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Structured Root-Check LDPC Codes and PEG-Based Techniques for Block-Fading Channels

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Abstract

In this work, we propose structured Root-Check Low-Density Parity-Check (LDPC) codes and design techniques for block-fading channels. In particular, Quasi-Cyclic Root-Check LDPC codes, Irregular repeat-accumulate Root-Check LDPC codes and Controlled Doping Root-Check LDPC codes based on Progressive Edge Growth (PEG) techniques for block-fading channels are proposed. The proposed Root-Check LDPC codes are both suitable for channels under $F = 2, 3$ and 4 independent fading per codeword. The performance of the proposed codes is investigated in terms of Frame Error Rate (FER). The proposed Root-Check LDPC codes are capable of achieving the channel diversity and outperform standard LDPC codes. For block-fading channel with $F = 2$ our proposed PEG-based Root-Check LDPC codes outperform PEG-based LDPC codes by $7.5dB$.

Keywords: LDPC; Root-Check; PEG; Repeat-Accumulate; Block-Fading; Controlled Doping

1 Introduction

The most recent IEEE Wireless Local Area Network (WLAN) 802.11ad standard [1] argues that to achieve high throughput the devices must operate with LDPC codes. As wireless systems are subject to multi-path propagation and mobility, these systems are characterized by time-varying channels with fluctuating signal strength. In applications subject to delay constraints and slowly-varying channels, only limited independent fading realizations are experienced. In such conditions also known as non-ergodic scenarios, the channel capacity is zero since there is an irreducible probability, termed outage probability [2], that the transmitted data rate is not supported by the channel. A simple and useful model that captures the essential characteristics of non-ergodic channels is the block-fading channel [3]. It is especially important in wireless communications with slow time-frequency hopping (e.g., cellular networks and wireless local area networks) or multi-carrier modulation using Orthogonal Frequency Division Multiplexing (OFDM) [4]. Codes designed for block-fading channels are expected to achieve the channel diversity and to offer excellent coding gains.

1.1 Prior and Related Works

A family of LDPC codes called Root-Check for block-fading channels with $F = 2$ fading per codeword was proposed in [4]. Root-check codes are able to achieve the maximum diversity of a block-fading channel and have a performance near the limit of outage when decoded using the Sum Product Algorithm (SPA). Root-check codes are always designed with code rate $R = 1/F$, since the Singleton bound determines that this is the highest code rate possible to obtain the maximum diversity order [4]. Y. Li and M. Salehi in [5] have presented the construction of structured Root-LDPC codes by means of tiling circulant matrices, i.e., by designing Quasi-Cyclic Low-Density Parity-Check (QC-LDPC) codes. It is also shown that the QC-LDPC codes can perform as well as randomly generated Root-LDPC codes over block-fading channels. Uchoa et.al. in [6] proposed a PEG-based algorithm to design LDPC codes with Root-Check properties, thus providing Root-LDPC codes with larger girths. A strategy that imposes constraints on a PEG-based algorithm which are required by Root-Check LDPC codes was devised. This approach has provided better performance in terms of FER and BER than the works in [4, 5]. Duyck et. al. in [7] proposed the design of a random LDPC codes which are able to achieve full diversity in block-fading channels with $F = 2$ fadings. Healy and de Lamare in [8] extended the work in [7] for the case of block-fading channels with $F = 3$ and $F = 4$ fading per block transmitted.

1.2 Contributions

We propose in this work three structures to design Root-Check LDPC codes which are: Quasi-Cyclic, Repeat and Accumulate and Controlled Doping. Preliminary results toward PEG-based algorithm to design QC-LDPC codes with Root-Check properties for block-fading channel with $F = 3, 4$ fading per codeword were reported in [9]. Here, in this work we present a more detailed analysis of Quasi-Cyclic Root-Check based LDPC codes. Furthermore, initial results for a PEG-based algorithm to design irregular repeat-accumulate (IRA) LDPC codes with Root-Check properties for block-fading channels were discussed in [10]. Here, we present a more detailed analysis of Irregular Repeat-Accumulate and Accumulate IRAA Root-Check structure for $F = 2, 3$ independent fading.

In general, the parity check bits of Root-Check LDPC codes are not full diversity. Boutros in [11] proposed a controlled doping via high order Root-Check LDPC codes, which are able to guarantee full diversity for the parity check bits. Such kind of design becomes really important when Iterative Detection and Decoding (IDD) [12] is used in a Multiple-Input Multiple-Output (MIMO) systems [13, 14]. In IDD systems the detector and the decoder exchange their extrinsic information in an iterative way. Therefore, if the parity bits are not full diversity the overall IDD system performance will lead to a degradation in terms of Bit Error Rate (BER) instead of improvements as stated in [12].

In this paper we also propose a novel full diversity controlled doping Root-Check RA-based LDPC codes for Block-Fading channels of $F = 2, 3, 4$ fading which includes the code rates $R = \frac{1}{2}$, $R = \frac{1}{3}$ and $R = \frac{1}{4}$.

The main contributions of this work can be summarized as:

- Root-Check LDPC codes for Block-Fading channels including structured, unstructured, controlled doping, and RA designs are developed.

- New PEG-based algorithms for several Root-Check code structures are presented.
- A comprehensive simulation study of Root-Check LDPC codes and design algorithms is detailed.

The rest of this paper is organized as follows. In Section II we describe the system model. In Section III we discuss the prior and related works on the design of Root-Check LDPC codes and their structure. In Section IV the proposed PEG-based Quasi-Cyclic Root-Check LDPC codes, Irregular repeat-accumulate Root-Check LDPC codes and Controlled Doping Root-Check LDPC codes and their structure are presented. In Section V a discussion of which Root-Check LDPC code is more appropriate for a specific scenario is provided. Section VI the simulation results are shown, while Section VII concludes the paper.

2 System Model

Consider a block fading channel, where F is the number of independent fading blocks per codeword of length N. Following [5], the t-th received symbol is given by:

$$r_t = h_f s_t + n_{g_t}, \tag{1}$$

where $t = \{1, 2, \cdots, N\}$, $f = \{1, 2, \cdots, F\}$, f and t are related by $f = \lceil F \frac{t}{N} \rceil$, where $\lceil \phi \rceil$ returns the smallest integer not smaller than ϕ, h_f is the real Rayleigh fading coefficient of the f-th block, s_t is the transmitted signal, and n_{g_t} is additive white Gaussian noise with zero mean and variance $N_0/2$. In this paper, we assume that the transmitted symbols s_t are binary phase shift keying (BPSK) modulated. We assume that the receiver has perfect channel state information, and that the SNR is defined as E_b/N_0, where E_b is the energy per information bit. The information transmission rate is $R = K/N$, where K is the number of information bits per codeword of length N. For the case of a block-fading channel, we consider $R = 1/F$, since then it is possible to design a practical diversity achieving code [5]. The performance of a communication system in a non-ergodic block-fading channel can be investigated by means of the outage probability [5], which is defined as:

$$P_{out} = \mathcal{P}(\mathcal{I} < \mathcal{R}), \tag{2}$$

where $\mathcal{P}(\phi)$ is the probability of event ϕ and $\mathcal{I}$ is the mutual information. The mutual information I_G, for Gaussian channel inputs is [5]:

$$I_G = \frac{1}{F} \sum_{f=1}^{F} \frac{1}{2} \log_2 \left(1 + 2R \frac{E_b}{N_0} h_f^2 \right), \tag{3}$$

so that an outage occurs when the average accumulated mutual information among blocks is smaller than the attempted information transmission rate.

3 Root-Check LDPC Codes

In this section, the parity check matrix (PCM) of the most relevant Root-Check LDPC codes are discussed. The number of fadings considered are $F = 2, 3$ and 4 which correspond to code rates $R = \frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$.

3.1 Random Root-Check LDPC Codes

Here, we will introduce some definitions and the notation adopted in this work. The LDPC code in systematic form is specified by its sparse PCM $\mathbf{H}$:

$$\mathbf{H} = [\mathbf{I}_{N-K}\ \mathbf{P}], \tag{4}$$

where $\mathbf{I}_{N-K}$ is the identity matrix of size (N-K) and $\mathbf{P}$ is an (N-K)-by-K matrix. Then the generator matrix for the code is:

$$\mathbf{G} = [\mathbf{P}^T\ \mathbf{I}_K], \tag{5}$$

where $(\cdot)^T$ refers to the transpose operation.

