

# Blind adaptive decision feedback CDMA receivers for dispersive channels

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Blind adaptive decision feedback (DF) receivers for direct sequence code division multiple access (DS-CDMA) systems in dispersive channels are proposed. Blind adaptive stochastic gradient algorithms are developed for use with the constrained minimum variance and constrained constant modulus receivers along with successive and parallel DF structure.

*Introduction:* Blind adaptive linear receivers for DS-CDMA have been proposed in recent years to suppress multiple access interference (MAI) [1–4]. Blind solutions for flat channels based on the minimum variance (MV) and the constant modulus (CM) criterion have been reported in [1] and [2]. Recently, constrained minimum variance (CMV) and constrained constant modulus (CCM) adaptive linear receivers [3, 4] were proposed in order to operate in dispersive channels. Although relatively simple, decision feedback (DF) structures can perform significantly better than linear systems [5,6] and the work on blind adaptive DF receivers offers only single-path channel solutions [7]. In this Letter we propose blind adaptive stochastic gradient (SG) algorithms for frequency selective channels in DS-CDMA systems based on constrained optimisation techniques using the MV and CM criteria.

*DS-CDMA system model:* Let us consider the uplink of a synchronous DS-CDMA BPSK system with  $K$  users,  $N$  chips per symbol and  $L_p$  propagation paths. Assuming that the channel is constant during each symbol interval and the receiver is synchronised with the main path, the composite received signal, after coherent demodulation and filtering by a chip-pulse matched filter, is sampled at chip rate to yield the  $N + L_p - 1$  received vector

$$\mathbf{r}(i) = \sum_{k=1}^K \mathbf{h}_k(i) * A_k \mathbf{s}_k \mathbf{b}_k(i) + \mathbf{n}(i) \quad (1)$$

used by the blind CDMA receiver to detect the  $i$ th transmitted symbol. In (1),  $\mathbf{n}(i) = [n_1(i), \dots, n_{N+L_p-1}(i)]^T$  is the Gaussian noise vector with  $E[\mathbf{n}(k)\mathbf{n}^T(i)] = \sigma^2 \mathbf{I}$ ,  $A_k$  is the amplitude for user  $k$ ,  $\mathbf{b}_k(i) = [b_k(i), \dots, b_k(i - L_s + 1)]^T$  is the  $k$ th user symbol vector, where  $b_k \in \{\pm 1\}$ , and  $L_s$  is the intersymbol interference (ISI) span. The symbol  $*$  denotes convolution, the  $k$ th user channel vector is  $\mathbf{h}_k(i) = [h_{k,0}(i), \dots, h_{k,L_p-1}(i)]^T$  and the  $(L_s \times N) \times L_s$  diagonal matrix  $\mathbf{S}_k$  with non-overlapping versions of the signature of user  $k$  is described by

$$\mathbf{S}_k = \begin{bmatrix} \mathbf{s}_k & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{s}_k \end{bmatrix} \quad (2)$$

where  $\mathbf{s}_k = [a_k(1), \dots, a_k(N)]^T$  is the binary  $\{\pm 1\}$  signature sequence for the  $k$ th user.

*Constrained decision feedback receivers:* Consider the  $(N + L_p - 1) \times L_p$  constraint matrix  $\mathbf{C}_k$  that contains shifted versions of the signature sequences of user  $k$

$$\mathbf{C}_k = \begin{bmatrix} a_k(1) & & \mathbf{0} \\ \vdots & \ddots & \\ a_k(N) & & \vdots \\ \mathbf{0} & \ddots & a_k(N) \end{bmatrix} \quad (3)$$

The DF receiver design is equivalent to determining a feedforward filter  $\mathbf{w}_k$  with  $N + L_p - 1$  elements and a feedback filter  $\mathbf{f}_k$  with  $K$  elements that provide an estimate of the desired symbol:

$$z_k(i) = \mathbf{w}_k^T(i)\mathbf{r}(i) - \mathbf{f}_k^T(i)\hat{\mathbf{b}}(i), \quad k = 1, 2, \dots, K \quad (4)$$

where  $\hat{b}_k = \text{sgn}(z_k)$ ,  $\hat{\mathbf{b}}(i)$  is the  $K \times 1$  vector of decisions,  $\mathbf{w}_k$  and  $\mathbf{f}_k$  are optimised by a given criterion (i.e. the MV or CM cost function), subject to the multipath constraint  $\mathbf{C}_k^T \mathbf{w}_k(i) = \mathbf{g}_k(i)$  where  $\mathbf{g}_k(i)$  is a parameter vector to be optimised. For successive DF (S-DF) [5],

the  $K \times K$  matrix  $\mathbf{f}(i) = [\mathbf{f}_1(i), \dots, \mathbf{f}_K(i)]^T$  is strictly lower triangular, whereas for parallel DF (P-DF) [6]  $\mathbf{f}(i)$  is full and constrained to have zeros along the diagonal to avoid cancelling the desired symbols.

