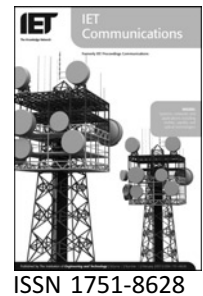


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# Blind adaptive and iterative interference cancellation receiver structures based on the constant modulus criterion in multipath channels

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**Abstract:** Blind adaptive and iterative interference cancellation (IC) receiver structures for direct sequence code division multiple access systems in multipath channels are proposed. A code-constrained constant modulus design criterion based on constrained optimisation techniques and adaptive algorithms for receiver and channel parameter estimation are described for successive IC (SIC) and parallel IC (PIC) detectors and a new hybrid IC (HIC) scheme in scenarios subject to multipath fading. The proposed HIC structure combines the strengths of linear, SIC and PIC receivers and is shown to outperform the conventional linear, SIC and PIC structures. A novel iterative detection approach that generates different cancellation orders and selects the most likely symbol estimate on the basis of the instantaneous minimum constant modulus criterion is also proposed and combined with the new HIC structure to further enhance performance. Simulation results for an uplink scenario assess the algorithms, the proposed blind adaptive IC detectors against existing receivers and evaluate the effects of error propagation of the new cancellation techniques.

## 1 Introduction

Multisuser detection is a set of techniques that deals with the suppression of multiaccess interference (MAI), increasing the capacity and the performance of code division multiple access (CDMA) systems [1]. The optimal multisuser detector has been proposed by Verdú in [2]; however, its prohibitive complexity makes its deployment unfeasible and motivated the development of several suboptimal schemes that are amenable to implementation: the linear [3] and decision feedback [4] receivers, the successive interference canceller (SIC) [5] and the multistage detector or parallel interference canceller (PIC) [6]. These suboptimal receivers require the estimation of various parameters in order to carry out interference mitigation.

In most practical scenarios such as those subject to multipath fading channels, the parameter estimation of the

receiver has to be computed adaptively in order to track the time-varying channel conditions. Among the existing adaptive parameter estimation techniques, one can broadly divide them into two classes: supervised and unsupervised (blind) methods. In this context, blind adaptive parameter estimation methods have been reported in [7–9] along with linear detectors and have proven to be very valuable techniques that can alleviate the need for training sequences, increasing the throughput and efficiency of wireless networks.

For uplink scenarios, SIC [5, 10, 11] and PIC [6, 12–16] receivers, which are relatively simple and perform interference cancellation by sequentially or iteratively removing the multiaccess interference (MAI), are known to provide significant gains over RAKE and linear detectors. In this regard, the work on SIC and PIC detectors is very limited with respect to blind parameter estimation in multipath, despite the effectiveness of these structures for the uplink.

The goal of this paper is to propose blind adaptive and iterative (IT) receiver structures that employ algorithms based on the code-constrained constant modulus (CCM) criterion. First, we describe a CCM design criterion for the receiver and derive computationally efficient stochastic gradient (SG) and recursive least squares (RLS) type algorithms for receiver and channel parameter estimation. Secondly, we present SIC and PIC detectors and a new hybrid IC scheme, denoted as HIC, that employ a linear receiver front end and an SG amplitude estimation algorithm. The new HIC scheme uses a linear detection front end with an SIC architecture and multiple stages for IC such as PIC receivers, gathering the strengths of the linear, SIC and PIC receivers. The third contribution is a novel IT detection method for the proposed SIC receiver that generates different cancellation orders and selects the most likely symbol estimate on the basis of the instantaneous minimum constant modulus (CM) criterion. The IT detection with the SIC scheme is then combined with multiple IC stages, resulting in an IT detection with the new HIC structure that is shown to achieve very substantial gains over conventional receivers.

This paper is organised as follows. Section 2 briefly describes the direct sequence CDMA (DS-CDMA) communication system model. The linearly constrained SIC, PIC and HIC receivers, the CCM design criterion, the amplitude estimation and the blind channel estimator are presented in Section 3. Section 4 is devoted to the proposed IT-SIC and HIC detection based on parallel arbitrated branches and the CM criterion, whereas Section 5 is dedicated to the derivation of adaptive SG algorithms and RLS-type algorithms. Section 5 presents and discusses the simulation results and Section 7 gives the concluding remarks of this work.

## 2 DS-CDMA system model

Let us consider the uplink of a symbol synchronous binary phase-shift keying (BPSK) DS-CDMA system with  $K$  users,  $N$  chips per symbol and  $L_p$  propagation paths. It should be remarked that a synchronous model is assumed for simplicity, although it captures most of the features of more realistic asynchronous models with small to moderate delay spreads. The baseband signal transmitted by the  $k$ th active user to the base station is given by

$$x_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k(i) p_k(t - iT) \quad (1)$$

where  $b_k(i) \in \{\pm 1\}$  denotes the  $i$ th symbol for user  $k$ , the real-valued spreading waveform and the amplitude associated with user  $k$  are  $p_k(t)$  and  $A_k$ , respectively. The spreading waveforms are expressed by  $p_k(t) = \sum_{i=1}^N a_k(i) \phi(t - iT_c)$ , where  $a_k(i) \in \{\pm 1/\sqrt{N}\}$ ,  $\phi(t)$  is the chip waveform,  $T_c$  is the chip duration and  $N = T/T_c$  is the processing gain. Assuming that the receiver is

synchronised with the main path, the coherently demodulated composite received signal is

$$r(t) = \sum_{k=1}^K \sum_{l=0}^{L_p-1} h_{k,l}(t) x_k(t - \tau_{k,l}) + n(t) \quad (2)$$

where  $h_{k,l}(t)$  and  $\tau_{k,l}$  are, respectively, the channel coefficient and the delay associated with the  $l$ th path and the  $k$ th user. Assuming that  $\tau_{k,l} = lT_c$  (the delays are multiples of the chip duration), the channel is constant during each symbol interval and the spreading codes are repeated from symbol to symbol, the received signal  $r(t)$  after filtering by a chip-pulse-matched filter and sampled at chip rate yields the  $M$ -dimensional received vector

$$r(i) = \sum_{k=1}^K \mathbf{H}_k(i) A_k \mathbf{P}_k \mathbf{b}_k(i) + \mathbf{n}(i) \quad (3)$$

where  $M = N + L_p - 1$ ,  $\mathbf{n}(i) = [n_1(i) \dots n_M(i)]^T$  is the complex Gaussian noise vector with  $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma^2 \mathbf{I}$ , where  $(\cdot)^T$  and  $(\cdot)^H$  denote transpose and Hermitian transpose, respectively,  $E[\cdot]$  stands for expected value, the user symbol vector is  $\mathbf{b}_k(i) = [b_k(i + L_s - 1) \dots b_k(i) \dots b_k(i - L_s + 1)]^T$ , the amplitude of user  $k$  is  $A_k$ , the channel vector of user  $k$  is  $\mathbf{h}_k(i) = [h_{k,0}(i) \dots h_{k,L_p-1}(i)]^T$ ,  $(2L_s - 1)$  is the intersymbol interference (ISI) span, and the  $((2L_s - 1) \times N) \times (2L_s - 1)$  diagonal matrix  $\mathbf{S}_k$  with  $N$ -chip shifted versions of the signature of user  $k$  is given by

