In this work, we propose a novel adaptive reduced-rank strategy based on joint interpolation, decimation and filtering (JIDF) for large multiuser multiple-input multiple-output (MIMO) systems. In this scheme, a reduced-rank framework is proposed for linear receive processing and multiuser interference suppression according to the minimization of the bit error rate (BER) cost function. We present a structure with multiple processing branches that performs dimensionality reduction, where each branch contains a group of jointly optimized interpolation and decimation units, followed by a linear receive filter. We then develop stochastic gradient (SG) algorithms to compute the parameters of the interpolation and receive filters along with a low-complexity decimation technique. Simulation results are presented for time-varying environments and show that the proposed MBER-JIDF receive processing strategy and algorithms achieve a superior performance to existing methods at a reduced complexity.

Index Terms— Adaptive filtering, minimum-BER, reduced-rank techniques, massive MIMO, stochastic gradient algorithms.

1. INTRODUCTION

Large MIMO systems have received significant attention in the recent years since they can substantially increase the system capacity and improve the quality and reliability of wireless links [1]. Different configurations have been investigated for large MIMO systems, such as distributed and centralized MIMO schemes. Key applications of these systems include wireless cellular, local area [2, 3, 4] and multi-beam satellite networks [5]. The problem of detecting a desired user in a large multiuser MIMO system presents many signal processing challenges including the need for algorithms with the ability to process large-dimensional received data, fast and accurate adjustment of parameters, scalable computational complexity and the development of cost-effective interference mitigation schemes.

In this context, reduced-rank signal processing is a key tool for large systems which can provide faster training, a better tracking performance and an increased robustness against interference as compared to standard methods. A number of reduced-rank techniques have been developed to design the dimensionality reduction matrix and the reduced-rank receive filter [6]-[15]. Among the first schemes are the eigendecomposition-based (EIG) algorithms [6, 7] and the multistage Wiener filter (MWF) investigated in [8]-[10]. EIG and MWF have faster convergence speed compared to the full rank adaptive algorithms with a much smaller filter size, but their computational complexity is high. A strategy based on the joint and iterative optimization (JIO) of a subspace projection matrix and a reduced-rank filter has been reported in [12, 13, 14, 15].

However, most of the contributions to date are either based on the minimization of the mean square error (MSE) and/or the minimization of the bit error rate (BER) cost function. We present a structure with multiple processing branches that performs dimensionality reduction, where each branch contains a group of jointly optimized interpolation and decimation units, followed by a linear receive filter. We then develop stochastic gradient (SG) algorithms to compute the parameters of the interpolation and receive filters along with a low-complexity decimation technique. Simulation results are presented for time-varying environments and show that the proposed MBER-JIDF receive processing strategy and algorithms achieve a superior performance to existing methods at a reduced complexity.
where the $N_f \times 1$ channel vectors $b_{k,n}(i)$, for $f = 1, \ldots, M$, consist of independent and identically distributed complex Gaussian variables with zero mean and unit variance, $n(i) = [n_1(i), \ldots, n_M(i)]^T$ is the complex Gaussian noise vector with zero mean and $E[n(i)n^H(i)] = \sigma^2 I$, where $\sigma^2$ is the noise variance. $(.)^T$ and $(.)^H$ denote transpose and Hermitian transpose, respectively.

In the following, we explain the design of reduced-rank receive processing schemes which minimize the BER. In a reduced-rank algorithm, an $M \times D$ subspace projection matrix $S_D$ is applied to the received data to extract the most important information of the data by performing dimensionality reduction, while $1 \leq D \leq M$. A $D \times 1$ projected received vector is obtained as $\tilde{r}(i) = S_D^HR_i(i)$, where it is the input to a $D \times 1$ filter $\tilde{w}$. The filter output is given by $\tilde{x}_{k,n}(i) = \tilde{w}^H \tilde{r}(i) = \tilde{w}^H S_D^H r(i)$. Assuming that we use binary signalling, the estimated symbol of user $k$ is given by $b_{k,n}(i) = \text{sign}(\tilde{r}(i))$, where the operator $\text{sign}(\cdot)$ retains the real part of the argument and $\text{sign}(\cdot)$ is the sign function. The probability of error for user $k$ is given by