*Adaptive algorithms for DF receivers:* We describe normalised SG algorithms for the blind estimation of the channel, the feedforward and feedback sections of DF receivers using the CM and MV criteria along with constrained optimisation techniques. In terms of computational complexity these algorithms require  $O(N + L_p - 1 + K)$  operations per user in order to suppress MAI and ISI and about  $O((L_p)^2 + (L_p)(N + L_p - 1))$  [3] for channel estimation.

*A. Normalised constrained minimum variance (NCMV) algorithm:* Consider the proposed MV cost function

$$J_k = (\mathbf{w}_k^T(i)\mathbf{r}(i) - \mathbf{f}_k^T(i)\hat{\mathbf{b}}(i))^2 + \boldsymbol{\lambda}^T (\mathbf{C}_k^T \mathbf{w}_k(i) - \mathbf{g}_k(i)) \quad (5)$$

where  $\boldsymbol{\lambda}$  is a vector of Lagrange multipliers. An SG solution to (5) is devised by taking the gradient terms with respect to  $\mathbf{w}_k(i)$ ,  $\mathbf{f}_k(i)$  and  $\mathbf{g}_k(i)$  as given by  $\mathbf{w}_k(i+1) = \mathbf{w}_k(i) - \mu_w \nabla J_k(\mathbf{w}_k(i))$ ,  $\mathbf{f}_k(i+1) = \mathbf{f}_k(i) - \mu_f \nabla J_k(\mathbf{f}_k(i))$  and  $\mathbf{g}_k(i+1) = \mathbf{g}_k(i) + \mu_h \nabla J_k(\mathbf{g}_k(i))$  which will adaptively minimise  $J_k$  with respect to  $\mathbf{w}_k(i)$  and  $\mathbf{f}_k(i)$ , and maximise  $J_k$  with respect to  $\mathbf{g}_k(i)$ . Using the constraints and following the approach in [3], we arrive at the update rules for the adaptive estimation of  $\mathbf{w}_k(i)$ ,  $\mathbf{f}_k(i)$  and  $\mathbf{g}_k(i)$ :

$$\mathbf{w}_k(i+1) = \mathbf{P}_k(\mathbf{w}_k(i) - \mu_w z_k(i)\mathbf{r}(i)) + \mathbf{C}_k(\mathbf{C}_k^T \mathbf{C}_k)^{-1} \mathbf{g}_k(i) \quad (6)$$

$$\mathbf{f}_k(i+1) = \mathbf{f}_k(i) - \mu_f z_k(i)\hat{\mathbf{b}}(i) \quad (7)$$

$$\mathbf{g}_k(i+1) = \mathbf{g}_k(i) - \frac{\mu_h}{\mu_w} \left( \mathbf{I} - \frac{\mathbf{g}_k(i)\mathbf{g}_k^T(i)}{\mathbf{g}_k^T(i)\mathbf{g}_k(i)} \right) (\mathbf{C}_k^T \mathbf{C}_k)^{-1} \times (\mathbf{C}_k^T (\mathbf{w}_k(i) - \mu_w z_k(i)\mathbf{r}(i)) - \mathbf{g}_k(i)) \quad (8)$$

where  $\mathbf{P}_k = \mathbf{I} - \mathbf{C}_k(\mathbf{C}_k^T \mathbf{C}_k)^{-1} \mathbf{C}_k^T$  is a matrix that projects the receiver's parameters onto another hyperplane to ensure the constraints. To guarantee that  $\|\mathbf{g}_k(i)\| = 1$  at each iteration we normalise  $\mathbf{g}_k(i+1)$  by making  $\mathbf{g}_k(i+1) \leftarrow \mathbf{g}_k(i+1) / \|\mathbf{g}_k(i+1)\|$ . A normalised version of this algorithm is developed by substituting (6) and (7) into the MV cost function, differentiating it with respect to  $\mu_w$  and  $\mu_f$ , setting it to zero and solving the new equation. Hence, the variable step size mechanisms are  $\mu_w(i) = (\mu_{0_w}(1 - \mu_f(i)\hat{\mathbf{b}}^T(i)\hat{\mathbf{b}}(i))) / (\mathbf{r}^T(i)\mathbf{P}_k\mathbf{r}(i))$  and  $\mu_f(i) = (\mu_{0_f}(1 - \mu_w(i)\mathbf{r}^T(i)\mathbf{P}_k\mathbf{r}(i))) / (\hat{\mathbf{b}}^T(i)\hat{\mathbf{b}}(i))$ , where  $\mu_{0_w}$  and  $\mu_{0_f}$  are the convergence factors for  $\mathbf{w}_k$  and  $\mathbf{f}_k$ , respectively. Note that  $\mathbf{g}_k(i)$  adaptively estimates the channel vector  $\mathbf{h}_k(i)$  as in [3].