$$\mathbf{P}_k = \begin{bmatrix} \mathbf{p}_k & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_k & \ddots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{p}_k \end{bmatrix} \quad (4)$$

where  $\mathbf{p}_k = [a_k(1) \dots a_k(N)]^T$  is the signature sequence for the  $k$ th user, and the  $M \times ((2L_s - 1)N)$  channel matrix  $\mathbf{H}_k(i)$  for user  $k$  is

$$\mathbf{H}_k(i) = \begin{bmatrix} h_{k,0}(i) & \dots & h_{k,L_p-1}(i) & \dots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & h_{k,0}(i) & \dots & h_{k,L_p-1}(i) \end{bmatrix} \quad (5)$$

where  $h_{k,l}(i) = h_{k,l}(iT_c)$ . The MAI comes from the non-orthogonality between the received signature sequences, whereas the ISI span  $L_s$  depends on the length of the channel impulse response, which is related to the length of the chip sequence. For  $L_p = 1, L_s = 1$  (no ISI), for  $1 < L_p \leq N, L_s = 2$  and for  $N < L_p \leq 2N, L_s = 3$ .

### 3 Linearly constrained IC receiver structures

Let us describe the design of synchronous blind linearly constrained IC schemes. The proposed IC schemes in this paper have a blind linear receiver front end which is based on the CCM cost function [8, 9]. In this section, we propose an SIC receiver and an HIC receiver.

#### 3.1 SIC receivers

At each symbol, an SIC receiver select the strongest user (decreasing power level order) and then sequentially regenerates and cancels the interference contribution of every user at each level. For the SIC, the number of levels refers to the number of users  $K$ . The received signal at the  $k$ th SIC level is described by

$$\mathbf{r}_k(i) = \mathbf{r}(i) - \sum_{j=1}^{k-1} \hat{A}_j(i) \hat{b}_j(i) \hat{s}_j(i), \quad k = 1, 2, \dots, K \quad (6)$$

and  $\mathbf{r}_1(i) = \mathbf{r}(i)$ , for  $k = 1$  (the first user),  $\hat{A}_j(i)$  is the  $j$ th user amplitude estimate, the  $M \times 1$  effective signature estimate is  $\hat{s}_j(i) = \mathbf{C}_j \hat{b}_j(i)$ , where  $\hat{b}_j(i)$  is the channel estimate at level  $j$  and  $\mathbf{C}_k$  is an  $M \times L_p$  convolution matrix containing one-chip shifted versions of the  $k$ th user signature sequence  $\mathbf{p}_k$

$$\mathbf{C}_k = \begin{bmatrix} a_k(1) & & \mathbf{0} \\ \vdots & \ddots & a_k(1) \\ a_k(N) & & \vdots \\ \mathbf{0} & \ddots & a_k(N) \end{bmatrix} \quad (7)$$

The SIC technique exploits the power level difference between the users and performs very well in scenarios without tight power control. However, it generally leads to non-uniform performance among the users [5, 10, 11].

The proposed SIC receiver detects users in a multilevel fashion using a blind receiver front end. The receiver front-end design determines an finite impulse response (FIR) filter  $\mathbf{w}_k(i)$  with  $M$  coefficients for user  $k$ , which provides a first estimate of the desired symbol as given by

$$\hat{b}_k(i) = \text{sgn}(\Re[\mathbf{w}_k^H(i) \mathbf{r}_k(i)]), \quad k = 1, 2, \dots, K \quad (8)$$

where  $\Re(\cdot)$  selects the real part,  $\text{sgn}(\cdot)$  is the signum function and  $\mathbf{r}_k(i)$  is the received signal at the  $k$ th SIC level given by (6). Consider the received signal  $\mathbf{r}(i)$  and the  $M \times L_p$  matrix  $\mathbf{C}_k$ .

The CCM receiver parameter vector  $\mathbf{w}_k(i)$  results from the minimisation of the CM cost function [8, 9]

$$J_{\text{CM}} = E[|\mathbf{w}_k^H(i) \mathbf{r}_k(i)|^2 - 1]^2 \quad (9)$$

subject to the linear constraints  $\mathbf{C}_k^H \mathbf{w}_k(i) = \nu \mathbf{b}_k(i)$ , where  $\nu$  is a constant to ensure the convexity of the CM-based receiver (see 9.1 for further discussion). Assuming that the channel vector  $\mathbf{b}_k$  is known, the expression for the CCM receiver is [9]

$$\mathbf{w}_k(i) = \mathbf{R}_k^{-1}(i) [\mathbf{d}_k(i) - \mathbf{C}_k (\mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{C}_k)^{-1} \times (\mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{d}_k(i) - \nu \mathbf{b}_k(i))] \quad (10)$$

where

$$\mathbf{R}_k(i) = E[|z_k(i)|^2 \mathbf{r}_k(i) \mathbf{r}_k^H(i)] \quad (11)$$

$$\mathbf{d}_k(i) = E[z_k^*(i) \mathbf{r}_k(i)] \quad (12)$$

and

$$z_k(i) = \mathbf{w}_k^H(i) \mathbf{r}_k(i) \quad (13)$$

The asterisk denotes complex conjugation. We also note that the right-hand side of (10) is still a function of  $\mathbf{w}_k(i)$  and the channel  $\mathbf{b}_k(i)$ . Adaptive methods for the estimation of  $\mathbf{w}_k(i)$  and  $\mathbf{b}_k(i)$  are presented in Section 5. The channel estimation procedure adopted here computes

$$\hat{\mathbf{b}}_k(i) = \min_{\mathbf{b}} \mathbf{b}^H \mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{C}_k \mathbf{b} \quad (14)$$

subject to  $\|\hat{\mathbf{b}}_k\| = 1$  and whose solution is the eigenvector corresponding to the minimum eigenvalue of the  $L_p \times L_p$  matrix  $\mathbf{C}_k^H \mathbf{R}_k^{-1} \mathbf{C}_k$ . The use of  $\mathbf{R}_k$  given by (11) instead of  $\mathcal{R}_k = E[\mathbf{r}_k(i) \mathbf{r}_k^H(i)]$ , as in [17], avoids the estimation of both  $\mathcal{R}_k$  and  $\mathbf{R}_k$ , and shows no performance loss as verified in our studies (see 9.2 for further discussion). Unlike the original channel estimator of [17], the proposed channel estimation scheme exploits SIC at the  $k$ th stage to improve channel estimation. It should be remarked that SIC renders itself naturally to exploit IC and enhance channel estimates.

