos^{-1} \left( \frac{d}{2\pi\lambda} \hat{\Theta}_{k,m}(i) \right) \quad (10)$$

and then we can construct an estimate of the array manifold for user  $k$  with the DoA of each path  $m$  and sensor  $l$  as expressed by the  $JL_p \times 1$  vector  $\mathbf{a}(\hat{\Theta}_k(i)) = [1 \dots 1, e^{-j\hat{\Theta}_{k,1}(i)} \dots e^{-j\hat{\Theta}_{k,L_p}(i)}, \dots, e^{-j(J-1)\hat{\Theta}_{k,1}(i)} e^{-j(J-1)\hat{\Theta}_{k,L_p}(i)}]^T$ . An estimate of the spatial signature can be formed by  $\hat{\mathbf{s}}_k^a(i) = \mathcal{F}_k \mathbf{a}(\hat{\Theta}_k(i)) = [\hat{d}_k^{(1)}(i) \dots \hat{d}_k^{(JM)}(i)]^T$ . An analysis of the consistency, the necessary and sufficient conditions of channel identifiability of the method and a study of the capacity gains provided by the antenna array over a single sensor approach [23, 24] is included in the Appendix. Furthermore, although the method proposed in (7)–(10) leads to accurate DoA estimates, it does not provide improved channel estimates as compared to the algorithms in [23, 24]. To further enhance the channel estimates we describe two space-time channel estimation (SP-TE) algorithms in the next section.

### 4 SP-TE algorithms

Here we consider two algorithms that exploit the antenna array in order to enhance the channel estimates obtained

from (7) and use them as the starting point. The first algorithm employs the DoA to build a spatial signature estimate and formulate a new ST blind estimation algorithm. The second algorithm is very simple and consists in averaging the estimates obtained by the method of Section 3.

#### 4.1 Algorithm 1

Consider the  $JM \times L_p$  ST constraint matrix  $\mathcal{S}_k$  with one-chip shifted versions of the spatial signature  $\mathbf{s}_k^a(i) = \mathcal{F}_k \mathbf{a}(\Theta_k(i)) = [d_k^{(1)}(i) \dots d_k^{(JM)}(i)]^T$  of user  $k$

$$\mathcal{S}_k = \begin{bmatrix} \hat{d}_k^{(1)} & \mathbf{0} & & & \\ \vdots & \ddots & \hat{d}_k^{(1)} & & \\ \hat{d}_k^{(JM)} & \vdots & & & \\ \mathbf{0} & \ddots & \hat{d}_k^{(JM)} & & \end{bmatrix} = \begin{bmatrix} \mathbf{C}_k \\ \mathbf{C}_k \mathbf{z}(\Theta_{k,1}) \\ \vdots \\ \mathbf{C}_k \mathbf{z}(\Theta_{k,J-1}) \end{bmatrix} \quad (11)$$

where  $\mathbf{z}(\Theta_{k,l}) = \text{diag}(e^{-j\Theta_{k,l}}, \dots, e^{-j\Theta_{k,L_p}})$ . From (3) and (11), the  $k$ th user received signal without noise can be alternatively expressed by  $\mathbf{x}_k = A_k b_k \mathcal{S}_k \mathbf{h}_k$ . Again, since the subspaces  $\mathcal{V}_s$  and  $\mathcal{V}_n$  are orthogonal, we have the condition  $\mathbf{V}_n^H \mathbf{x}_k = \mathbf{V}_n^H \mathcal{S}_k \mathbf{h}_k = \mathbf{0}$  and hence  $\mathbf{h}_k^H \mathcal{S}_k^H \mathbf{V}_n \mathbf{V}_n^H \mathcal{S}_k \mathbf{h}_k = 0$  that allows the recovery of  $\mathbf{h}_k$  as the eigenvector corresponding to the smallest eigenvalue of the matrix  $\mathcal{S}_k^H \mathbf{V}_n \mathbf{V}_n^H \mathcal{S}_k$ , provided  $\mathbf{V}_n$  is known. Using (6), the estimates  $\hat{\mathbf{R}}$  and  $\hat{\mathcal{S}}_k$ , the ST channel estimate  $\hat{\mathbf{h}}_k$  can be computed by the following proposed optimisation

$$\hat{\mathbf{h}}_k(i) = \arg \min_{\mathbf{h}} \mathbf{h}^H \hat{\mathcal{S}}_k^H \mathbf{R}^{-p}(i) \hat{\mathcal{S}}_k \mathbf{h} \quad (12)$$

subject to  $\|\hat{\mathbf{h}}_k(i)\| = 1$ , whose solution is the eigenvector corresponding to the minimum eigenvalue of the  $L_p \times L_p$  matrix  $\hat{\mathcal{S}}_k^H \mathbf{R}^{-p}(i) \hat{\mathcal{S}}_k$  that can be obtained using SVD.