The variable node degree sequence D_s is defined to be the set of column weights of $\mathbf{H}$ as designed, and is prescribed by the variable node degree distribution $\lambda(x)$ as described in [15]. Moreover, D_s is arranged in non-decreasing order. The first proposed Root-Check LDPC codes were devised by Boutros et. al. in [4]. Therefore, the general structure of the PCM for a random Root-Check LDPC code for $F = 2$ can be defined as

$$\mathbf{H} = \begin{array}{c} \\ 1c \\ 2c \end{array} \begin{array}{c} \begin{array}{cccc} 1i & 2i & 1p & 2p \end{array} \\ \begin{pmatrix} \mathbf{I} & \mathbf{H}_{2i} & \mathbf{0} & \mathbf{H}_{2p} \\ \mathbf{H}_{1i} & \mathbf{I} & \mathbf{H}_{1p} & \mathbf{0} \end{pmatrix} \end{array}, \tag{6}$$

where the nodes (1i and 2i) represent the systematic symbols that are sent over two independent fading, the same happens to nodes (1p and 2p) which are the parity symbols; (1c and 2c) are the check nodes. In the PCM $\mathbf{H}$, there are eight sub-matrices of size $\frac{N}{4} \times \frac{N}{4}$. $\mathbf{I}$ is an identity sub-matrix, $\mathbf{O}$ is a null sub-matrix, $\mathbf{H}_{1i}$ and $\mathbf{H}_{2i}$ are sub-matrices of Hamming weight 2 connected to the systematic symbols, $\mathbf{H}_{1p}$ and $\mathbf{H}_{2p}$ are also sub-matrices of Hamming weight 3 connected to the parity symbols. In a similar fashion, it can be devised for the case of $F = 3$ as stated in [4].

3.2 Quasi-Cyclic Root-Check LDPC Codes

Following the idea of Boutros et. al. in [4], Li and Salehi in [5] devised a Quasi-Cyclic Root-Check LDPC Codes. The PCM $\mathbf{H}$ of a QC-LDPC code can be defined as [16]:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{0,0} & \mathbf{H}_{0,1} & \cdots & \mathbf{H}_{0,w-1} \\ \mathbf{H}_{1,0} & \mathbf{H}_{1,1} & \cdots & \mathbf{H}_{0,w-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{c-1,0} & \mathbf{H}_{c-1,1} & \cdots & \mathbf{H}_{c-1,w-1} \end{bmatrix}, \tag{7}$$

where $\mathbf{H}_{ij}$ is an $n \times n$ circulant or all-zeros matrix, and c and w are two positive integers with $c < w$. The null space of $\mathbf{H}$ gives a QC-LDPC code over $GF(2)$ of length $N = wn$. The rank of $\mathbf{H}$ is at most cn. Hence the code rate is at least $\frac{w-c}{w}$.

For the case of Quasi-Cyclic Root-Check LDPC codes the PCM follows the same idea as (6), although the sub-matrices become a set of Quasi-Cyclic matrices. Consequently, $\mathbf{I}$ becomes

$$\mathbf{I}_{(top-left)} = \begin{vmatrix} \mathbf{I}_{0,0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{1,1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{2,2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{3,3} \end{vmatrix}, \tag{8}$$

$\mathbf{H}_{1i}$ as

$$\mathbf{H}_{1i} = \begin{vmatrix} \mathbf{I}_{4,0} & \mathbf{I}_{4,1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{5,1} & \mathbf{I}_{5,2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{6,2} & \mathbf{I}_{6,3} \\ \mathbf{I}_{7,0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{7,3} \end{vmatrix} \tag{9}$$

and for $\mathbf{H}_{1p}$ we define it as

$$\mathbf{H}_{1p} = \begin{vmatrix} \mathbf{0} & \mathbf{I}_{4,5} & \mathbf{I}_{4,6} & \mathbf{I}_{4,7} \\ \mathbf{I}_{5,4} & \mathbf{0} & \mathbf{I}_{5,6} & \mathbf{I}_{5,7} \\ \mathbf{I}_{6,4} & \mathbf{I}_{6,5} & \mathbf{0} & \mathbf{I}_{6,7} \\ \mathbf{I}_{7,4} & \mathbf{I}_{7,5} & \mathbf{I}_{7,6} & \mathbf{0} \end{vmatrix}, \tag{10}$$

where in each $\mathbf{I}_{i,j}$ is a circulant permutation matrix and each $\mathbf{0}$ is a null matrix. The example presented in Equations (8), (9) and (10) are for a regular Root-Check QC-LDPC code $C(3,6)$. Root-Check QC-LDPC codes were proposed with the aim of providing fast encoding and to save memory to store the generator matrix. Li and Salehi in [5] have shown that the QC-LDPC codes can perform as well as randomly generated Root-LDPC codes [4] over block-fading channels.

3.3 Unstructured Full Diversity LDPC Codes

Duyck et. al. in [7] proposed the design of random LDPC codes which are able to achieve full diversity in block-fading channels with $F = 2$ fading. The principle proposed in [7] is to design random LDPC codes with code rate $R \cong 0.5$ to avoid stopping sets in the systematic bits. Moreover, the systematic bits must have degree $d_v = 2$ to avoid stopping sets.

The design of such LDPC codes was achieved by requiring that the number of check nodes in the graph be greater than $\frac{N}{2}$, i.e., that the rate be less than $\frac{1}{2}$, and that the weight of the first $\frac{N}{2}$ variable nodes is 2 and that the graph be constructed by the PEG algorithm [17], which maximises cycle length at each placement, ensuring under these conditions no cycles in the sub-graph comprised of the first $\frac{N}{2}$ variable nodes alone. The requirement of recoverability for the worst-case scenario is equivalent to the requirement that no systematic variable node $v_{syst} \in \mathbf{V}_{syst}$, affected by α_1, is an element of any stopping set found among the variable nodes $\mathbf{V}_1 \cup \{\mathbf{V}_2 \cup \mathbf{V}_3 \cup \cdots \mathbf{V}_F\} \backslash \mathbf{V}_i$. This requirement must hold for all $i = 2, \cdots, F$ for the systematic variable nodes to be recoverable on the block binary erasure channel and thus for the code to achieve full diversity on the block fading channel. The parity-check matrix for this general case, with variable node subset labels and the corresponding fading coefficients are given in Fig. 1.

3.3.1 Unstructured Full Diversity Rate $\frac{1}{3}$

In (11) is shown a code graph for the case of $F = 3$ fading per codeword [8] by means of imposing null matrices on the parity-check matrix, along with restrictions on the cycles present in the sub-graphs of the code. The structured matrices $[\mathbf{H}_{\alpha,1}\mathbf{H}_2]$ and $[\mathbf{H}_{\alpha,2}\mathbf{H}_3]$ must be constructed by the PEG algorithm, as in [7], ensuring the extrinsic connections to $\mathbf{V}_2$ and $\mathbf{V}_3$, respectively. The constraints on the code sub-graphs result in the variable nodes of $\mathbf{V}_1$ having weight 4. The distribution of the nodes in $\mathbf{V}_2$ and $\mathbf{V}_3$ is unconstrained and may be irregular. In addition to this weight constraint, each of the sub-matrices $[\mathbf{H}_{\alpha,1}\mathbf{H}_2]$ and $[\mathbf{H}_{\alpha,2}\mathbf{H}_3]$ are constrained to have rate less than $\frac{1}{2}$, and so the final graph will have rate less than $\frac{1}{3}$.

$$\mathbf{H}_{BF3} = \begin{matrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \end{matrix} \begin{bmatrix} \mathbf{H}_{\alpha_1,1} & \mathbf{H}_{\alpha_2} & \mathbf{0} \\ \mathbf{H}_{\alpha_1,2} & \mathbf{0} & \mathbf{H}_{\alpha_3} \end{bmatrix} \tag{11}$$

3.3.2 Unstructured Full Diversity Rate $\frac{1}{4}$

The code graph achieving the requirements on stopping sets among $\mathbf{V}_1, \cdots, \mathbf{V}_4$ containing systematic variable nodes is presented in (12) [8]. We can see that with each additional fading coefficient considered, a straightforward graph expansion is carried out, effectively nesting the $F-1$ diversity achieving graph in the code capable of full diversity performance on the channel with F fading coefficients.

$$\mathbf{H}_{BF4} = \begin{matrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \end{matrix} \begin{bmatrix} \mathbf{H}_{\alpha_1,1} & \mathbf{H}_{\alpha_2} & \mathbf{0} & \mathbf{0} \\ \mathbf{H}_{\alpha_1,2} & \mathbf{0} & \mathbf{H}_{\alpha_3} & \mathbf{0} \\ \mathbf{H}_{\alpha_1,3} & \mathbf{0} & \mathbf{0} & \mathbf{H}_{\alpha_4} \end{bmatrix} \tag{12}$$

4 Proposed PEG-Based Root-Check LDPC Codes

In this section, the proposed PEG-Based Root-Check LDPC codes are discussed. The number of fadings considered are $F = 2, 3$ and 4 which correspond to code rates $R = \frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$.