$$P_k = \int_0^1 \text{Q} \left( \frac{\sqrt{\sum_{k,n} (b_{k,n}(i) R[x_{k,n}(i)])^2}}{2w^H S_D^H S_D w} \right) \rho \sqrt{2 \pi w^H S_D^H S_D w} \text{d}x_{k,n}$$

where $\rho$ is the radius parameter of the kernel density estimate, $\text{Q}(\cdot)$ is the Gaussian error function. The problem we are interested in solving is how to devise a cost-effective algorithm to adjust the parameters of $S_D$ and $\tilde{w}$ based on minimizing the probability of error with reduced length component filters.

### 3. PROPOSED MBER-JIDF REDUCED-RANK LINEAR RECEIVE PROCESSING SCHEME

In this section, we detail the proposed MBER reduced-rank linear receive processing scheme based on joint interpolation, decimation and filtering, which comes from two observations. The first is that rank reduction can be performed by reconstructing new samples with interpolators and eliminating (decimating) samples that are not useful in the filtering process [14]. The second comes from the structure of the dimensionality reduction matrix, whose columns are a set of vectors formed by the interpolators and decimators.

#### 3.1. Overview of the MBER-JIDF Scheme

We design the subspace projection matrix $S_D$ by considering interpolation and decimation. In this case, the receive filter length is substantially reduced, which results in significantly reduced computational complexity and very fast training for large MIMO systems. The proposed MBER-JIDF scheme for the $n$-th symbol of the $k$-th user is depicted in Fig. 1. The $M \times 1$ received vector $r(i)$ is processed by a framework with $B$ branches, where each branch contains an interpolator and a decimation unit, followed by a reduced-rank receive filter. In the $l$-th branch, the received vector is operated by the interpolator $p(i) = [p_{1,l}(i), \ldots, p_{I,l}(i)]^T$ with filter length $I$, $I < M$, where the output of the interpolator of the $l$-th branch is expressed by

$$\tilde{r}_l(i) = P^H_l(i) r(i)$$

where the $M \times M$ Toeplitz convolution matrix $P_l(i)$ is given by

$$P_l(i) = \begin{pmatrix} p_{1,l}(i) & 0 & \ldots & 0 \\ \vdots & p_{1,l}(i) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & p_{r,l}(i) \end{pmatrix}.$$  

In order to facilitate the description of the scheme, we introduce an alternative way to represent the vector $\tilde{r}_l(i)$,

$$\tilde{r}_l(i) = P^H_l(i) r(i) = R'(i) p^*_l(i)$$

where the $M \times I$ matrix $R'(i)$ with the samples of $r(i) = [r_0(i), \ldots, r_{M-1}(i)]^T$ has a Hankel structure [26] given by

$$R'(i) = \begin{pmatrix} r_0(i) & r_1(i) & \ldots & r_{I-1}(i) \\ \vdots & \vdots & \ddots & \vdots \\ r_{M-I}(i) & r_{M-I+1}(i) & \ldots & r_{M-1}(i) \\ \vdots & \vdots & \ddots & 0 \\ r_{M-2}(i) & r_{M-1}(i) & 0 & 0 \\ r_{M-1}(i) & 0 & 0 & 0 \end{pmatrix}.$$  