The amplitude estimation procedure considers the optimisation  $\hat{A}_j(i) = \min_A E[\|\hat{A}_j(i) \hat{b}_j(i) \hat{s}_j(i) - \mathbf{r}_j(i)\|^2]$  and employs the following adaptive recursion

$$\hat{A}_j(i+1) = \hat{A}_j(i) - \mu (\hat{A}_j(i) \hat{s}_j^H(i) \hat{s}_j(i) - \hat{b}_j^*(i) \mathbf{r}_j^H(i) \hat{s}_j(i)) \quad (15)$$

#### 3.2 HIC receivers

The proposed blind IT-HIC receivers employ the SIC in the first stage followed by multiple PIC stages. The symbol estimates of the SIC stage, given by (8), are used as initial decisions and are further refined by the PIC stages. The  $k$ th user received signal at the SIC stage which corresponds to the first stage of HIC receiver is given by (6), whereas for the remaining stages of the HIC receiver structure (PIC stages), the received signal is given by

$$\mathbf{r}_{k,m}(i) = \mathbf{r}(i) - \sum_{j \neq k} \hat{A}_j(i) \hat{b}_{j,m-1}(i) \hat{s}_j(i), \quad m \geq 2 \quad (16)$$

where  $\hat{b}_{j,1}(i) = \hat{b}_j(i)$  is the the symbol estimates of the SIC given by (8) and  $\hat{b}_{j,m-1}(i)$  is the detected symbol at stage  $m - 1$  ( $m \geq 2$ ) for user  $j$  and symbol  $i$ , which are given by

$$\hat{b}_{k,m}(i) = \text{sgn}(\Re[\mathbf{w}_k^H(i)\mathbf{r}_{k,m}(i)]), \quad k = 1, 2, \dots, K, \quad (17)$$

$$m \geq 2$$

We can clearly see the PIC concept expressed by (16) in contrast to the SIC concept in (6). In (16), for a given stage  $m$  and a desired user  $k$ , the MAI estimate due to all but the desired user is subtracted simultaneously from the received signal, whereas in (6), this interference cancellation is performed in a sequential fashion.

The amplitude and channel estimation procedures as well as the design of the detection filters  $\mathbf{w}_k(i)$  are accomplished at the first stage (SIC) and used throughout the HIC architecture. It should be remarked that SIC renders itself naturally to exploit IC and enhance channel estimates.

### 4 Iterative IC and detection based on parallel arbitration and CM criterion

Here, we describe an IT-SIC detection scheme based on the computation of different orderings and their exploitation to enhance receiver performance. The proposed IT-SIC scheme is then combined with multiple stages, resulting in the proposed IT-HIC scheme that combines the strengths of linear, SIC, PIC and the proposed IT scheme.

#### 4.1 IT-SIC receivers

In the proposed IT-SIC detection scheme, depicted in Fig. 1, the blind channel estimates are provided to the bank of RAKE receivers, whose outputs are used to compute different orderings for serial cancellation. The new IT approach generates  $Q$  different orderings for interference cancellation, which are carried out in the following way. For ordering vector  $\mathbf{v}_q$  ( $1 \leq q \leq Q$ ) with  $K$  elements, which contains the

cancellation order of each user, the receiver structure switches to the corresponding  $k$ th user blind adaptive linear detector and performs interference suppression.

The interference suppression is carried out at each stage by a linear detector whose parameter vector  $\mathbf{w}_k(i)$  is designed with the CCM criterion as given by (10) and provides an estimate of the transmitted symbol through (8), where  $\mathbf{r}_k(i)$  is the observed vector at stage  $k$ . After linear interference suppression, the user  $k$  signal  $\mathbf{x}_k(i)$  is reconstructed, with the aid of channel and amplitude estimates, and subtracted from the observation signal  $\mathbf{r}_k(i)$  at the  $k$ th stage. This procedure is repeated for the  $K$  users and the  $Q$  different orderings, yielding  $Q$  detection candidates for every symbol  $i$  and user  $k$ . The received signal for the proposed IT scheme at the  $k$ th level and ordering  $q$  is described by

$$\mathbf{r}_k^q(i) = \mathbf{r}(i) - \sum_{j=1}^{k-1} \hat{A}_{\mathbf{v}_q(j)}(i) \hat{b}_{\mathbf{v}_q(j)}(i) \hat{\mathbf{s}}_{\mathbf{v}_q(j)}(i) \quad (18)$$

where  $\mathbf{v}_q(m)$  is the  $m$ th index of the ordering vector  $\mathbf{v}_q$ . The amplitude estimation considers the optimisation  $\hat{A}_{\mathbf{v}_q(j)} = \min_A E[\|A \hat{b}_{\mathbf{v}_q(j)}(i) \hat{\mathbf{s}}_{\mathbf{v}_q(j)}(i) - \mathbf{r}_j^q(i)\|^2]$  and employs the following adaptive recursion

$$\hat{A}_{\mathbf{v}_q(j)}(i+1) = \hat{A}_{\mathbf{v}_q(j)}(i) - \mu(\hat{A}_{\mathbf{v}_q(j)}(i) \hat{\mathbf{s}}_{\mathbf{v}_q(j)}^H(i) \hat{\mathbf{s}}_{\mathbf{v}_q(j)}(i) - \hat{b}_{\mathbf{v}_q(j)}^*(i) \mathbf{r}_j^{qH}(i) \hat{\mathbf{s}}_{\mathbf{v}_q(j)}(i)) \quad (19)$$

The strategy to select the ordering vectors  $\mathbf{v}_q$  is to provide sufficiently different local maxima of the likelihood function. Unlike the related work of Barriac and Madhow [18] that employed matched filters as the starting point, we adopt blind linear receivers as the initial condition and the instantaneous CM cost function as the candidate selection criterion. In addition, we also consider the optimum ordering algorithm as a generalisation of our scheme. Let us consider the ordering vector for a  $Q = 4$  branch IT scheme. The first vector  $\mathbf{v}_1$  corresponds to a conventional SIC with users following a decreasing power order and the remaining ordering vectors  $\mathbf{v}_q$  are permuted versions of  $\mathbf{v}_1$  as given by

$$\mathbf{v}_q = \mathbf{M}_q \mathbf{v}_1 \quad (20)$$

where

$$\mathbf{M}_1 = \mathbf{I}_K, \quad \mathbf{M}_2 = \begin{bmatrix} \mathbf{0}_{K/4,3K/4} & \mathbf{I}_{3K/4} \\ \mathbf{I}_{K/4} & \mathbf{0}_{K/4,3K/4} \end{bmatrix}$$

$$\mathbf{M}_3 = \begin{bmatrix} \mathbf{0}_{K/2} & \mathbf{I}_{K/2} \\ \mathbf{I}_{K/2} & \mathbf{0}_{K/2} \end{bmatrix}, \quad \mathbf{M}_4 = \begin{bmatrix} 0 & \dots & 1 \\ \vdots & \dots & \vdots \\ 1 & \dots & 0 \end{bmatrix} \quad (21)$$

where  $\mathbf{0}_{m,n}$  denotes an  $m \times n$ -dimensional matrix of zeros and the structures of the matrices  $\mathbf{M}_q$  correspond to phase shifts regarding the cancellation order of the users. Specifically,