4.1 QC PEG-Based Root-Check LDPC Codes

Preliminary results on the design of a PEG-based Quasi- Cyclic Root-Check LDPC codes for Block-Fading channel with $F = 3, 4$ fadings per codeword were presented by Uchoa et. al. in [9]. The codes generated by this strategy can achieve a significant performance in terms of FER with respect to the theoretical limit. These codes can save up to $3dB$ in terms of signal to noise ratio to achieve the same FER when compared to other codes.

A Root-Check LDPC code requires a designer to divide both variable and check nodes in F equal parts. Following the non-systematic Root-Check structure reported in [4], the PCM becomes:

$$\mathbf{H} = [\mathbf{S}_1\mathbf{P}_1, \cdots, \mathbf{S}_F\mathbf{P}_F], \tag{13}$$

where the subscripts represent the variable nodes (systematic and parity, respectively) under a specific fading block. A systematic PCM can be defined as being $\mathbf{H} = [\mathbf{A}|\mathbf{B}]$, where $\mathbf{A}$ is connected to the systematic and $\mathbf{B}$ is connected to the parity check bits, respectively. So, if we consider the structure in (13) in systematic form, the PCM becomes $\mathbf{H} = [\mathbf{S}_1, \cdots, \mathbf{S}_F\mathbf{P}_1, \cdots, \mathbf{P}_F]$. In order to obtain the generator matrix, the sub-matrix $\mathbf{B}$ formed by parity matrices $\mathbf{P}_1, \cdots, \mathbf{P}_F$ must be a non-singular matrix, which means it is invertible under $GF(2)$ [5].

To design a practical code for $F = 3$ which is able to achieve the channel diversity, the rate of such code must be $R = \frac{1}{F} = \frac{1}{3}$. As a result, the PCM for $R = \frac{1}{3}$ can be defined as in (14),

$$\mathbf{H} = \begin{bmatrix} \mathbf{I_{0,0}} & \mathbf{H_{0,1}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H_{0,6}} & \mathbf{H_{0,7}} & \mathbf{H_{0,8}} \\ \mathbf{I_{1,0}} & \mathbf{0} & \mathbf{H_{1,2}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H_{1,7}} & \mathbf{H_{1,8}} \\ \mathbf{H_{2,0}} & \mathbf{I_{2,1}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H_{2,8}} \\ \mathbf{0} & \mathbf{I_{3,1}} & \mathbf{H_{3,2}} & \mathbf{H_{3,3}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{H_{4,0}} & \mathbf{0} & \mathbf{I_{4,2}} & \mathbf{H_{4,3}} & \mathbf{H_{4,4}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H_{5,1}} & \mathbf{I_{5,2}} & \mathbf{H_{5,3}} & \mathbf{H_{5,4}} & \mathbf{H_{5,5}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{14}$$

where $\mathbf{H}_{ij}$ and $\mathbf{I}_{ij}$ are $n \times n$ circulant matrices, while $\mathbf{0}$ is an all-zeros matrix. The notation $\mathbf{I}_{ij}$ was used to reinforce that such connections are the Root-Check connections [4]. The restrictions that should be imposed are only the $\mathbf{I}_{ij}$ to be placed in the positions described in (14) and the upper and down triangular sub-matrices in the parity part, $\mathbf{B}$, of $\mathbf{H}$. In order to perform a PEG-based design the only restriction imposed is that the sub-matrices $\mathbf{I}_{ij}$ and the upper and down sub-matrices of (14) are kept. The other sub-matrices can be placed following a quasi-cyclic PEG-based algorithm.

Similarly to the case of $R = \frac{1}{3}$, the PCM structure for $F = 4$ with code rate $R = \frac{1}{4}$ the PCM can be defined as in (15).

$$\mathbf{H} = \begin{bmatrix} \mathbf{I_{0,0}} & \mathbf{H_{0,1}} & \mathbf{0} & \mathbf{H_{0,3}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H_{0,10}} & \mathbf{0} & \mathbf{H_{0,12}} & \mathbf{0} & \mathbf{H_{0,14}} & \mathbf{0} \\ \mathbf{I_{1,0}} & \mathbf{0} & \mathbf{H_{1,2}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H_{1,11}} & \mathbf{0} & \mathbf{H_{1,13}} & \mathbf{0} & \mathbf{H_{1,15}} \\ \mathbf{I_{2,0}} & \mathbf{H_{2,1}} & \mathbf{0} & \mathbf{H_{2,3}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H_{2,12}} & \mathbf{0} & \mathbf{H_{2,14}} & \mathbf{0} \\ \mathbf{H_{3,0}} & \mathbf{I_{3,1}} & \mathbf{H_{3,2}} & \mathbf{H_{3,3}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H_{3,13}} & \mathbf{0} & \mathbf{H_{3,15}} \\ \mathbf{0} & \mathbf{I_{4,1}} & \mathbf{H_{4,2}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H_{4,14}} & \mathbf{0} \\ \mathbf{H_{5,0}} & \mathbf{I_{5,1}} & \mathbf{0} & \mathbf{H_{5,3}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H_{5,15}} \\ \mathbf{H_{6,0}} & \mathbf{0} & \mathbf{I_{6,2}} & \mathbf{0} & \mathbf{H_{6,4}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H_{7,1}} & \mathbf{I_{7,2}} & \mathbf{0} & \mathbf{0} & \mathbf{H_{7,5}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I_{8,2}} & \mathbf{H_{8,3}} & \mathbf{H_{8,4}} & \mathbf{0} & \mathbf{H_{8,6}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{H_{9,0}} & \mathbf{0} & \mathbf{H_{9,2}} & \mathbf{I_{9,3}} & \mathbf{0} & \mathbf{H_{9,5}} & \mathbf{0} & \mathbf{H_{9,7}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H_{10,1}} & \mathbf{H_{10,2}} & \mathbf{I_{10,3}} & \mathbf{H_{10,4}} & \mathbf{0} & \mathbf{H_{10,6}} & \mathbf{0} & \mathbf{H_{10,8}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{H_{11,0}} & \mathbf{H_{11,1}} & \mathbf{0} & \mathbf{I_{11,3}} & \mathbf{0} & \mathbf{H_{11,5}} & \mathbf{0} & \mathbf{H_{11,7}} & \mathbf{0} & \mathbf{H_{11,9}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{15}$$

The restrictions that should be imposed to design a PEG-based QC-Root-Check LDPC codes for rate $R = \frac{1}{4}$ are similar to the case of $R = \frac{1}{3}$.

4.1.1 Proposed Design Algorithm

Here, we introduce some definitions and notations. Then, we present the pseudo-code of our proposed algorithm for PEG-based Quasi-Cyclic Root-Check LDPC codes. A block-fading channel with $F = 3$ and $F = 4$ are considered. In extending to a greater number of fadings, $F > 4$, the general structure presented is maintained, with the systematic VNs for each fading possessing Root-Check identity matrices connecting to parity VNs in each of the other fading blocks only, ensuring the upper and lower triangular sections of parity bits observed in (14) and (15). The placement of the remaining cyclic sub-matrices is required to maintain this relationship and provide satisfactory final code degree distribution. The LDPC code in systematic form is specified by its sparse PCM $\mathbf{H} = [\mathbf{A} \mid \mathbf{B}]$, where $\mathbf{A}$ is a matrix of size M-by-K, and $\mathbf{B}$ is an M-by-M matrix. The generator matrix for the code is $\mathbf{G} = [(\mathbf{B}^{-1}\mathbf{A})^T \mid \mathbf{I}_K]$, $\mathbf{I}_K$ is an identity matrix of size K.

The variable node degree sequence D_s is defined as the set of column weights of the designed $\mathbf{H}$, and is prescribed by the variable node degree distribution $\lambda(x)$ as described in [15]. Moreover, D_s is arranged in non-decreasing order. The proposed algorithm, called QC-PEG Root-Check, constructs $\mathbf{H}$ by operating progressively on variable nodes to place the edges required by D_s. The Variable Node (VN) of interest is labelled v_j and the candidate check nodes are individually referred to as c_i. The PEG Root-Check algorithm chooses a check node c_i to connect to the variable node of interest v_j by expanding a constrained sub-graph from v_j up to maximum depth l. The set of check nodes found in this sub-graph are denoted $N^l_{v_j}$ while the set of check nodes of interest, those not currently found in the sub-graph, are denoted $\overline{N^l_{v_j}}$. For the QC-PEG Root-Check algorithm, a check node is chosen at random from the minimum weight check nodes of this set.