#### 4.2 Algorithm 2

A low complexity and simple alternative for enhancing the estimates obtained with (6) is to average the channel estimates over the  $J$  sensors as given by

$$\hat{\mathbf{h}}_k(i) = \frac{1}{J} \sum_{l=1}^J \mathbf{z}^*(\Theta_{k,l-1}) \hat{\mathbf{h}}_{k,l-1}(i), \quad J \geq 2 \quad (13)$$

Note that this strategy avoids another SVD on an  $L_p \times L_p$  matrix and can still improve the channel estimates. Alternatively, one can implement this algorithm to enhance the channel estimates through

$$\hat{\mathbf{h}}_{k,m}(i) = \frac{1}{J} \sum_{l=1}^J \hat{\mathbf{h}}_{k,m}^{(l-1)}(i) e^{j(l-1)\hat{\Theta}_{k,m}(i)} \quad (14)$$

### 5 Simulation experiments

We assess the mean squared error (MSE) performance of the proposed techniques for a differential BPSK DS-CDMA system with  $J = 1, 2, 3$  antenna elements. With respect to DoA estimation, the proposed method is compared with the MUSIC-based approach of [11] and the ESPRIT with maximum overlapped subarrays of [17], that employ  $J = 4$  antenna elements. For the comparison of channel estimates,

the proposed ST-CE algorithms are evaluated against the blind channel estimator of [23, 24]. The system employs random sequences of length  $N = 32$ , has perfect power control and synchronisation (as in [23, 24]) and the channel estimate phase ambiguity is resolved. The DoAs of each arrival path of the interferers are uniformly distributed in  $(0, 2\pi/3)$  and  $\mathbf{R}$  is estimated as  $\hat{\mathbf{R}}(i) = \sum_{m=1}^i \alpha^{i-m} \mathbf{r}(m) \mathbf{r}^H(m)$ , where  $\alpha = 0.998$  and includes ISI. The MSE is defined by  $\text{MSE}(i) = 1/L \sum_{n=1}^L \|\mathbf{h}_k(i) - \hat{\mathbf{h}}_k^n(i)\|^2$  for the channel estimates and  $\text{MSE}(i) = 1/L \sum_{n=1}^L 1/L_p \sum_{t=1}^{L_p} |e^{j\hat{\Theta}_{k,m}(i)} - e^{j\hat{\Theta}_{k,m}^n(i)}|^2$  for the DoA estimates, where  $L = 1000$  is the number of independent runs. All channels assume that  $L_p = 6$  is an upper bound and have a profile with three paths with relative powers 0, -3 and -6 dB, where for each run and user the spacing between paths is taken from a discrete uniform random variable between 1 and 2 chips.

In Figs. 1 to 3, we depict the DoA and CE performance for a non-fading scenario, where the system starts with  $K = 8$  users and the DoAs of the desired signal are 30, 45 and 60° for the first, second and third channel paths, respectively. In this regard, we will use the notation [30; 45, 60] for simplicity. At time 1500, the DoAs of the desired user are suddenly changed to [60, 30, 45] degrees and eight interferers enter the system. The results show that the proposed DoA estimation method can provide accurate DoA estimates, outperforming the MUSIC [15] and ESPRIT [17] based-algorithms that operate on despread signals with a higher number of sensors ( $J = 4$  against  $J = 2, 3$  with  $p = 1$ ). In our studies, we also noticed that for most practical scenarios, the proposed technique is superior to the existing approaches [15, 17] for the same  $J$ , leading to savings in sensors for the same performance. With respect to CE, the results in Figs. 2 and 3 show that the proposed ST-CE algorithms lead to improved channel estimates as compared to those of the CE [23, 24] and that increasing  $p$  enhances performance. In terms of computational complexity, the new method requires  $O((JM)^3)$  for the inversion of  $\mathbf{R}$ ,  $O((JL_p)^3)$  for (7) and  $O(JL_p)$  for (8) and (9), while the single-sensor approach of [23, 24] requires  $O(M^3)$  to invert the covariance matrix and  $O(L_p^3)$  to compute the SVD and identify the channel. The proposed Algs. I and II require additional  $O(L_p^3)$  and  $O(JL_p)$  to refine the channel estimates.