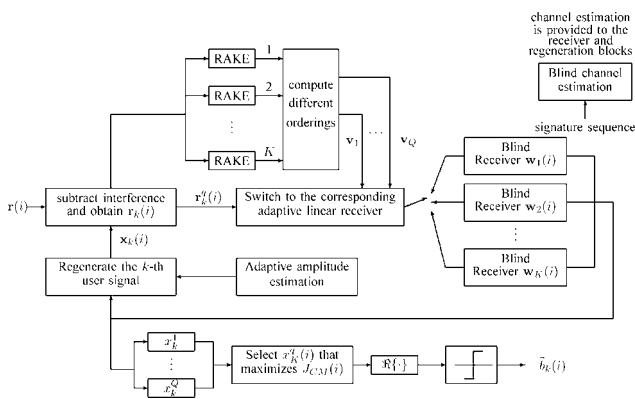


Figure 1 Proposed blind adaptive IT-SIC receiver block diagram

these matrices perform the cancellation with the following order:  $\mathbf{M}_1$  follows  $\mathbf{v}_1$ ;  $\mathbf{M}_2$  with indices  $K/4, K/4 + 1, \dots, K, 1, \dots, K/4 - 1$  of  $\mathbf{v}_1$ ;  $\mathbf{M}_3$  with indices  $K/2, K/2 + 1, \dots, K, 1, \dots, K/2 - 1$  of  $\mathbf{v}_1$ ;  $\mathbf{M}_4$  with the reverse order of  $\mathbf{v}_1$ . For more branches, additional phase shifts are applied with respect to user cancellation ordering. Note that different update orders were tested although they did not result in performance improvements.

The CM cost function is then used to select the symbol estimates and their corresponding amplitudes, resulting in a performance very close to the minimum mean square error (MMSE), as indicated in our studies. The final output  $\hat{b}_k^f(i)$  of the proposed IT-SIC receiver chooses the best estimate of the  $Q$  candidates according to

$$\hat{b}_k^{(f)}(i) = \text{sgn}[\Re(\arg \min_{1 \leq q \leq Q} \text{CM}_k^q(i))] \quad (22)$$

where the best estimate is the value  $z_k^q(i) = \mathbf{w}_k^H(i) \mathbf{r}_k^q(i)$  that minimises  $\text{CM}_k^q(i) = (|z_k^q(i)|^2 - 1)^2$ . The parameter  $Q$  must be chosen by the designer and our studies indicate that  $Q = 4$  achieves most of the gains of the new structure and offers a good trade-off between performance and complexity. In this regard, the optimal ordering algorithm is given by  $\hat{b}_k^{(f)}(i) = \text{sgn}[\Re(\arg \min_{1 \leq q \leq K!} \text{CM}_k^q(i))]$ , where the number of candidates is  $Q = K!$  and is clearly very complex for practical systems. The IT method employs the same filters, namely  $\mathbf{w}_k$ , for the  $Q$  orderings to compute the detection candidates and requires additional arithmetic operations.

### 4.2 IT-HIC receivers

The proposed IT-HIC receiver employs the IT-SIC in the first stage and multiple PIC stages to further refine the symbol estimates (Fig. 2). The received signal at the  $k$ th user of the IT-SIC stage which corresponds to the first stage of IT-HIC receiver is given by (18)–(21), whereas for the remaining stages of the IT-HIC structure, the received signal is given by

$$\mathbf{r}_{k,m}(i) = \mathbf{r}(i) - \sum_{j \neq k} \hat{A}_j(i) \hat{b}_{j,m-1}(i) \hat{\mathbf{s}}_j(i), \quad m \geq 2 \quad (23)$$

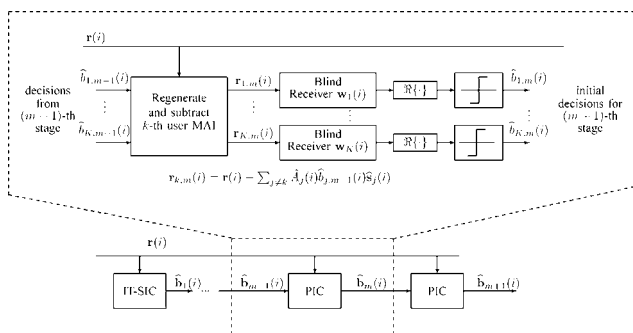


Figure 2 Proposed blind adaptive IT-HIC receiver block diagram

where  $\hat{b}_{j,1}(i) = \hat{b}_j^{(f)}(i)$  which means that IT-HIC uses the symbol estimates of the IT-SIC given by (22) as initial decisions and  $\hat{b}_{j,m-1}(i)$  is the detected symbol at stage  $m - 1$  ( $m \geq 2$ ) for user  $j$  and symbol  $i$ , which are given by

$$\hat{b}_{k,m}(i) = \text{sgn}[\Re[\mathbf{w}_k^H(i) \mathbf{r}_{k,m}(i)]], \quad k = 1, 2, \dots, K, \quad m \geq 2 \quad (24)$$

The amplitude estimation and channel estimation procedures as well as the design of the detection filters  $\mathbf{w}_k(i)$  are accomplished at the IT-SIC stage and used throughout the IT-HIC architecture.

## 5 Blind adaptive constrained algorithms

Owing to the mobile radio propagation channel, which is subject to impairments like fading and multipath propagation, the use of adaptive versions of multiuser receivers, coupled with dynamic estimation of the channel parameters, may present significant gains while they present limited complexity. Here, we describe SG and RLS algorithms for the blind estimation of the channel and the parameter vector  $\mathbf{w}_k$  of the SIC receivers using the CCM criterion.