4.1.2 Pseudo-code for the QC-PEG-Root-Check Algorithm

Initialization: A matrix of size $M \times N$ is created with the circulant identity matrices $\mathbf{I}_{i,j}$ in the positions shown in (14) and (15) and zeros in all other positions. We define the indicator vectors $\mathbf{z}_1, \cdots, \mathbf{z}_{F^2}$ for the cases $R = \frac{1}{3}$, $R = \frac{1}{4}$ respectively as:

$$
\begin{aligned}
\mathbf{z}_1 &= [\mathbf{0}_{1\times\frac{2N}{9}}, \mathbf{1}_{1\times\frac{N}{9}}, \mathbf{0}_{1\times\frac{N}{9}}, \mathbf{1}_{1\times\frac{N}{9}}, \mathbf{0}_{1\times\frac{N}{9}}]^T, \\
\mathbf{z}_2 &= [\mathbf{1}_{1\times\frac{N}{9}}, \mathbf{0}_{1\times\frac{4N}{9}}, \mathbf{1}_{1\times\frac{N}{9}}]^T, \\
\mathbf{z}_3 &= [\mathbf{0}_{1\times\frac{N}{9}}, \mathbf{1}_{1\times\frac{N}{9}}, \mathbf{0}_{1\times\frac{N}{9}}, \mathbf{1}_{1\times\frac{N}{9}}, \mathbf{0}_{1\times\frac{2N}{9}}]^T, \\
\mathbf{z}_\chi &= [\mathbf{0}_{1\times\frac{(i-1)\cdot N}{9}}, \mathbf{1}_{1\times\frac{(7-i)N}{9}}]^T \quad \text{for } \chi = 4, 5, 6, \\
\mathbf{z}_\gamma &= [\mathbf{1}_{1\times\frac{(i-6)\cdot N}{9}}, \mathbf{0}_{1\times\frac{(12-i)N}{9}}]^T \quad \text{for } \gamma = 7, 8, 9,
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\mathbf{z}_1 &= [\mathbf{0}_{1\times\frac{3N}{16}}, \mathbf{1}_{1\times\frac{N}{16}}, \mathbf{0}_{1\times\frac{N}{16}}, \mathbf{1}_{1\times\frac{2N}{16}}, \mathbf{0}_{1\times\frac{2N}{16}}, \\
&\quad \mathbf{1}_{1\times\frac{N}{16}}, \mathbf{0}_{1\times\frac{N}{16}}, \mathbf{1}_{1\times\frac{N}{16}}]^T, \\
\mathbf{z}_2 &= [\mathbf{1}_{1\times\frac{N}{16}}, \mathbf{0}_{1\times\frac{N}{16}}, \mathbf{1}_{1\times\frac{N}{16}}, \mathbf{0}_{1\times\frac{4N}{16}}, \mathbf{1}_{1\times\frac{N}{16}}, \\
&\quad \mathbf{0}_{1\times\frac{2N}{16}}, \mathbf{1}_{1\times\frac{2N}{16}},]^T, \\
\mathbf{z}_3 &= [\mathbf{0}_{1\times\frac{N}{16}}, \mathbf{1}_{1\times\frac{N}{16}}, \mathbf{0}_{1\times\frac{N}{16}}, \mathbf{1}_{1\times\frac{2N}{16}}, \mathbf{0}_{1\times\frac{4N}{16}}, \\
&\quad \mathbf{1}_{1\times\frac{2N}{16}}, \mathbf{0}_{1\times\frac{N}{16}}]^T, \\
\mathbf{z}_4 &= [\mathbf{1}_{1\times\frac{N}{16}}, \mathbf{0}_{1\times\frac{N}{16}}, \mathbf{1}_{1\times\frac{2N}{16}}, \mathbf{0}_{1\times\frac{N}{16}}, \mathbf{1}_{1\times\frac{N}{16}}, \\
&\quad \mathbf{0}_{1\times\frac{2N}{16}}, \mathbf{1}_{1\times\frac{N}{16}}, \mathbf{0}_{1\times\frac{3N}{16}},]^T, \\
\mathbf{z}_\chi &= [\mathbf{0}_{1\times\frac{(i+1)N}{16}}, \mathbf{v}_{ALT(0:\frac{(11-i)N}{16}-1)}]^T \\
&\quad \text{for } \chi = 5, \cdots, 10, \\
\mathbf{z}_\gamma &= [\mathbf{v}_{ALT(\frac{(17-i)N}{16}:\frac{7N}{16}-1)}, \mathbf{0}_{1\times\frac{(22-i)N}{16}}]^T \\
&\quad \text{for } \gamma = 11, \cdots, 16,
\end{aligned}
\tag{17}
$$

$$
\mathbf{v}_{ALT} = [\mathbf{1}_{1\times\frac{N}{16}}, \mathbf{0}_{1\times\frac{N}{16}}, \mathbf{1}_{1\times\frac{N}{16}}, \mathbf{0}_{1\times\frac{N}{16}}, \mathbf{1}_{1\times\frac{N}{16}}, \mathbf{0}_{1\times\frac{N}{16}}, \mathbf{1}_{1\times\frac{N}{16}}] \tag{18}
$$

These indicator vectors are modelled on that of the original PEG algorithm [17], indicating submatrices for which placement is permitted, thus imposing the form of (14) or (15). The degree sequence as defined for LDPC codes must be altered to take into account the structure imposed by Root-Check codes, namely the circulant identity matrices, $\mathbf{I}_{i,j}$, of (14) and (15). The pseudo-code for our proposed QC-PEG Root-Check algorithm is detailed in Algorithm 1, where the indicator vector, $\mathbf{z}_i$, is taken from (16), (17) for constructing codes of rate $R = \frac{1}{3}$, $R = \frac{1}{4}$, respectively.

Algorithm 1 QC-PEG Root-Check Algorithm

for $j = 1 : F^2$ **do**
  for $k = 0 : D_s(j) - 1$ **do**
    if $j \geq \frac{N}{F}$ & $k == 0$ **then**
      Place edge at random among minimum weight submatrices permitted by the indicator $\mathbf{z}_j$, with a random first edge placement within the chosen submatrix, in column $\frac{(j-1)\cdot N}{F^2}$-*th*.
      Place remaining edges in the submatrix by circulant shift of the first placement.
      Null the entry in the indicator vector $\mathbf{z}_j$ in the position of the chosen submatrix, preventing further placements in that submatrix.
    else
      Expand the PEG subtree from the $\frac{(j-1)\cdot N}{F^2}$-*th* VN to depth *l* such that the tree contains all CNs allowed by the indicator vector **or** the number of nodes in the tree does not increase with an expansion to the *(l+1)-th* level.
      Place edge connecting the $\frac{(j-1)\cdot N}{F^2}$-*th* VN to a CN chosen randomly from the set of minimum weight nodes which were added to the subtree at the last tree expansion.
      Place remaining edges in the submatrix by circulant shift of the first placement.
      Null the entry in the indicator vector $\mathbf{z}_j$ in the position of the chosen submatrix, preventing further placements in that submatrix.
    end if
  end for
end for

4.2 RA Based Root-Check LDPC Codes

Preliminary results on the design of Irregular Repeat Accumulate (IRA) LDPC codes PEG-based with Root-Check properties were reported in [10]. We considered a block-fading channel with $F = 2$ and $F = 3$. Here, in this section we synthesize the most relevant information on the design of IRA Root-Check LDPC codes and Irregular Repeat-Accumulate and Accumulate (IRAA) Root-Check LDPC codes.

A repeat-accumulate (RA) code consists of a serial concatenation, through an interleaver, of a single rate $1/q$ repetition code with an accumulator having transfer function $\frac{1}{1+D}$, where q is the number of repetitions for each group of K information bits. Fig. 2 shows a typical repeat-accumulate code block diagram. The implementation of the transfer function $\frac{1}{1+D}$ is identical to an accumulator, although the accumulator value can be only 0 or 1 since the operations are over the binary field [18, pp. 267-279]. As discussed in [18, pp. 267-279], to ensure a large minimum Hamming distance, the interleaver should be designed so that consecutive 1s at its input are widely separated at its output. The RA based codes proposed in [10] were systematic. The main limitation of RA codes is the code rate, which cannot be higher than $\frac{1}{2}$.

Irregular repeat-accumulate (IRA) codes generalize the concept of RA codes by changing the repetition rate for each group of K information bits and performing a linear combination of the repeated bits which are sent through the accumulator. Furthermore, IRA codes are typically systematic. IRA codes allow flexibility in the choice of the repetition rate for each information bit so that high-rate codes may be designed. Their irregularity allows operation closer to the capacity limit [18, pp. 267-279].