A mobile fading channel scenario is considered in Figs. 4 and 5. The channels experienced by different users are i.i.d., have the same profile as before, but are subject to fading according to Clarke's model [25]. The results are shown in terms of the normalised Doppler frequency  $f_d T$  (cycles/symbol). The curves indicate that the proposed techniques achieve an excellent performance for channel and DoA estimation and tracking. In particular, the proposed DoA estimator with  $J = 2$  slightly outperforms the MUSIC of [15] with  $J = 4$  and achieves a performance close to the ESPRIT of [17] with  $J = 4$ , whereas the proposed DoA method with  $J = 3$  outperforms the other analysed techniques. With regard to CE, the proposed ST-CE algorithms show a performance significantly superior to the existing CE algorithm of [23, 24].

In Fig. 5, we illustrate the bit error ratio (BER) of RAKE detectors designed with the channel and DoA estimates provided by the new method and algorithms for a mobile fading channel. Unlike the other scenarios, that had perfect power control, the power distribution among the interferers for each run follows a log-normal distribution with associated standard deviation of 3 dB and the desired user

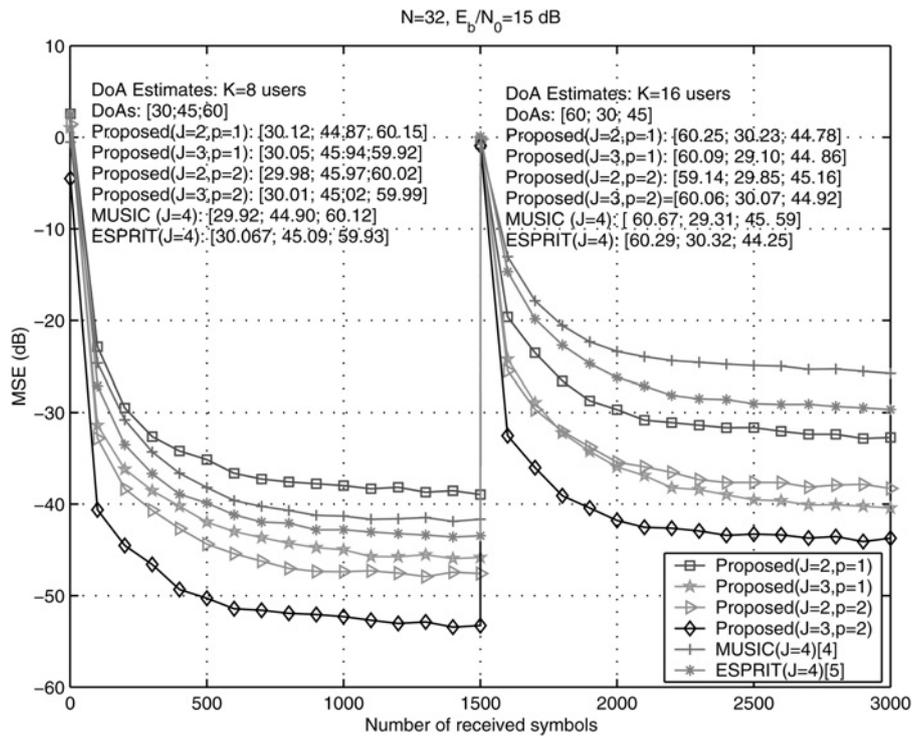


Fig. 1 Performance of DoA estimation against number of received symbols

DoA estimates at time instants 1500 and 3000 for the proposed technique with  $p = 1, 2$  and the methods in [15, 17] are shown above the curves