### 5.1 Constrained constant modulus (CCM) SG algorithm

To derive an CCM-SG algorithm, let us consider the Lagrangian cost function

$$\mathcal{L}_{\text{CCM}} = (|z_k(i)|^2 - 1)^2 + 2\Re[(\mathbf{C}_k^H \mathbf{w}_k(i) - \mathbf{b}_k(i))^H \boldsymbol{\lambda}] \quad (25)$$

where  $z_k(i) = \mathbf{w}_k^H(i) \mathbf{r}_k(i)$  and  $\boldsymbol{\lambda}$  is a vector of Lagrange multipliers. An SG solution to (25) can be obtained by taking the gradient terms with respect to  $\mathbf{w}_k(i)$  which yields the following recursion for  $\mathbf{w}_k(i)$

$$\mathbf{w}_k(i + 1) = \mathbf{w}_k(i) - \mu_{w} \nabla_{\mathbf{w}_k^*} \mathcal{L}_{\text{CCM}} \quad (26)$$

Enforcing the constraints on  $\mathbf{w}_k$  to be  $\mathbf{C}_k^H \mathbf{w}(i + 1) = \mathbf{b}_k(i)$  and solving for the Lagrange multipliers, we obtain

$$\mathbf{w}_k(i + 1) = \mathbf{I}_k(\mathbf{w}_k(i) - \mu_{w} e_k(i) z_k^*(i)) + \mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1} \mathbf{b}_k(i) \quad (27)$$

where  $e_k(i) = (|z_k(i)|^2 - 1)$ ,  $\mathbf{I}_k = \mathbf{I} - \mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1} \mathbf{C}_k^H$ . The normalised version of this algorithm is adopted in order to make easier the choice of the step size, also guaranteeing stability. The algorithm utilises  $\mu_{w}(\mu_{0_w}(|z_k(i)| + 1)/|z_k(i)| e_k(i) \mathbf{r}_k^H(i) \mathbf{I}_k \mathbf{r}_k(i))$ , where  $\mu_{0_w}$  is the convergence factor. To estimate the channel and avoid the SVD on  $\mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{C}_k$  required in (14), we compute the estimates



$\mathbf{\Omega}_k(i) = \mathbf{C}_k^H \hat{\mathbf{\Psi}}_k(i)$ , where  $\hat{\mathbf{\Psi}}_k(i)$  is an estimate of the matrix  $\mathbf{R}_k^{-1}(i)\mathbf{C}_k$  generated by the following recursion

$$\hat{\mathbf{\Psi}}_k(i) = \alpha \hat{\mathbf{\Psi}}_k(i-1) + \mu_b (\hat{\mathbf{\Psi}}_k(i-1) - \mathbf{r}_k(i) \mathbf{r}_k^H(i) \hat{\mathbf{\Psi}}_k(i-1)) \quad (28)$$

with  $\hat{\mathbf{\Psi}}_k(0) = \mathbf{C}_k$  and  $0 < \alpha < 1$ . To estimate the channel, an iteration of a variant of the power method [17] is used

$$\hat{\mathbf{b}}_k(i) = (\mathbf{I} - \gamma(i) \mathbf{\Omega}_k(i)) \hat{\mathbf{b}}_k(i-1) \quad (29)$$

where  $\gamma(i) = 1/\text{tr} \mathbf{\Omega}_k(i)$ , where tr stands for trace. We make  $\hat{\mathbf{b}}_k(i) \leftarrow \hat{\mathbf{b}}_k(i)/\|\hat{\mathbf{b}}_k(i)\|$  to normalise the channel.

### 5.2 Blind adaptive CCM RLS-type (CCM-RLS) algorithm

Given the solution for  $\mathbf{w}_k$  in (10), we develop an algorithm that estimates the matrices  $\mathbf{R}_k^{-1}$  and  $(\mathbf{C}_k^H \mathbf{R}_k^{-1} \mathbf{C}_k)^{-1}$  recursively, reducing the computational complexity. Using the matrix inversion lemma and Kalman RLS recursions, we have

$$\mathbf{G}_k(i) = \frac{\alpha^{-1} \hat{\mathbf{R}}_k^{-1}(i-1) \mathbf{z}_k(i) \mathbf{r}_k(i)}{1 + \alpha^{-1} \mathbf{r}_k^H(i) \mathbf{z}_k(i) \hat{\mathbf{R}}_k^{-1}(i-1) \mathbf{z}_k^*(i) \mathbf{r}_k(i)} \quad (30)$$

$$\hat{\mathbf{R}}_k^{-1}(i) = \alpha^{-1} \hat{\mathbf{R}}_k^{-1}(i-1) - \alpha^{-1} \mathbf{G}_k(i) \mathbf{z}_k^*(i) \mathbf{r}_k^H(i) \hat{\mathbf{R}}_k^{-1}(i-1) \quad (31)$$

where  $\mathbf{G}_k$  is the Kalman gain vector with dimension  $M \times 1$ ,  $\hat{\mathbf{R}}_k$  is the estimate of the matrix  $\mathbf{R}_k$  and  $0 < \alpha < 1$  is the forgetting factor. At each processed symbol, the matrix  $\hat{\mathbf{R}}_k^{-1}(i)$  is updated and we employ another recursion to estimate  $\mathbf{\Gamma}_k^{-1}(i) = (\mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{C}_k)^{-1}$  as described by

$$\hat{\mathbf{\Gamma}}_k^{-1}(i) = \frac{\hat{\mathbf{\Gamma}}_k^{-1}(i-1)}{1 - \alpha} - \frac{\hat{\mathbf{\Gamma}}_k^{-1}(i-1) \gamma_k(i) \mathbf{\gamma}_k^H(i) \hat{\mathbf{\Gamma}}_k^{-1}(i-1)}{(1 - \alpha)^2 / \alpha + (1 - \alpha) \gamma_k^H(i) \hat{\mathbf{\Gamma}}_k^{-1}(i) \gamma_k(i)} \quad (32)$$

where  $\mathbf{\gamma}_k(i) = \mathbf{C}_k^H \mathbf{r}_k(i) \mathbf{z}_k(i)$ . To estimate the channel and avoid the SVD on  $\mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{C}_k$ , we compute an estimate of  $\mathbf{\Gamma}_k(i) = \mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{C}_k$  as  $\hat{\mathbf{\Gamma}}_k(i) = \mathbf{C}_k^H \hat{\mathbf{R}}_k^{-1}(i) \mathbf{C}_k$  and employ the variant of the power method introduced in [17]

$$\hat{\mathbf{b}}_k(i) = (\mathbf{I} - \gamma(i) \hat{\mathbf{\Gamma}}_k(i)) \hat{\mathbf{b}}_k(i-1) \quad (33)$$

where  $\gamma(i) = 1/\text{tr} \hat{\mathbf{\Gamma}}_k(i)$ . We make  $\hat{\mathbf{b}}_k(i) \leftarrow \hat{\mathbf{b}}_k(i)/\|\hat{\mathbf{b}}_k(i)\|$  to normalise the channel. The CCM linear receiver is then designed as described by

$$\hat{\mathbf{w}}_k(i) = \hat{\mathbf{R}}_k^{-1}(i) [\hat{\mathbf{d}}_k(i) - \mathbf{C}_k \hat{\mathbf{\Gamma}}_k^{-1}(i) (\mathbf{C}_k^H \hat{\mathbf{R}}_k^{-1}(i) \hat{\mathbf{d}}_k(i) - \mathbf{v} \hat{\mathbf{b}}_k(i))] \quad (34)$$

where  $\{\hat{\mathbf{d}}_k(i+1) = \alpha \hat{\mathbf{d}}_k(i) + (1 - \alpha) \mathbf{z}_k^*(i) \mathbf{r}_k(i)\}$  corresponds to an estimate of  $\mathbf{d}_k(i)$ . In terms of computational complexity, the CCM-RLS algorithm requires  $O(M^2)$  to suppress MAI and ISI and  $O(L_p^2)$  to estimate the channel, against  $O(M^3)$  and  $O(L_p^3)$  required by (7) and direct SVD, respectively.