The PCM for a systematic RA and IRA codes has the form $\mathbf{H} = [\mathbf{H}_u \ \mathbf{H}_p]$, where $\mathbf{H}_p$ is a square dual-diagonal matrix given by

$$\mathbf{H}_p = \begin{bmatrix} 1 & & & & \\ 1 & 1 & & & \\ & \ddots & \ddots & & \\ & & 1 & 1 & \\ & & & 1 & 1 \end{bmatrix}. \tag{19}$$

For RA codes, $\mathbf{H}_u$ is a regular matrix having column weight q and row weight 1. For IRA codes, $\mathbf{H}_u$ has irregular columns and rows weights. The Generator Matrix (GM) can be obtained as $\mathbf{G} = \left[\mathbf{I}_K \ \mathbf{H}_u^T \mathbf{H}_p^{-T}\right]$, where $\mathbf{I}_K$ is an identity matrix of dimension $K \times K$. In matrix notation $\mathbf{H}_p^{-T}$ can be described as

$$\mathbf{H}_p^{-T} = \begin{bmatrix} 1 & 1 & \cdots & & 1 \\ & 1 & 1 & \cdots & 1 \\ & & \ddots & & \vdots \\ & & & 1 & 1 \\ & & & & 1 \end{bmatrix}. \tag{20}$$

4.2.1 IRA Root-Check Rate $\frac{1}{2}$

The design of a Root-Check with an IRA structure imposes some constraints in terms of PCM to guarantee the Root-Check properties. Following the notation adopted in [19], for the case of a systematic Rate $\frac{1}{2}$ with $F = 2$, the PCM must be like

$$\mathbf{H} = \begin{bmatrix} \mathbf{I}_{\frac{N}{4}} & \mathbf{H}_2 & \mathbf{0}_{\frac{N}{4}} & \mathbf{H}_3 \\ \mathbf{H}_2 & \mathbf{I}_{\frac{N}{4}} & \mathbf{H}_3 & \mathbf{0}_{\frac{N}{4}} \end{bmatrix}, \tag{21}$$

where $\mathbf{H}_2$ and $\mathbf{H}_3$ are $\frac{N}{4} \times \frac{N}{4}$ sub-matrices with Hamming weight two and three, respectively, while $\mathbf{0}_{\frac{N}{4}}$ is a null sub-matrix with dimension $\frac{N}{4} \times \frac{N}{4}$. Therefore, to impose the RA structure and Root-Check properties the PCM of an IRA Root-Check is

$$\mathbf{H} = \begin{bmatrix} \mathbf{I}_{\frac{N}{4}} & \mathbf{H}_2 & \mathbf{0}_{\frac{N}{4}} & \mathbf{H}_p \\ \mathbf{H}_2 & \mathbf{I}_{\frac{N}{4}} & \mathbf{H}_p & \mathbf{0}_{\frac{N}{4}} \end{bmatrix}, \tag{22}$$

where $\mathbf{H}_p$ is a dual diagonal matrix with dimension $\frac{N}{4} \times \frac{N}{4}$.

4.2.2 IRA Root-Check Rate $\frac{1}{3}$

For the case of Rate $\frac{1}{3}$ with $F = 3$, we followed a similar structure to the one adopted in [4,9]. The accumulator used is a transfer function given by $\frac{1}{1+D+D^{\frac{N}{9}}}$ as suggested by [20]. As a result, $\mathbf{H}_p$ must be redefined as

$$\mathbf{H}_p = \begin{bmatrix} \mathbf{H}_{p1} \\ \mathbf{H}_{p2} \end{bmatrix}, \tag{23}$$

$$\mathbf{H}_{p1} = \begin{bmatrix} 1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 1 & 1 & 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{24}$$

$$\mathbf{H}_{p2} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & 1 & 1 & 0 & 0 \\ \vdots & 0 & \ddots & 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}, \tag{25}$$

where $\mathbf{H}_{p1}$ and $\mathbf{H}_{p2}$ are sub-matrices with dimensions $\frac{N}{9} \times \frac{2N}{9}$. Therefore, the PCM $\mathbf{H} = [\mathbf{H}_u|\mathbf{H}_p]$ for this particular case of an IRA Root-Check Rate $\frac{1}{3}$ as

$$\mathbf{H} = \begin{bmatrix} \mathbf{I}_{\frac{N}{9}} & \mathbf{H}_1 & \mathbf{0}_{\frac{N}{9}} & | & \mathbf{0} & \mathbf{H}_{p2} & \mathbf{0} \\ \mathbf{I}_{\frac{N}{9}} & \mathbf{0}_{\frac{N}{9}} & \mathbf{H}_1 & | & \mathbf{0} & \mathbf{0} & \mathbf{H}_{p1} \\ \mathbf{H}_1 & \mathbf{I}_{\frac{N}{9}} & \mathbf{0}_{\frac{N}{9}} & | & \mathbf{H}_{p1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0}_{\frac{N}{9}} & \mathbf{I}_{\frac{N}{9}} & \mathbf{H}_1 & | & \mathbf{0} & \mathbf{0} & \mathbf{H}_{p2} \\ \mathbf{H}_1 & \mathbf{0}_{\frac{N}{9}} & \mathbf{I}_{\frac{N}{9}} & | & \mathbf{H}_{p2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0}_{\frac{N}{9}} & \mathbf{H}_1 & \mathbf{I}_{\frac{N}{9}} & | & \mathbf{0} & \mathbf{H}_{p1} & \mathbf{0} \end{bmatrix}, \tag{26}$$

where $\mathbf{H}_1$ and $\mathbf{I}_{\frac{N}{9}}$ are sub-matrices with dimensions $\frac{N}{9} \times \frac{N}{9}$ and $\mathbf{H}_1$ is a sub-matrix with Hamming weight equal to 1. The null sub-matrices $\mathbf{0}$ on the right hand side of (26) have dimensions $\frac{N}{9} \times \frac{2N}{9}$ while on the left hand side the dimensions are $\frac{N}{9} \times \frac{N}{9}$.

4.3 IRAA Root-Check Design

The general structure of an Irregular Repeat-Accumulate and Accumulate (IRAA) encoder can be seen in Fig. 3. In this figure, some b extra parity bits are indicated in addition to the normal p parity bits. The b parity bits can be punctured to obtain a higher code rate. For instance, in general an IRAA code has rate 1/3 without puncturing, while by puncturing b parity-checks a code with rate 1/2 can be obtained.

The PCM of an IRAA LDPC code can be represented by

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_u & \mathbf{H}_p & \mathbf{0} \\ \mathbf{0} & \prod_1 & \mathbf{H}_p \end{bmatrix}, \tag{27}$$

where $\prod_1$ must be a sub-matrix with rows and columns with Hamming weight one.

In order to obtain IRAA Root-Check LDPC codes some constraints must be imposed on the standard IRAA design. We have noticed that the IRAA Root-Check LDPC codes led to a more flexible rate compatible code. For further details refer to [10].

4.3.1 IRAA Root-Check Rate $\frac{1}{2}$

We applied the Root-Check structure from (22) in (27) to obtain the following PCM for rate 1/2

$$\mathbf{H} = \begin{bmatrix} \mathbf{I}_{\frac{N}{9}} & \mathbf{H}_2 & \mathbf{0}_{\frac{N}{9}} & \mathbf{H}_p & \mathbf{0} & \mathbf{0}_{\frac{N}{9}} \\ \mathbf{H}_2 & \mathbf{I}_{\frac{N}{9}} & \mathbf{H}_p & \mathbf{0}_{\frac{N}{9}} & \mathbf{0}_{\frac{N}{9}} & \mathbf{0}_{\frac{N}{9}} \\ \mathbf{0}_{\frac{N}{9}} & \mathbf{0}_{\frac{N}{9}} & \multicolumn{2}{c}{\multirow{2}{*}{$\prod_1$}} & \mathbf{0}_{\frac{N}{9}} & \mathbf{H}_p \\ \mathbf{0}_{\frac{N}{9}} & \mathbf{0}_{\frac{N}{9}} & & & \mathbf{H}_p & \mathbf{0}_{\frac{N}{9}} \end{bmatrix}, \tag{28}$$

where $\mathbf{I}_{\frac{N}{9}}$, $\mathbf{H}_2$, $\mathbf{H}_p$ and $\mathbf{0}_{\frac{N}{9}}$ are all $\frac{N}{9} \times \frac{N}{9}$ in dimension, while $\prod_1$ is $\frac{N}{3} \times \frac{N}{3}$. The key point to guarantee the full diversity property is the puncturing procedure. Instead of puncturing b parity bits we have punctured p. The reason why puncturing p

instead of b guarantees the full diversity is due to the fact that the Root-Check structure of the code is kept unchanged.

4.3.2 IRAA Root-Check Rate $\frac{1}{3}$

For the case of rate 1/3 we considered the design done in (26) and we apply the constraints in (27) to obtain the following PCM

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_u & | & \mathbf{H}_p & \mathbf{0} \\ \mathbf{0} & | & \prod_1 & \mathbf{H}_p \end{bmatrix}. \qquad (29)$$

It must be noted that without puncturing the code rate is 1/5.