The dimensionality reduction is performed by a decimation unit with $D \times M$ dimension matrices $T_l$ that projects $\tilde{r}_l(i)$ onto $D \times 1$ vectors $\tilde{x}_{k,n}(i)$ with $D$ is the rank. The $D \times 1$ vector $\tilde{x}_{k,n}(i)$ for the $l$-th branch is given by

$$\tilde{x}_{k,n}(i) = T_l P^H_l(i) r(i) = T_l \tilde{r}_l(i) = T_l R'(i) p^*_l(i)$$

where $S_D(i)$ denotes the equivalent subspace projection matrix corresponding to the $l$-th branch. The output of the reduced-rank receive filter $\tilde{w}(i)$ corresponding to the $l$-th branch is given by $\tilde{x}_{k,n}(i) = \tilde{w}^H(i) \tilde{r}_l(i)$, which is used in the minimization of the error probability for branch $l$. The hard decision for the $l$-th branch is given by $b_{k,n}(i) = \text{sign}(\tilde{r}(i))$. The proposed scheme employs $B$ parallel branches of interpolators and decimators. The optimum branch is selected according to

$$l_{opt} = \arg \min_{1 \leq l \leq B} \frac{P_e^{(i)}}{P_e^T}$$

where $P_e^{(i)} = Q \left( \frac{\sqrt{\sum_{k,n} (b_{k,n}(i) R[x_{k,n}(i)])^2}}{\rho(\tilde{w}^H S_D^H S_D \tilde{w})} \right)$

The output of the scheme is given by $b_{k,n}(i) = \text{sign}(\tilde{r}(i))$.  

#### 3.2. Design of the Decimation Unit

In this work, the elements of the decimation matrix only take the value 0 or 1. This corresponds to the decimation unit simply keeping or discarding the samples. The optimal decimation scheme exhaustively explores all possible patterns which select $D$ samples out of $M$ samples. In this case, the scheme can be viewed as a combinatorial problem and the total number of patterns is $B = M(M-1) \ldots (M-D+1)$. 

$$R'(i) = \begin{pmatrix} r_0(i) & r_1(i) & \ldots & r_{I-1}(i) \\ \vdots & \vdots & \ddots & \vdots \\ r_{M-I}(i) & r_{M-I+1}(i) & \ldots & r_{M-1}(i) \\ \vdots & \vdots & \ddots & 0 \\ r_{M-2}(i) & r_{M-1}(i) & 0 & 0 \\ r_{M-1}(i) & 0 & 0 & 0 \end{pmatrix}.$$
However, the optimal decimation scheme is too complex for practical use. We introduce a low-complexity suboptimal method to generate the decimation matrix. It employs a structure formed in the following way

$$
T_l = \begin{bmatrix}
t_{l,1} & t_{l,2} & \ldots & t_{l,D}
\end{bmatrix}^T
$$

where the $M \times 1$ vector $t_{l,d}$ denotes the $d$-th basis vector of the $l$-th decimation unit, $d = 1, \ldots, D$, $l = 1, \ldots, B$, and its structure is given by

$$
t_{l,d} = \begin{bmatrix} 0, \ldots, 0, 1, 0, \ldots, 0 \end{bmatrix}^T \quad (9)
$$

where $q_{l,d}$ is the number of zeros before the nonzero element. Note that it is composed of a single 1 and $M$ − 1 0s. We set the value of $q_{l,d}$ in a deterministic way which can be expressed as $q_{l,d} = \lfloor \frac{M}{2} \rfloor \times (d - 1) + (l - 1)$. The simulation results will show that the proposed reduced-rank scheme with the suboptimal decimation unit design method works very well. In the following section, we will introduce the proposed adaptive algorithms for the interpolator filter $p_l(i)$ and the reduced-rank receive filter $\hat{w}(i)$.

### 4. PROPOSED ADAPTIVE ALGORITHMS

In this section, we develop the MBER based adaptive SG algorithms to update the interpolator and the reduced-rank filters for each branch. We then provide a computational complexity analysis of the proposed and conventional adaptive reduced-rank algorithms.