## 6 Performance evaluation and comparisons

To assess the performance of the proposed receiver and algorithms, we conducted several simulations. We consider the CCM and the constrained minimum variance (CMV) [7] blind receiver design criteria with SG and RLS algorithms for parameter estimation, and the linear [7–9], the traditional PIC detector [6, 12–16] and the new SIC and HIC receivers with and without IT detection. For the traditional PIC receiver, the received signal at the  $m$ th stage is described by

$$\mathbf{r}_{k,m}(i) = \mathbf{r}(i) - \sum_{j \neq k} \hat{A}_j(i) \hat{b}_{j,m-1}(i) \hat{\mathbf{s}}_j(i), \quad m \geq 1 \quad (35)$$

where  $\mathbf{r}_{k,1}(i) = \mathbf{r}(i)$ , for  $m = 1$  (the first stage),  $\hat{b}_{j,m-1}(i)$  is the detected symbol at stage  $m - 1$  for user  $j$  and symbol  $i$ ,  $\hat{A}_j(i)$  is the  $j$ th user amplitude estimate, the  $M \times 1$  effective signature estimate is  $\hat{\mathbf{s}}_j(i) = \mathbf{C}_j \hat{\mathbf{b}}_j(i)$ . The channel estimate,  $\hat{\mathbf{b}}_k(i)$ , for user  $k$ , is computed using a modified version of (14) which is given by

$$\hat{\mathbf{b}}_k(i) = \min_b \mathbf{b}^H \mathbf{C}_k^H \tilde{\mathbf{R}}_k^{-1}(i) \mathbf{C}_k \mathbf{b} \quad (36)$$

where  $\tilde{\mathbf{R}}_k(i)$  is defined as in (11) and (13) using  $\mathbf{r}(i)$  given by (3) in lieu of  $\mathbf{r}_k(i)$ . The adaptive recursions for  $\mathbf{w}_k(i)$  and  $\hat{\mathbf{b}}_k(i)$  can be derived in a straightforward way from Section 5, again using  $\mathbf{r}(i)$  in lieu of  $\mathbf{r}_k(i)$  in the definition of  $\mathbf{z}_k(i)$  and in expressions (25)–(34).

The amplitude estimation procedure considers the optimisation  $\hat{A}_j(i) = \arg \min_A E[\|\hat{A}_j(i) \hat{\mathbf{s}}_j(i) - \mathbf{r}(i)\|^2]$  and employs the following adaptive recursion

$$\hat{A}_j(i+1) = \hat{A}_j(i) - \mu (\hat{A}_j(i) \hat{\mathbf{s}}_j^H(i) \hat{\mathbf{s}}_j(i) - \hat{b}_j^*(i) \mathbf{r}^H(i) \hat{\mathbf{s}}_j(i)) \quad (37)$$

As in the SIC, IT-SIC, HIC and IT-HIC, the channel and amplitude estimation procedures as well as the design of the filters  $\mathbf{w}_k$  are performed only at the first ‘pure PIC’ stage.

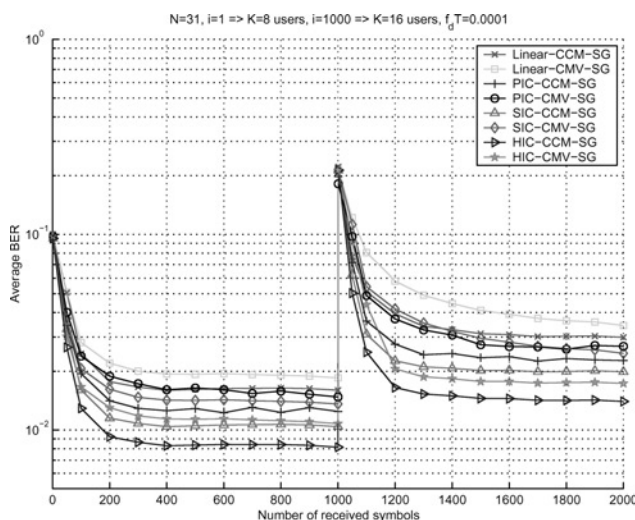
The PIC and HIC receivers, as well as their IT versions, have been designed with two stages because our studies revealed that this captured most of the gains of the structures and provided a good trade-off between performance and complexity. The DS-CDMA system employs random sequences of length  $N = 16$  and Gold

sequences of length  $N = 31$ , SG algorithms are normalised, all parameters are optimised for each scenario, and simulations are averaged over 100 experiments. The channels experienced by different users are i.i.d. whose coefficients for each user are obtained with Clarke's model [19]. The results are shown in terms of the normalised Doppler frequency  $f_d T$  (cycles/symbol) and use three-path channels with relative powers given by 0, -3 and -6 dB, where the spacing between paths for each run is obtained from a discrete uniform random variable between 1 and 2 chips. The channel estimation algorithms of [17] model the channel as an FIR filter and we employ a filter with six taps as an upper bound for the experiments. In all the figures, the bit error rate (BER) is averaged over the users.

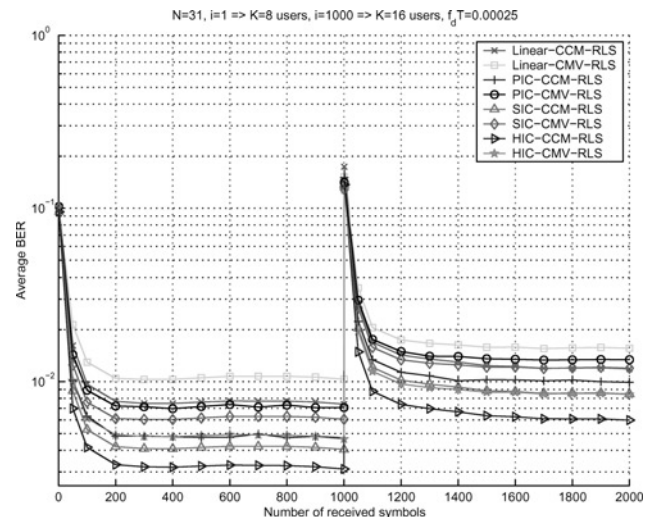
To examine the convergence of the algorithms, we use BER against received symbols plots and consider a non-stationary scenario where users enter and exit the system. In the experiments, shown in Figs. 3 and 4, the system starts with eight users all with an average  $E_b/N_0 = 15$  dB and whose power distribution among the users for each run follows a log-normal distribution with associated standard deviation of 1.5 dB. At 1000 transmitted symbols, two users exit and 10 users enter the system and power distribution among the users for each run is loosen and follows a log-normal distribution with associated standard deviation of 3 dB.