4.3.3 Pseudo-code for the IRA-PEG Root-Check Algorithm

Initialization: A matrix of size $M \times N$ is created with the identity matrices $\mathbf{I}_K$ and parity matrices $\mathbf{H}_p$ in the positions shown in (22), (26), (28), (29) and zeros in all other positions. We define the indicator vectors $\mathbf{z}_1, \cdots, \mathbf{z}_F$ for the cases $R = \frac{1}{2}$, $R = \frac{1}{3}$ respectively as:

$$\begin{aligned} \mathbf{z}_1 &= [\mathbf{0}_{1\times\gamma}, \mathbf{1}_{1\times\gamma}]^T, \\ \mathbf{z}_2 &= [\mathbf{1}_{1\times\gamma}, \mathbf{0}_{1\times\gamma}]^T, \end{aligned} \qquad (30)$$

$$\begin{aligned} \mathbf{z}_1 &= [\mathbf{0}_{1\times2\chi}, \mathbf{1}_{1\times\chi}, \mathbf{0}_{1\times\chi}, \mathbf{1}_{1\times\chi}, \mathbf{0}_{1\times\chi}]^T, \\ \mathbf{z}_2 &= [\mathbf{1}_{1\times\chi}, \mathbf{0}_{1\times4\chi}, \mathbf{1}_{1\times\chi}]^T, \\ \mathbf{z}_3 &= [\mathbf{0}_{1\times\chi}, \mathbf{1}_{1\times\chi}, \mathbf{0}_{1\times\chi}, \mathbf{1}_{1\times\chi}, \mathbf{0}_{1\times2\chi}]^T \\ &, \end{aligned} \qquad (31)$$

where $\gamma = \frac{N}{2}$ for the case of IRA, while for IRAA design $\gamma = \frac{N}{4}$. We have $\chi = \frac{N}{9}$ for the case of IRA, while for IRAA design $\chi = \frac{N}{15}$. In addition, for rate $R = \frac{1}{2}$ under IRAA design $\mathbf{z}_i = [\mathbf{z}_i, \mathbf{0}_{4\times\gamma}]$, while for rate $R = \frac{1}{3}$ under IRAA design $\mathbf{z}_i = [\mathbf{z}_i, \mathbf{0}_{6\times\chi}]$.

These indicator vectors are modelled on that of the original PEG algorithm [17], indicating sub-matrices for which placement is permitted, thus imposing the form of (22), (26), (28), (29). The degree sequence as defined for LDPC codes must be altered to take into account the structure imposed by Root-Check codes, namely, the identity matrices $\mathbf{I}_K$ and the parity matrices $\mathbf{H}_p$, of (22), (26), (28) and (29). The pseudo-code for our proposed IRA-PEG Root-Check algorithm is detailed in Algorithm 2, where the indicator vector $\mathbf{z}_i$ is taken from (30) and (31) for constructing codes of rate $R = \frac{1}{2}$, $R = \frac{1}{3}$ respectively.

4.4 Controlled Doping Root-LDPC Codes Design

Boutros in [11] a controlled doping via high order Root-Check LDPC codes was proposed. Such Root-Check LDPC codes are able to guarantee full diversity for the parity check bits. First of all, we have made some modifications in the original doped Root-Check LDPC code PCM described in [11].

Algorithm 2 PEG Root-Check LDPC Algorithm

for $j = 1 : K$ **do**
    for $k = 0 : D_s(j) - 1$ **do**
        Expand the PEG tree from the *j-th* VN to depth *l* such that the tree contains all CNs allowed by the indicator vector **or** the number of nodes in the tree does not increase with an expansion to the *(l+1)-th* level.
        Place the edge connecting the *j-th* VN to a CN chosen randomly from the set of minimum weight nodes which were added to the sub-tree at the last tree expansion.
    end for
end for

4.4.1 Controlled Doping Root-LDPC Codes $R = \frac{1}{2}$

The modifications we have made was to take the advantages of easy encodability of IRA-based LDPC codes. Furthermore, a PEG-based design to improve the local girth of the generated LDPC codes was considered. The purpose of controlled and uncontrolled doping is to improve the energy coefficient of information bits after solving parity bits. Then, the parity bit should transmit a high-confidence message to a new information bit. Uncontrolled doping corresponds to a diversity population evolution (DPE) steady-state parameter $p_\infty = 7.82\%$ for a $C(3,6)$ regular Root-LDPC code [11]. Controlled doping can achieve a fraction p_∞ as high as 100%. The sub-matrix (19) is modified by introducing a smaller identity (or a permutation) matrix for parity bits. Therefore, in order to obtain a Root-Check LDPC code with 50% of controlled doping, H_p is redefined as

$$\mathbf{H}_p = \begin{bmatrix} \mathbf{I}_{\frac{N}{8}} & \mathbf{O}_{\frac{N}{8}} \\ \mathbf{P}_{\frac{N}{8}} & \mathbf{DD}_{\frac{N}{8}} \end{bmatrix}, \tag{32}$$

where $\mathbf{I}$ is an identity matrix, $\mathbf{O}$ is a null matrix, $\mathbf{P}$ is a permutation matrix with Hamming weight 1, $\mathbf{DD}$ is a dual diagonal matrix and all sub-matrices of $\mathbf{H}_p$ are $\frac{N}{8} \times \frac{N}{8}$ in dimension. Accordingly, the final PCM becomes

$$\mathbf{H} = \left[\begin{array}{c|c|cc|cc} \mathbf{I}_{\frac{N}{4}} & \mathbf{H}_{2i} & \multicolumn{2}{c|}{\mathbf{0}_{\frac{N}{4}}} & \begin{matrix} \mathbf{I}_{\frac{N}{8}} \\ \mathbf{P}_2 \end{matrix} & \begin{matrix} \mathbf{0}_{\frac{N}{8}} \\ \mathbf{DD}_{\frac{N}{8}} \end{matrix} \\ \hline \mathbf{H}_{1i} & \mathbf{I}_{\frac{N}{4}} & \begin{matrix} \mathbf{I}_{\frac{N}{8}} \\ \mathbf{P}_1 \end{matrix} & \begin{matrix} \mathbf{0}_{\frac{N}{8}} \\ \mathbf{DD}_{\frac{N}{8}} \end{matrix} & \multicolumn{2}{c}{\mathbf{0}_{\frac{N}{4}}} \end{array}\right], \tag{33}$$

where subscripts in $\mathbf{P}_1$ and $\mathbf{P}_2$ means that are distinct permutation sub-matrices with hamming weight 1. The sub-matrices $\mathbf{H}_{1i}$ and $\mathbf{H}_{2i}$ are in dimension $\frac{N}{4} \times \frac{N}{4}$. $\mathbf{P}_1$ and $\mathbf{P}_2$ are in dimension $\frac{N}{8} \times \frac{N}{8}$. The PEG algorithm will work through the sub-matrices $\mathbf{H}_{1i}$ and $\mathbf{H}_{2i}$.

4.4.2 Controlled Doping Root-LDPC Codes $R = \frac{1}{3}$

The PCM for the case of code rate $R = \frac{1}{3}$ has followed a similar design as for an IRA Root-Check code rate $R = \frac{1}{3}$ in (26). Therefore, the PCM for the proposed PEG controlled doping Root-Check LDPC code (PEG-CDRC LDPC) has the structure as presented in (34),

$$\mathbf{H} = \left[\begin{array}{ccc|cccccc} \mathbf{I} & \mathbf{H}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{H}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{H}_2 & \mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{H}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_3 & \mathbf{DD} \\ \mathbf{H}_2 & \mathbf{0} & \mathbf{I} & \mathbf{P}_1 & \mathbf{DD} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2 & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{P}_2 & \mathbf{DD} & \mathbf{0} & \mathbf{0} \end{array}\right], \tag{34}$$

where the subscripts of $\mathbf{P}_i$ in (34) means that are distinct permutation sub-matrices. The sub-matrices of eq. (34) are all $\frac{N}{9} \times \frac{N}{9}$ in dimension. In addition, the left hand side of (34) are connected to the systematic symbols while the right hand side are connected to the parity check bits.