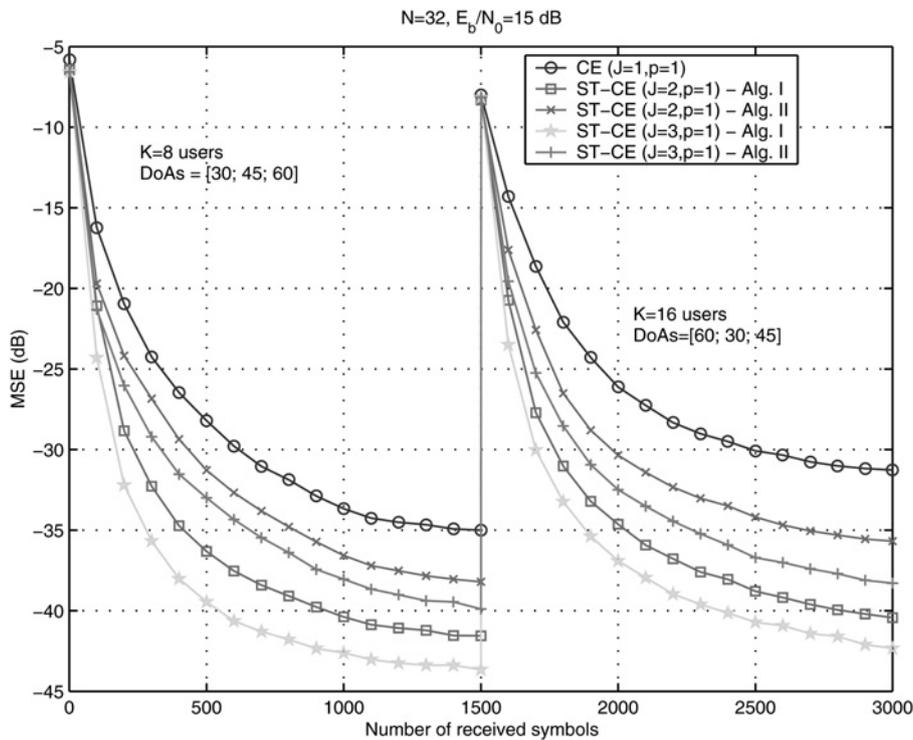


Fig. 2 Performance of CE against number of received symbols for  $p = 1$

power level corresponds to  $E_b/N_0 = 12$  dB. We compare the performance of the RAKE with the CE of [23, 24], the ST-RAKE for  $J = 2, 3$  with the proposed DoA and CE algorithms (Alg. I and Alg. II) and the ST-RAKE with known DoAs and channels. The detected symbols of the RAKE receiver for the adopted differential binary phase-shift

keying (BPSK) modulation are obtained as follows

$$\hat{b}_k(i) = \text{sgn}[\Re(z_k(i-1)^* z_k(i))] \quad (15)$$

where  $\text{sgn}(\cdot)$  is the slicer and  $z_k(i) = (\mathcal{S}_k(i) \hat{\mathbf{h}}_k(i))^H \mathbf{r}(i)$ , where  $\mathcal{S}_k$  is the matrix with one-chip shifted versions of the estimated spatial signature defined in (11).