The results show that the CCM algorithms converge to a lower BER than CMV-based techniques and HIC receivers achieve a performance superior to SIC, PIC and linear structures. An important feature of the proposed HIC schemes is that they gather the strengths of the other schemes and its additional complexity is linear with the number of users.

Let us now consider the proposed IT-SIC receiver, evaluate the number of arbitrated branches that should be

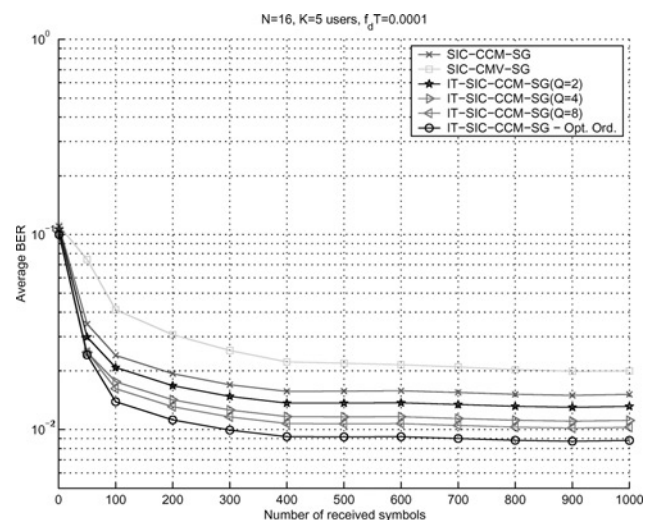


**Figure 3** BER convergence performance of blind SG algorithms and receivers

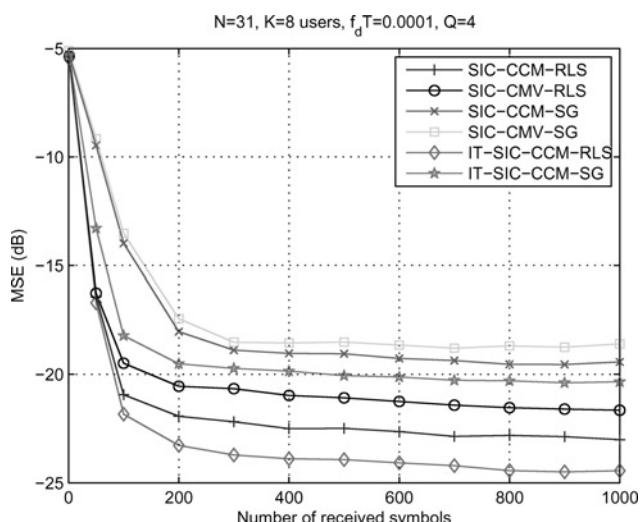


**Figure 4** BER convergence performance of blind RLS-type algorithms and receivers

used in the ordering algorithm and account for the impact of additional branches upon performance. We carry out a comparison of the proposed low-complexity user ordering algorithm against the optimal ordering approach, briefly described in Section 4.1, that tests  $K!$  possible branches and selects the most likely estimate on the basis of the instantaneous CM cost function. We designed the IT-SIC receivers with  $Q = 2, 4, 8$  parallel branches and compared their BER performance against the number of symbols with SIC and IT-SIC with optimal ordering, as depicted in Fig. 5. The results show that the proposed low-complexity ordering algorithm achieves a performance close to the optimal ordering, while keeping the complexity reasonably low for practical utilisation. It can be noted from the curves that the performance of the new IT-SIC improves as the number of parallel branches increase and that the gains in performance obtained through additional



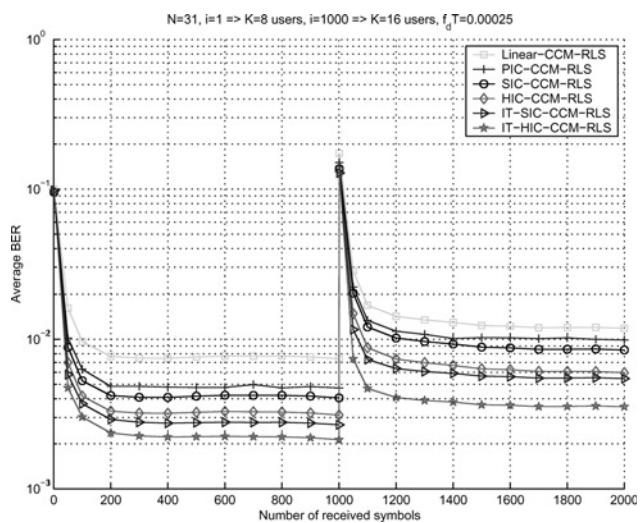
**Figure 5** BER convergence of IT-SIC receivers with blind SG algorithms for different  $Q$



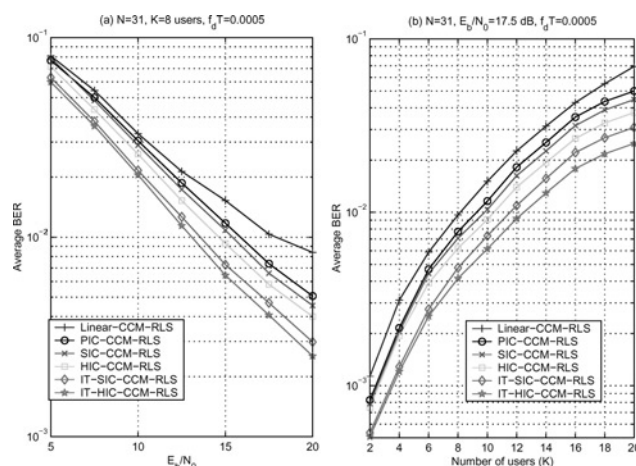
**Figure 6** Channel estimate performance for different SIC receivers

branches decrease as  $Q$  is increased, resulting in marginal improvements for more than  $Q = 4$  branches. For this reason, we adopt  $Q = 4$  for the remaining experiments because it presents a very attractive trade-off between performance and complexity. In Fig. 6, we illustrate the channel estimation performance in terms of mean square error for a system with  $K = 8$  users. The number of parallel branches for the IT receivers is  $Q = 4$ . The results show that the proposed IT-SIC receiver structure can provide improved channel estimates compared with non-IT-SIC receivers.