4.4.3 Controlled Doping Root-LDPC Codes $R = \frac{1}{4}$

For the case of rate $\frac{1}{4}$ with $F = 4$, we have adopted a different type of structure to guarantee the full diversity of the code. In addition, we have also rearranged the rows of the PCM in order to obtain a non singular sub-matrix in the parity check part which corresponds to the right hand side of (35), where the subscripts of $\mathbf{P}_i$ in (35) means that are distinct permutation sub-matrices. The sub-matrices of eq. (35) are all $\frac{N}{16} \times \frac{N}{16}$ in dimension. Moreover, the left hand side of (35) are connected to the systematic symbols while the right hand side are connected to the parity check bits.

$$\mathbf{H} = \left[\begin{array}{cccc|cccccccccccc} \mathbf{H}_1 & \mathbf{I} & \mathbf{H}_1 & \mathbf{H}_1 & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{H}_1 & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{P}_1 & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{H}_1 & \mathbf{0} & \mathbf{H}_1 & \mathbf{I} & \mathbf{P}_2 & \mathbf{P}_3 & \mathbf{DD} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{H}_1 & \mathbf{0} & \mathbf{H}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1 & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_4 & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1 & \mathbf{H}_1 & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_5 & \mathbf{P}_6 & \mathbf{DD} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{H}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{H}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_7 & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{H}_1 & \mathbf{H}_1 & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_8 & \mathbf{P}_9 & \mathbf{DD} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{H}_1 & \mathbf{0} & \mathbf{H}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{H}_1 & \mathbf{I} & \mathbf{0} & \mathbf{H}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_{10} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{H}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_{11} & \mathbf{P}_{12} & \mathbf{DD} \end{array}\right], \tag{35}$$

4.5 Proposed Design Algorithm

Here, we introduce some definitions and a specific notation. Then, we present the pseudo-code of our proposed algorithm. In this work, the scenarios of a block-fading channel with $F = 2$, $F = 3$ and $F = 4$ are considered. In extending to a greater number of fading, $F > 4$, the general structure presented is maintained.

4.5.1 Pseudo-code for the PEG-CDRC LDPC Algorithm

Initialization: A matrix of size $M \times N$ is created with the identity matrices $\mathbf{I}$, dual diagonal matrices $\mathbf{DD}$ and parity matrices $\mathbf{H}_i$ in the positions shown in (33), (34),

(35) and zeros in all other positions. We define the indicator vectors $\mathbf{z}_1, \cdots, \mathbf{z}_F$ for the cases $R = \frac{1}{2}$, $R = \frac{1}{3}$ and $R = \frac{1}{4}$ respectively as:

$$\begin{aligned} \mathbf{z}_1 &= [\mathbf{0}_{1\times\gamma}, \mathbf{1}_{1\times\gamma}]^T, \\ \mathbf{z}_2 &= [\mathbf{1}_{1\times\gamma}, \mathbf{0}_{1\times\gamma}]^T, \end{aligned} \tag{36}$$

$$\begin{aligned} \mathbf{z}_1 &= [\mathbf{0}_{1\times2\chi}, \mathbf{1}_{1\times\chi}, \mathbf{0}_{1\times\chi}, \mathbf{1}_{1\times\chi}, \mathbf{0}_{1\times\chi}]^T, \\ \mathbf{z}_2 &= [\mathbf{1}_{1\times\chi}, \mathbf{0}_{1\times4\chi}, \mathbf{1}_{1\times\chi}]^T, \\ \mathbf{z}_3 &= [\mathbf{0}_{1\times\chi}, \mathbf{1}_{1\times\chi}, \mathbf{0}_{1\times\chi}, \mathbf{1}_{1\times\chi}, \mathbf{0}_{1\times2\chi}]^T, \end{aligned} \tag{37}$$

$$\begin{aligned} \mathbf{z}_1 &= [\mathbf{1}_{1\times3\zeta}, \mathbf{0}_{1\times5\zeta}, \mathbf{1}_{1\times\zeta}, \mathbf{0}_{1\times\zeta}, \mathbf{1}_{1\times\zeta}, \mathbf{0}_{1\times\zeta}]^T, \\ \mathbf{z}_2 &= [\mathbf{0}_{1\times3\zeta}, \mathbf{1}_{1\times3\zeta}, \mathbf{0}_{1\times2\zeta}, \mathbf{1}_{1\times2\zeta}, \mathbf{0}_{1\times2\zeta}]^T, \\ \mathbf{z}_3 &= [\mathbf{1}_{1\times\zeta}, \mathbf{0}_{1\times\zeta}, \mathbf{1}_{1\times\zeta}, \mathbf{0}_{1\times2\zeta}, \mathbf{1}_{1\times3\zeta}, \mathbf{0}_{1\times4\zeta}]^T, \\ \mathbf{z}_4 &= [\mathbf{1}_{1\times\zeta}, \mathbf{0}_{1\times2\zeta}, \mathbf{1}_{1\times\zeta}, \mathbf{0}_{1\times5\zeta}, \mathbf{1}_{1\times3\zeta}]^T, \end{aligned} \tag{38}$$

where $\gamma = \frac{N}{2}$, $\chi = \frac{N}{9}$ and $\zeta = \frac{N}{16}$.

These indicator vectors are modelled on the basis of the original PEG algorithm [17], indicating sub-matrices for which placement is permitted, thus imposing the form of (33), (34) and (35). The degree sequence as defined for LDPC codes must be altered to take into account the structure imposed by Root-Check codes, namely, the identity matrices $\mathbf{I}$, the permutation matrices $\mathbf{P}_i$, the dual diagonal matrices $\mathbf{DD}$ and the parity matrices $\mathbf{H}_i$, of (33), (34) and (35). The pseudo-code for our proposed CDRC-LDPC algorithm is detailed in Algorithm 2, where the indicator vector $\mathbf{z}_i$ is taken from (36), (37) and (38) for constructing codes of rate $R = \frac{1}{2}$, $R = \frac{1}{3}$ and $R = \frac{1}{4}$ respectively.

5 Discussion

In this section we analyse the advantages and disadvantages of different types of PEG-based Root-Check LDPC codes discussed in the previous sections.

In terms of performance the PEG-based Root-Check LDPC codes are able to get closer to the outage curve than their counterpart Root-Check LDPC codes. However, the complexity of encoding standard PEG-based Root-Check LDPC codes can be prohibitive for some hardware implementations.

The Quasi-Cyclic PEG-based Root-Check LDPC codes have the advantage of performing better than Quasi-Cyclic Root-Check LDPC codes and both codes require low memory to store the PCM. Moreover, Quasi-Cyclic based LDPC codes can be encoded by using simple shift registers.

RA PEG-based Root-Check LDPC codes have the advantage of being simple to encode and also simple to design the PCM. Furthermore, the parity part of an RA-based PCM is a dual diagonal which is straightforward to obtain the generator matrix [18, pp. 267-279]. Such codes perform very close to the channel capacity which is usually upper-bounded by the outage curve. In addition, RA-based LDPC codes can provide: low complexity to encode, simplicity on the design of the PCM and low memory is required to store them. On the other hand, the main limitation of RA-based codes is the code rate, which cannot be higher than $\frac{1}{2}$.

In the case of Unstructured Full Diversity LDPC codes, they draw an important path in terms of designing the PCM which avoids the constraints that must be imposed to produce Root-Check based LDPC codes. Nonetheless, they require a more complex encoding process which is the same complexity as the case of Random LDPC codes.

As discussed previously, the PEG Controlled Doping Root-Check LDPC codes are able to guarantee full diversity for the parity check bits. These LDPC codes are relevant for the case of IDD in MIMO systems. The results presented in [21] demonstrate how useful are PEG-CDRC LDPC codes for MIMO systems in a block-fading channel. In addition, our proposed PEG-CDRC LDPC codes have the advantage of being RA-based encodable which are simple to encode and the PCM is easily designed.

6 Simulations

The performance of the proposed PEG-based Root-Check LDPC codes for block-fading channels with $F = 2$, $F = 3$ and $F = 4$ independent fading blocks is analysed. The block length of the codes for rates $R = \frac{1}{2}$ and $R = \frac{1}{4}$ is $L = 1024$ while for rate $R = \frac{1}{3}$ the block length is $L = 900$. The standard SPA algorithm is employed at the decoder with a maximum of 5 iterations for rate $R = \frac{1}{2}$ and for rates $R = \frac{1}{3}$ and $R = \frac{1}{4}$ a maximum of 20 iterations were used. The Gaussian outage limit in (3) is drawn in dashed line in each figure for reference.

In Fig. 4 we compare the FER performance among the proposed PEG-CDRC LDPC codes, IRA PEG Root-Check LDPC code, IRAA PEG Root-Check LDPC codes, PEG-Root-Check LDPC, Random Root-Check LDPC and PEG based LDPC [17] codes, all for $R = \frac{1}{2}$. From the results, it can be noted that the proposed PEG-CDRC LDPC code, IRAA-PEG Root-Check LDPC code and PEG-Root-Check LDPC code achieve the same FER performance. Moreover, note that all Root-Check-based codes are able to achieve the full diversity order of the channel, while (non root-check) PEG LDPC codes fail to achieve full diversity. The PEG-based Root-Check LDPC codes outperform the PEG LDPC code by about $7.5dB$.