#### 4.1. Adaptive MBER-JIDF Algorithms

Firstly, we derive the gradient terms for the reduced-rank filter and the interpolation vector. By taking the gradient of (8) with respect to $\hat{w}$ and after further mathematical manipulations we obtain

$$
\frac{\partial P^e_l}{\partial \hat{w}} = -\exp\left(\frac{-||\hat{w}^T l_i ||^2}{2\sigma^2} \right) \text{sign} \{ b_{k,n}(i) \} \\
\times \left( \frac{S_{D,l}^H \hat{w}}{\hat{w}^H S_{D,l} \hat{w}} - \frac{S_{D,l}^H \hat{S}_{D,l} \hat{w}}{\hat{w}^H \hat{S}_{D,l} \hat{w}} \right).
$$

(10)

To derive the gradient terms for the interpolator $p_l(i)$, we need to express the output of the $l$-th branch $x_{k,n}(i)$ as a function of $p_l(i)$, which is given by

$$
x_{k,n}(i) = \hat{w}^H(i) T_l(i) R(i) p_l(i) = p_l^H(i) u(i)
$$

(11)

where $u(i) = R^T(i) T_l^T(i) \hat{w}^*(i)$ is an $I \times 1$ vector. We let $u(i) = [u_1(i), \ldots, u_I(i)]^T$ and rewrite the error probability cost function $P_e^l(i)$ as follows

$$
P_e^l(i) = Q \left( \text{sign} \{ b_{k,n}(i) \} R[p_{l,1}u_1 + p_{l,2}u_2 + \ldots + p_{l,I}u_I] \mu \right)
$$

where the function $g(p_{l,1}, p_{l,2}, \ldots, p_{l,I})$ is given by

$$
g(p_{l,1}, p_{l,2}, \ldots, p_{l,I}) = \hat{w}_1 p_{l,1} + \ldots + \hat{w}_I p_{l,I} + \hat{w}_D p_{l,D} + \hat{w}_0 L_p
$$

(13)

where $\hat{w}_d$ denotes the number of nonzero elements for row $d$ in the $D \times I$ matrix $T_l(i) R(i)$, $1 \leq d \leq D$, $I = \hat{w}_1 \geq \hat{w}_2 \geq \ldots \geq \hat{w}_D \geq 1$. Note that $g(p_{l,1}, p_{l,2}, \ldots, p_{l,I}) = \hat{w}^H D_p \hat{w}$, and we define $\hat{w} = [\hat{w}_1, \ldots, \hat{w}_D]^T$. By taking the gradient with respect to each element $p_{l,j}$ in vector $p_l(i)$, $j = 1, \ldots, I$, we obtain

$$
\frac{\partial P^e_l}{\partial p_{l,j}} = -\exp\left(\frac{-||\hat{w}^T l_i ||^2}{2\sigma^2} \right) \text{sign} \{ b_{k,n}(i) \} \\
\times \left( \frac{u_j(i)}{g^2(p_{l,1},\ldots, p_{l,I})} - \frac{\hat{S}_{D,l}^H \hat{w}}{\hat{w}^H \hat{S}_{D,l} \hat{w}} \right),
$$

(14)

where $u_j$ denotes the number of nonzero elements for column $j$ in the $D \times I$ matrix $T_l(i) R(i)$, $1 \leq j \leq I$, $D = \psi_1 \geq \psi_2 \geq \ldots \geq \psi_I \geq 1$. We stack the $I$ elements $\frac{\partial P^e_l}{\partial p_{l,j}}$ and obtain an $I \times 1$ gradient vector as $v_l = \left[ \frac{\partial P^e_l}{\partial p_{l,1}}, \frac{\partial P^e_l}{\partial p_{l,2}}, \ldots, \frac{\partial P^e_l}{\partial p_{l,I}} \right]^T$.