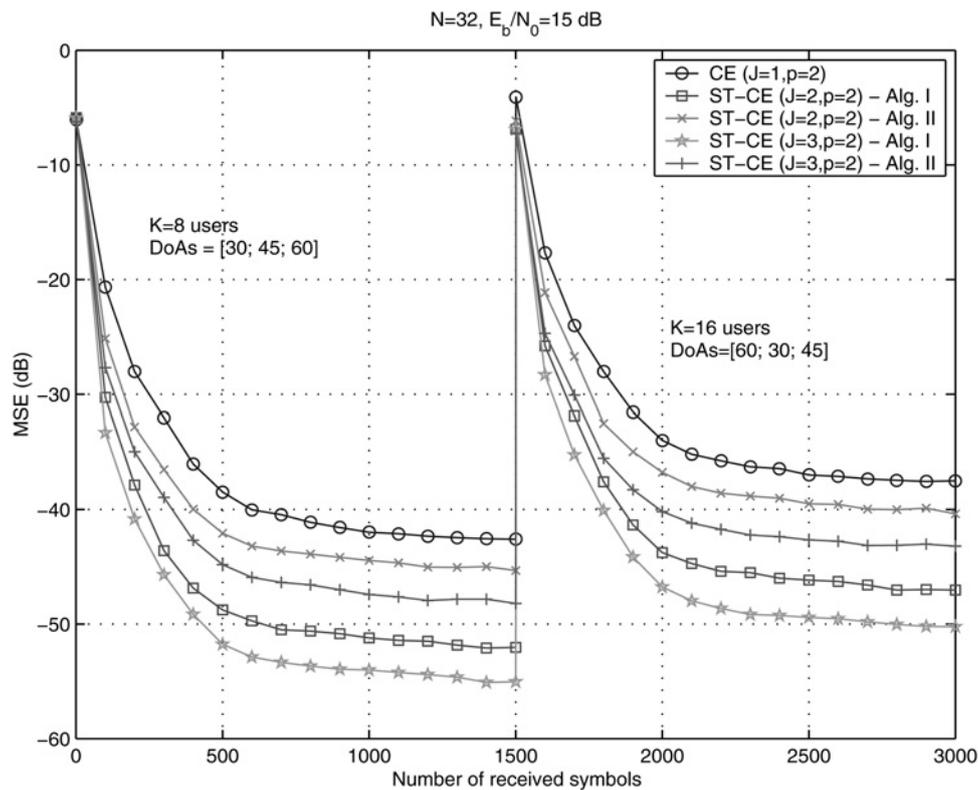


Fig. 3 Performance of CE against number of received symbols for  $p = 2$

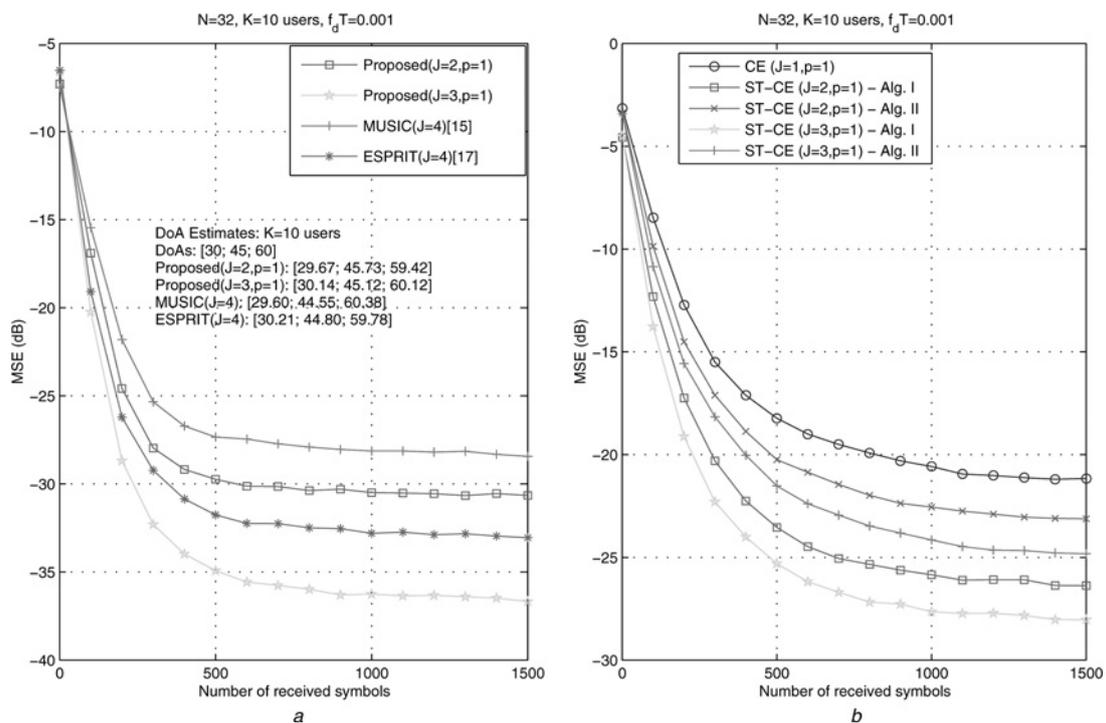


Fig. 4 Performance for a dynamic channel with  $f_d T = 0.001$  at  $E_b/N_0 = 15$  dB

a DoA

b CE against symbols

DoA estimates at time instant 1500 for the proposed technique with  $p = 1$  and the methods in [15, 17] are shown above the curves

The curves in Fig. 5 show that the proposed techniques achieve an excellent performance for channel and DoA estimation. Specifically, Alg. I can save up to 2 dB in comparison with Alg. II and accommodate more users for the same BER performance. We also notice that there is a

performance gap of up to 3 dB for  $J = 2$  between Alg. I and the ST-RAKE with known DoAs and channels, whereas this gap is reduced to 2 dB for  $J = 3$ , indicating the improvements obtained by the proposed DoA estimator and Alg. I with more sensors. In this context, it should be