In Fig. 5 we compare the FER performance between the proposed PEG-CDRC LDPC, QC-PEG-Root-Check LDPC, IRA PEG Root-Check LDPC code, IRAA PEG Root-Check LDPC codes, and QC-PEG LDPC codes, all for $R = \frac{1}{3}$. From the results, it can be seen that the best performance is achieved by the proposed Quasi-Cyclic PEG Root-Check LDPC code. IRA-PEG Root-Check LDPC and IRAA-PEG Root-Check LDPC have in average the same performance in terms of FER. The PEG-CDRC LDPC code is performing marginally worst than IRA and IRAA PEG Root-Check based LDPC codes. It was required to sacrifice the FER performance of the proposed PEG-CDRC LDPC codes to guarantee the full diversity of the parity check bits. Moreover, note that the proposed CDRC-LDPC code outperforms the QC-PEG LDPC code by about $2dB$ in terms of SNR. The proposed QC-PEG-Root-Check LDPC code outperforms the QC-PEG LDPC code by about $3.5dB$.

In Fig. 6 we compared the FER performance between the proposed PEG-CDRC LDPC, QC-PEG-Root-Check LDPC codes and QC-PEG LDPC codes all for $R = \frac{1}{4}$. The codeword length is $L = 1024$ bits. From the results, it can be noted that the proposed PEG-CDRC LDPC code outperforms the QC-PEG LDPC code by about $1.5dB$ while the proposed QC-PEG-Root-Check LDPC code outperforms the QC-PEG LDPC code by about $2.5dB$. In addition, note that only the PEG-based Root-Check LDPC codes are able to achieve the full diversity order of the channel.

Fig. 7 shows the average number of iterations required by the proposed PEG-CDRC LDPC codes, IRA PEG Root-Check LDPC code, IRAA PEG Root-Check LDPC codes, PEG-Root-Check LDPC, Random Root-Check LDPC and PEG based LDPC [17] codes, all for $R = \frac{1}{2}$. For the entire SNR region, in average, we can observe that the proposed PEG Root-Check based LDPC codes require less decoding iterations than standard PEG LDPC code. It must be mentioned that for medium to high SNR the average required number of iterations is less than 2 iterations. The average number of iterations, less than 2 at medium to high SNR, corroborates with hardware friendly capabilities of structured LDPC codes [5].

7 Conclusion

Novel PEG-based algorithms have been proposed to design Controlled Doping Root-Check LDPC codes, IRA Root-Check LDPC codes, IRAA Root-Check LDPC codes and Quasi-Cyclic Root-Check LDPC codes for $F \geq 2$ fading blocks. Based on simulations, the proposed methods were compared to non Root-Check LDPC codes. The results demonstrate that the Root-Check-based LDPC codes generated by our proposed algorithm outperform standard LDPC codes. Furthermore, for the case of rate $R = \frac{1}{2}$ the PEG-based Root-Check LDPC codes outperform the PEG LDPC code by about $7.5dB$. As mentioned before, the proposed PEG-CDRC LDPC codes are RA based LDPC codes which are simple to encode and the PCM can be easily designed.

Competing interests
The authors declare that they have no competing interests.

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References

1. IEEE-WLAN 802.11ad: Very High Throughput 60 GHz
2. Kambhampati, S.K., Lechner, G., Chan, T., Rasmussen, L.K.: Check splitting of root-check LDPC codes over ARQ block-fading channels. In: Proc. Australian Communications Theory Workshop'10, Canberra, Australia, pp. 123–127 (2010)
3. Rappaport, T.S.: Wireless Communications Principles and Practice, 1st ed edn. Prentice-Hall, New Jersey (1999)
4. Boutros, J.J., Guillen i Fabregas, A., Biglieri, E., Zemor, G.: Design and analysis of low-density parity-check codes for block-fading channels. In: Proc. IEEE ITA'07, pp. 54–62 (2007)
5. Li, Y., Salehi, M.: Quasi-cyclic ldpc code design for block-fading channels. In: Proc. IEEE CISS'10, pp. 1–5 (2010)
6. Uchôa, A.G.D., Healy, C., de Lamare, R.C., Souza, R.D.: Design of ldpc codes based on progressive edge growth techniques for block fading channels. IEEE Comms. Letters **15**, 1221–1223 (2011)
7. Duyck, D., Boutros, J.J., Moeneclaey, M.: Full diversity random ldpc codes. In: Communications and Vehicular Technology in the Benelux (SCVT), 2011 18th IEEE Symposium On, pp. 1–6 (2011)
8. Healy, C.T., Lamare, R.C.d.: Full diversity ldpc codes with a reduced structure for general block fading channels. In: Wireless Communication Systems (ISWCS 2013), Proceedings of the Tenth International Symposium On, pp. 1–5 (2013)

9. Uchôa, A.G.D., Healy, C., de Lamare, R.C., Souza, R.D.: Generalised Quasi-Cyclic LDPC codes based on progressive edge growth techniques for block fading channels. In: Proc. IEEE ISWCS'12, Paris, France, pp. 1–5 (2012)
10. Uchôa, A.G.D., Healy, C., de Lamare, R.C.: Repeat accumulate based constructions for LDPC codes on fading channels. In: Proc. IEEE ISWCS'13, Ilmenau, Germany, pp. 1–5 (2013)
11. Boutros, J.J.: Controlled doping via high-order rootchecks in graph codes. In: Proc. CTW'11, pp. 1–10 (2011)
12. Wang, X., Poor, H.V.: Iterative (turbo) soft interference cancellation and decoding for coded cdma. IEEE Trans. Commun. **47**(7), 1046–1061 (1999)
13. Golden, G.D., Foschini, C.J., Valenzuela, R.A., Wolniansky, P.W.: Detection algorithm and initial laboratory results using v-blast space-time communication architecture. Electron. Lett. **35**, 14–16 (1999)
14. Fa, R., de Lamare, R.C.: Multi-branch successive interference cancellation for MIMO spatial multiplexing systems: design, analysis and adaptive implementation. IET Commun. **5**, 484–494 (2011)
15. Richardson, T., Urbanke, R.: The capacity of low density parity check codes under message passing decoding. IEEE Trans. Inform. Theory **47**, 599–618 (2001)
16. Li, Z., Kumar, B.V.K.V.: A class of good quasi-cyclic low-density parity check codes based on progressive edge growth graph. In: IEEE ASILOMARSSC'04, pp. 1990–1994 (2004)
17. Hu, X.-Y., Eleftheriou, E., Arnold, D.M.: Regular and irregular progressive edge-growth tanner graphs. IEEE Trans. Inform. Theory **51**(1), 386–398 (2005)
18. Ryan, W.E., Lin, S.: Channel Codes Classical and Modern. Cambridge University Press, ??? (2009)
19. Uchôa, A.G.D., Healy, C., de Lamare, R.C., Souza, R.D.: LDPC codes based on progressive edge growth techniques for block fading channels. In: Proc. IEEE ISWCS'11, Aachen, Germany, pp. 1–5 (2011)
20. Johnson, S., Weller, S.: Interleaver and accumulator design for systematic repeat-accumulate codes. In: 6th Australian Communications Theory Workshop, pp. 1–7 (2005)
21. Uchôa, A.G.D., Healy, C., de Lamare, R.C., Li, P.: Iterative detection and decoding algorithms for block-fading channels using LDPC codes. In: Proceedings of WCNC 2014 (2014)

Figures

Figure 1 PCM unstructured general case. Parity-check matrix for the general case.

Figure 2 RA code block diagram. A systematic repeat-accumulate code block diagram, where K is the number of information bits and p denotes the parity bits.

Figure 3 IRAA code block diagram. An systematic irregular repeat-accumulate and accumulate code block diagram. Where K are the information bits, b and p are the parity bits.

Figure 4 FER performance $F = 2$. FER performance for the PEG-CDRC LDPC, IRA-PEG Root-Check LDPC, IRAA-PEG Root-Check LDPC, PEG-Root-Check LDPC, Random Root-Check LDPC and PEG LDPC codes over a block-fading channel with $F = 2$ and $L = 1024$ bits. The maximum number of iterations is 5.

Figure 5 FER performance $F = 3$. FER performance for the CDRC-LDPC, QC-PEG-Root-Check LDPC , IRA-PEG Root-Check LDPC, IRAA-PEG Root-Check LDPC and QC-PEG LDPC codes over a block-fading channel with $F = 3$ and $L = 900$ bits. The maximum number of iterations is 20.

Figure 6 FER performance $F = 4$. FER performance for the PEG-CDRC LDPC, QC-PEG-Root-Check LDPC and QC-PEG LDPC codes over a block-fading channel with $F = 4$ and $L = 1024$ bits. The maximum number of iterations is 20.

Figure 7 Iterations performance comparison for $F = 2$. Average number of required iterations for the proposed PEG-CDRC LDPC codes, IRA PEG Root-Check LDPC code, IRAA PEG Root-Check LDPC codes, PEG-Root-Check LDPC, Random Root-Check LDPC and PEG based LDPC codes with codeword length $L = 1024$ bits over a block-fading channel with $F = 2$. Maximum number of iterations 5.