The performance of the proposed IT-HIC and IT-SIC receivers against the other schemes designed with the CCM criterion is assessed in Fig. 7. The curves indicate that the proposed IT-HIC and IT-SIC structures are



**Figure 7** BER convergence of CCM receivers with blind RLS algorithms



**Figure 8** BER performance of blind RLS-type algorithms and receivers

superior to the remaining receiver architectures. In this regard, the IT-HIC slightly outperforms the IT-SIC, which is followed by the HIC, SIC, PIC and linear detectors. It is worth mentioning that an advantage of the HIC structure over the SIC is that it attempts to equalise the performance over the user population, whereas SIC schemes usually lead to non-uniform performance among the users.

The BER performance of the adaptive receivers and algorithms against  $E_b/N_0$  and  $K$  is illustrated in Fig. 8, where the power distribution among the users for each run follows a log-normal distribution with associated standard deviation of 1.5 dB. The results (for 1500 transmitted symbols) confirm that the proposed IT-HIC has the best performance, followed by the IT-SIC, HIC, SIC, PIC and the linear receivers. From the curves, we verify that the proposed IT detection used in conjunction with SIC and HIC architectures can save up to 3 dB in  $E_b/N_0$  for the same BER performance as compared with the proposed SIC and HIC receivers. In terms of system capacity, the IT approach with SIC and HIC schemes can accommodate up to four more users for the same  $E_b/N_0$  as compared with the HIC and SIC without IT. In comparison with other receiver techniques such as the PIC and linear, the gains in performance are even more substantial.

## 7 Concluding remarks

We have proposed blind adaptive and IT interference cancellation receiver structures for DS-CDMA systems in frequency selective channels. The new blind adaptive detectors were designed on the basis of a CCM criterion and adaptive algorithms were described for parameter estimation. We presented a new hybrid receiver architecture, denoted as HIC, based on the combination of SIC and PIC scheme. A new IT detection scheme based on the instantaneous CM criterion was also introduced and



shown to provide a substantial performance enhancement over conventional IC structures in scenarios of practical interest.

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## 9 Appendixes

### 9.1 Convergence properties

If we assume perfect interference cancellation at each SIC level, the  $k$ th user detection filter is obtained by the minimisation of (9) in a system with  $K' = K - k + 1$  active users. Under this assumption, the convexity of the cost function for each user filter can be addressed using the analysis carried out in [9, 20]. Following the lines of [9, 20] we arrive at the condition  $v^2 |A_k|^2 |\hat{\mathbf{b}}_k^H \mathbf{b}_k|^2 \geq 1/4$  that ensures the convexity of  $J_{\text{CM}}(\cdot)$  for the  $k$ th SIC level in the noiseless case. Since the extrema of the cost function can be considered for small noise level  $\sigma^2$ , a slight perturbation of the noise-free case [21], the cost function is also convex for small  $\sigma^2$  when  $v^2 |A_k|^2 |\hat{\mathbf{b}}_k^H \mathbf{b}_k|^2 \geq 1/4$ . Interestingly, if we assume ideal channel estimation ( $|\hat{\mathbf{b}}_k^H \mathbf{b}_k| = 1$ ) and  $v = 1$ , the result reduces to  $|A_k|^2 \geq 1/4$ , which is the same found in [22]. For larger values of  $\sigma^2$ , we remark that the

term  $\nu$  can be adjusted in order to make the cost function  $J_{\text{CM}}$  convex in (9), as pointed out in [21].

### 9.2 On the use of $\mathbf{R}_k$ for channel estimation

Using the perfect cancellation assumption as in 9.1, and following the lines in [20], it can be verified that for the  $k$ th SIC level, the correlation matrix  $\mathcal{R}_k$  can be approximated by  $\mathbf{R}_k$  defined in (11) multiplied by a scalar factor plus a noise-like term, that for sufficient  $E_b/N_0$  has an insignificant contribution. Therefore we conclude that the channel estimation can be performed using matrix  $\mathbf{R}_k$ , as is done in (14), in lieu of  $\mathcal{R}_k$ , since the properties of the matrix  $\mathcal{R}_k$  studied in [17] hold for  $\mathbf{R}_k$ .

### 9.3 Derivation of normalised step size: CCM-SG case

To derive a normalised step size for the algorithm in (27), let us write, dropping the time index for simplicity, the CM cost function  $J_{\text{CM}} = (|\mathbf{w}_k^H \mathbf{r}_k|^2 - 1)^2$  as a function of (27)

$$J_{\text{CM}} = (|\mathbf{I}_k(\mathbf{w}_k - \mu_w \mathbf{r}_k e_k z_k^*)^H \mathbf{r}_k + (\mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1} \mathbf{b}_k)^H \mathbf{r}_k|^2 - 1)^2 \quad (38)$$

If we substitute  $\mathbf{I}_k = \mathbf{I} - (\mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1} \mathbf{C}_k^H)$  into the first

term of (38) and use  $\mathbf{C}_k^H \mathbf{w}_k = \mathbf{b}_k$ , we can simplify (38) and obtain

$$J_{\text{CM}} = (|z_k - \mu_w e_k z_k \mathbf{r}_k^H \mathbf{I}_k \mathbf{r}_k|^2 - 1)^2 \quad (39)$$

Next, if we take the gradient of  $J_{\text{CM}}$  with respect to  $\mu_w$  and equal it to zero, we have

$$\begin{aligned} \nabla J_{\mu_w} &= 2(|z_k - \mu_w e_k z_k \mathbf{r}_k^H \mathbf{I}_k \mathbf{r}_k|^2 - 1) \\ \frac{d}{d\mu_w} |z_k - \mu_w e_k z_k \mathbf{r}_k^H \mathbf{I}_k \mathbf{r}_k|^2 &= 0 \end{aligned} \quad (40)$$

From the above expression, it is clear that this minimisation leads to four possible solutions, namely

$$\begin{aligned} \mu_w^{n.1} = \mu_w^{n.2} &= 1/e_k \mathbf{r}_k^H \mathbf{I}_k \mathbf{r}_k, & \mu_w^{n.3} &= (|z_k| - 1)/|z_k| e_k \mathbf{r}_k^H \mathbf{I}_k \mathbf{r}_k, \\ \mu_w^{n.4} &= \frac{(|z_k| + 1)}{|z_k| e_k \mathbf{r}_k^H \mathbf{I}_k \mathbf{r}_k} \end{aligned} \quad (41)$$

By computing the second derivative of (38), one can verify that it is always positive for the third and fourth solutions above, indicating the minimum point. Hence, we choose  $\mu_w = (|z_k| + 1)/|z_k| e_k \mathbf{r}_k^H \mathbf{I}_k \mathbf{r}_k$  and introduce again the convergence factor  $\mu_{0_w}$  so that the algorithms can operate with adequate step sizes that are usually small to ensure good performance and thus we have  $\mu_w = \mu_{0_w} (|z_k| + 1)/|z_k| e_k \mathbf{r}_k^H \mathbf{I}_k \mathbf{r}_k$ .