The interpolator and the reduced-rank receive filters are jointly optimized according to the BER criterion. The algorithm has been devised to start its operation in the training (TR) mode, and then to switch to the decision-directed (DD) mode. The proposed SG algorithms are obtained by substituting the gradient terms (10) and (14) in the expressions $\hat{w}(i + 1) = \hat{w}(i) - \mu_v \frac{\partial P^e_l}{\partial \hat{w}}$ and $p_l(i + 1) = p_l(i) - \mu_p \frac{\partial P^e_l}{\partial p_l}$. Subject to the constraint of $\hat{w}^H D_p \hat{w} = 1$, each time instant, the weights of the two quantities of branch $l$ are updated in an alternating way by using the following equations

$$
\hat{w}(i + 1) = \hat{w}(i) + \mu_v \exp\left(\frac{-||\hat{w}^T l_i ||^2}{2\sigma^2} \right) \text{sign} \{ b_{k,n}(i) \} \\
\times \left( \hat{S}_{D,l}^H D_p \hat{w} - \hat{S}_{D,l}^H \hat{w} \right)
$$

(15)

$$
p_l(i + 1) = p_l(i) - \mu_p \left[ \frac{\partial P^e_l}{\partial p_{l,1}}, \frac{\partial P^e_l}{\partial p_{l,2}}, \ldots, \frac{\partial P^e_l}{\partial p_{l,I}} \right]^T
$$

(16)

where each element in the gradient vector is given by

$$
\frac{\partial P^e_l}{\partial p_{l,j}} = -\exp\left(\frac{-||\hat{w}^T l_i ||^2}{2\sigma^2} \right) \text{sign} \{ b_{k,n}(i) \} \\
\times \left( \frac{u_j(i)}{g^2(p_{l,1},\ldots, p_{l,I})} - 1 \right)
$$

(17)
where \(\rho_b\) and \(\rho_p\) are the step-size values. Expressions (15) and (16) need initial values, \(\tilde{w}(0)\) and \(p_i(0)\), and we scale the interpolation vector by \(p_i \leftarrow \frac{p_i}{\sqrt{\sum_j |s_i^j|^2}}\) at each iteration. The scaling has an equivalent performance to using a constrained optimization with Lagrange multipliers although it is computationally simpler. The proposed MBER-JIDF algorithm are summarized in Table 1.

Table 1. Proposed adaptive MBER-JIDF algorithm.

<table>
<thead>
<tr>
<th>Step</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Set step-size values (\rho_b) and (\rho_p) and the no. of branches (B).</td>
</tr>
<tr>
<td>2</td>
<td>Initialize (\tilde{w}(0)) and (p_i(0)). Set (T_1, \ldots, T_B).</td>
</tr>
<tr>
<td>3</td>
<td>for each time instant (i) do</td>
</tr>
<tr>
<td>4</td>
<td>for (l) from 1 to (B) do</td>
</tr>
<tr>
<td>5</td>
<td>Update (p_l(i+1)) based on (T_l) and (16).</td>
</tr>
<tr>
<td>6</td>
<td>Scale the vector (p_i) using (p_i \leftarrow \frac{p_i}{\sqrt{\sum_j</td>
</tr>
<tr>
<td>7</td>
<td>Select the optimal branch based on (8).</td>
</tr>
<tr>
<td>8</td>
<td>Generate the estimated symbol.</td>
</tr>
<tr>
<td></td>
<td>Update (\tilde{w}(i+1)) based on the selected branch and (15)</td>
</tr>
</tbody>
</table>

4.2. Computational Complexity

In Table 2, we show the number of additions and multiplications of the proposed MBER-JIDF algorithm, the existing adaptive reduced-rank algorithms, the adaptive least-mean square (LMS) [25] and the full-rank algorithm on the BER criterion [16]. In the case of large MIMO systems, the parameters \(D\), \(I\) and \(B\) are chosen much smaller than \(M\) which results in a substantial complexity saving. In particular, for a configuration with \(M = 40, I = D = 8\) and \(B = 4\), the numbers of multiplications and additions for the proposed algorithm are upper bounded by 1825 and 1595, respectively. For the MWF-MBER algorithm they are 15594 and 11857, respectively. Compared to the existing reduced-rank algorithms, the MBER-JIDF algorithm reduces the computational complexity significantly.