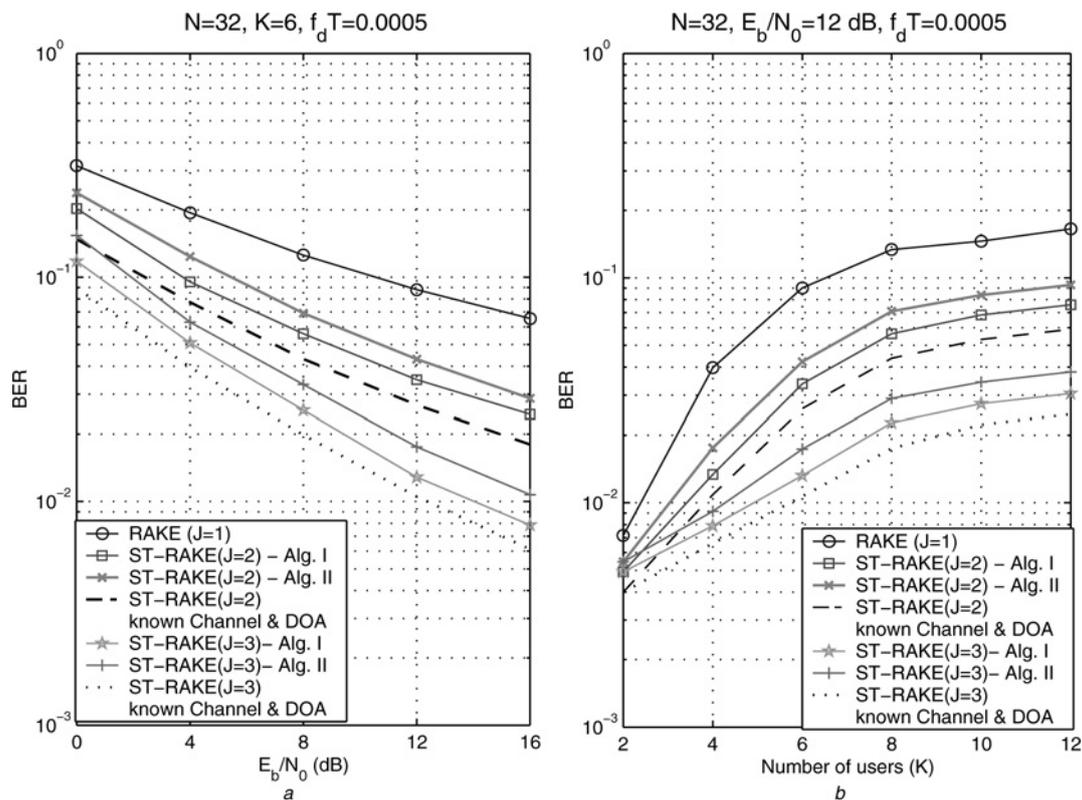


Fig. 5 BER performance of RAKE receivers

a Against  $E_b/N_0$   
b Against K

remarked that this performance gap is gradually reduced as the number of sensor elements is increased and the DoA estimates are refined.

## 6 Concluding remarks

A blind ST joint channel and DoA estimation method for DS-CDMA systems and two algorithms for enhancing the channel estimates are proposed. Simulations show significant improvements over existing techniques and a high accuracy achieved for DoA computation. An analysis of the consistency of the estimates provides guidelines with respect to the capacity improvement of the system. We also discussed and showed the necessary and sufficient conditions for the channel identifiability of the method.

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## 8 Appendix

Here we discuss necessary conditions with respect to the capacity in terms of maximum load as a function of the processing gain  $N$ , the propagation paths  $L_p$  and the number of antenna-array sensors  $J$  for the method to work and retrieve the parameters of interest. It should be remarked that these conditions were found to be accurate by several simulations and scenarios. Let  $r_s$  and  $r_n$  denote the signal and the noise subspace ranks, respectively, then the matrix  $V_n^H \mathcal{F}_k$  is of dimensions  $r_n \times JL_p$ . If the noise subspace  $V_n$  is the exact subspace then, because of  $V_n^H \mathcal{F}_k \mathcal{H}_k = \mathbf{0}$ , we conclude that the column rank of  $V_n^H \mathcal{F}_k$  can at most be  $JL_p - 1$ . In order to have a unique solution (times a phase ambiguity), the column rank of  $V_n^H \mathcal{F}_k$  must be exactly equal to  $JL_p - 1$ . Since a column rank of a square matrix is equal to its row rank in order to have a row rank equal to  $JL_p - 1$ , a necessary condition is to have at least  $JL_p - 1$  rows, that is,  $r_n \geq JL_p - 1$ . Since  $r_s + r_n = JM$  this yields  $r_s \leq JM - JL_p + 1$ . With regards to the signal subspace rank and assuming (for simplicity) symbol-by-symbol estimation and a synchronous system, the number of columns of  $V_s$  (which is an orthonormal basis for  $\mathbf{x}_k$ ) is  $r_s$  and is composed by the effective spatial signatures of all  $K$  users, that corresponds to a matrix with dimensions  $JM \times K$ , and ISI, that corresponds to a matrix with dimensions  $J(L_p - 1) \times K$ . Assuming that  $K < JN$ , we have that if