Table 2. Computational complexity of Algorithms.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Number of operations per symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-Rank-LMS</td>
<td>(2M + 1)</td>
</tr>
<tr>
<td>Full-Rank-MBER</td>
<td>(4M + 1)</td>
</tr>
<tr>
<td>EIG [7]</td>
<td>(O(M^2))</td>
</tr>
<tr>
<td>MBER-MWF [18]</td>
<td>((D + 1)M^2) (</td>
</tr>
<tr>
<td>MBER-JIDF</td>
<td>(MD + DB) (</td>
</tr>
</tbody>
</table>

5. SIMULATIONS

In this section, we evaluate the performance of the proposed MBER-JIDF reduced-rank algorithm and compare it with existing full-rank and reduced-rank algorithms. Monte-carlo simulations are conducted to verify the effectiveness of the MBER-JIDF adaptive reduced-rank SG algorithms. The number of receive antennas at the BS is \(M = 40\). The number of antennas per user is \(N_u = 2\). The coefficients of the channel matrix \(H_u(i)\) are computed according to Clarke’s model [27]. We have optimized the step sizes of each branch of the MBER-JIDF adaptive reduced-rank SG algorithms with the following rules, \(\mu_{w/p}(i + 1) = \left[\delta_1 \mu_{w/p}(i) + \delta_2 \times Q\left(\frac{\text{sign}(\hat{\beta}_u(i)) \|\hat{\beta}_u(i)\|}{\rho}\right)\right]^+\), where \([\cdot]^+\) denotes the truncation to the limits of a range. We tuned \(\delta_1 = 0.99\), \(\delta_2 = 1 \times 10^{-4}\), \(\mu_+ = 1 \times 10^{-2}\) and \(\mu_- = 1 \times 10^{-5}\) and set \(\rho = 2\sigma\) [16]. The step sizes for LMS adaptive full-rank, SG adaptive MBER full-rank and the other reduced-rank techniques are \(0.085, 0.05\) and \(0.035\), respectively. The initial full-rank, reduced-rank and interpolation filters are \([1, 0, \ldots, 0]^T\). The algorithms process 200 symbols in TR and 1000 symbols in DD.

Fig.2 (a) shows the BER performance of the desired user versus the number of received symbols for the proposed MBER-JIDF scheme and the conventional full rank and reduced-rank algorithms. We set the rank \(D = 8\), \(I = 8\), \(K = 4\), \(SNR = 15\) dB and \(f_d T = 1 \times 10^{-5}\). We can see that the proposed MBER-JIDF reduced-rank algorithms converge much faster than the conventional full rank and reduced-rank algorithms. Fig.2 (b) illustrates the steady-state BER performance of the desired user versus the number of users \(K\). We can see that the best performance is achieved by the proposed MBER-JIDF algorithms followed by the MWF-MBER algorithm, the full-rank MBER algorithm, the full-rank LMS algorithm and the eigen-decomposition-based algorithms. In particular, the MBER-JIDF algorithm using \(B = 4\) can accommodate up to four more users in comparison with the MWF-MBER algorithm [18], at the BER level of \(2 \times 10^{-2}\).

6. CONCLUSION

In this paper, we have proposed an adaptive reduced-rank linear receive processing scheme and MBER algorithms for interference suppression in large multiuser MIMO systems. For each branch, we designed a group of jointly optimized interpolation and decimation units, followed by linear receive filtering according to the minimization of the BER cost function. The final output is switched to the branch with the best performance based on the minimum error probability. We have developed an adaptive approach for their adaptive implementation. The results have shown that the proposed scheme significantly outperforms existing algorithms and supports systems with higher loads. Future work will consider non-linear detectors, higher order modulation and other MIMO configurations.
7. REFERENCES