$$K + 2 \min\{J(L_p - 1), K\} \leq JM - JL_p + 1 \\ = J(N - 1) + 1 \quad (16)$$

then the necessary condition on  $r_s$  is satisfied and an upper bound for the maximum load of the system with antenna arrays is

$$K \leq J \left[ N - \frac{(J - 1)}{J} 2 \min \left\{ \frac{(N - (J - 1)/J)}{3}, L_p - 1 \right\} \right] \quad (17)$$

which indicates an increase in the system capacity as

compared to the result  $K \leq N - 2 \min\{N/3, L_p - 1\}$  of [24], provided  $J \geq 2$ .

In order to determine  $\mathcal{H}_k$  such that the method has a unique non-trivial solution, we are required to firstly satisfy the conditions stated in the previous section. They correspond to the necessary conditions. In this part, we are interested in establishing the sufficient conditions for the identifiability of  $\mathcal{H}_k$ .

Let us first consider the effective ST signature  $\mathcal{F}_k \mathcal{H}_k$  and rewrite in an alternative form

$$\mathcal{F}_k \mathcal{H}_k = \mathbf{v}_k = \mathcal{X} \mathbf{e}_k / A_k \quad (18)$$

where  $\mathbf{e}_k = [1 \ 0, \dots, 0]^T$ . From the above we have

$$\dim\{\text{range}(\mathcal{F}_k) \cap \text{range}(\mathcal{X})\} = 1 \quad (19)$$

where  $\dim\{\cdot\}$  stands for the dimension of a subspace and  $\mathcal{X} = \sum_{k=1}^K E[\mathbf{x}_k \mathbf{x}_k^H]$ . The following theorem established the identifiability result for the proposed method.

*Theorem:* Let  $\mathcal{F}_k$  and  $\mathcal{X}$  be full-column rank matrices and  $\alpha$  be an arbitrary scalar. Then the equation

$$V_n^H \mathcal{F}_k \mathcal{H} = \mathbf{0} \quad (20)$$

has a non-trivial solution other than  $\alpha \mathcal{H}_k$  if and only if the following condition holds:

*Condition:* There exists two vectors  $\tilde{\mathcal{H}} \neq \alpha \mathcal{H}_k$  and  $\mathbf{q}_k$  such that

$$\mathcal{F}_k \tilde{\mathcal{H}} = \mathcal{X} \mathbf{q}_k \quad (21)$$

which is equivalent to

$$\dim\{\text{range}(\mathcal{F}_k) \cap \text{range}(\mathcal{X})\} = 1 \quad (22)$$

*Proof:* Let us consider

$$V_n^H \mathcal{F}_k \mathcal{H} = \mathbf{0} \quad (23)$$

Let (23) be satisfied with some  $\tilde{\mathcal{H}} \neq \alpha \mathcal{H}_k$ . As the matrix  $\mathcal{X}$  is full-column rank, the relation in (23) directly yields

$$V_n^H \mathcal{F}_k \tilde{\mathcal{H}} \in \text{range}(\mathcal{X}) \quad (24)$$

Hence, from the above we have (21). The relation in (22) follows from (21) along with (18) and the fact that the vectors  $\mathcal{H}_k$  and  $\tilde{\mathcal{H}}$  are linearly independent. This proves the condition in (21). Now, to prove the sufficiency part, let us assume that the condition (21) holds. From (21), it follows that  $\tilde{\mathcal{H}} \neq \alpha \mathcal{H}_k$  is a solution to (23). This completes the proof.  $\square$