*B. Normalised constrained CM (NCCM) algorithm:* Consider the proposed CM cost function

$$J_k = \frac{1}{4} ((\mathbf{w}_k^T(i)\mathbf{r}(i) - \mathbf{f}_k^T(i)\hat{\mathbf{b}}(i))^2 - 1)^2 + \boldsymbol{\lambda}^T (\mathbf{C}_k^T \mathbf{w}_k(i) - \mathbf{g}_k(i)) \quad (9)$$

An SG solution to (9) is obtained in an analogous form to the preceding Section. The update rules for the estimation of the parameters of the feedforward and feedback sections of the DF receiver and the channel are:

$$\mathbf{w}_k(i+1) = \mathbf{P}_k(\mathbf{w}_k(i) - \mu_w e_k(i)z_k(i)\mathbf{r}(i)) + \mathbf{C}_k(\mathbf{C}_k^T \mathbf{C}_k)^{-1} \mathbf{g}_k(i) \quad (10)$$

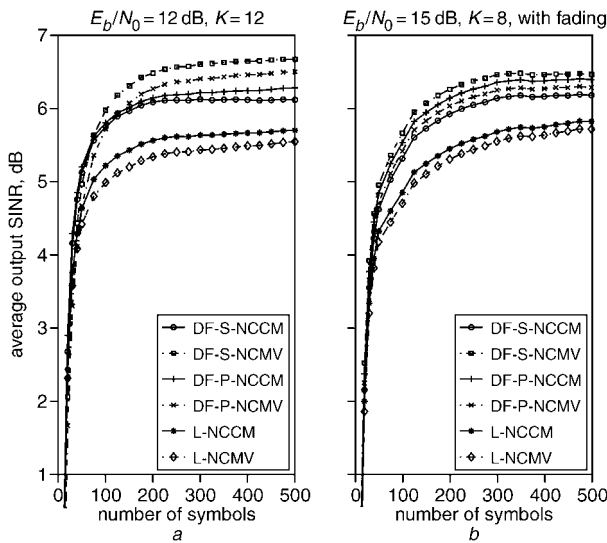
$$\mathbf{f}_k(i+1) = \mathbf{f}_k(i) - \mu_f e_k(i)z_k(i)\hat{\mathbf{b}}(i) \quad (11)$$

$$\mathbf{g}_k(i+1) = \mathbf{g}_k(i) - \frac{\mu_h}{\mu_w} \left( \mathbf{I} - \frac{\mathbf{g}_k(i)\mathbf{g}_k^T(i)}{\mathbf{g}_k^T(i)\mathbf{g}_k(i)} \right) (\mathbf{C}_k^T \mathbf{C}_k)^{-1} \times (\mathbf{C}_k^T (\mathbf{w}_k(i) - \mu_w e_k(i)z_k(i)\mathbf{r}(i)) - \mathbf{g}_k(i)) \quad (12)$$

where  $e_k(i) = z_k^2(i) - 1$ . A normalised version of this algorithm is derived similarly to the NCMV, and the adaptation mechanisms are  $\mu_w(i) = (\mu_{0_w}(z_k(i) - \mu_f(i)z_k(i)e_k(i)\hat{\mathbf{b}}^T(i)\hat{\mathbf{b}}(i+1))) / (z_k(i)e_k(i)\mathbf{r}^T(i)\mathbf{P}_k\mathbf{r}(i))$  and  $\mu_f(i) = (\mu_{0_f}(z_k(i) - \mu_w(i)z_k(i)e_k(i)\mathbf{r}^T(i)\mathbf{P}_k\mathbf{r}(i+1))) / (z_k(i)e_k(i)\hat{\mathbf{b}}^T(i)\hat{\mathbf{b}}(i))$ .

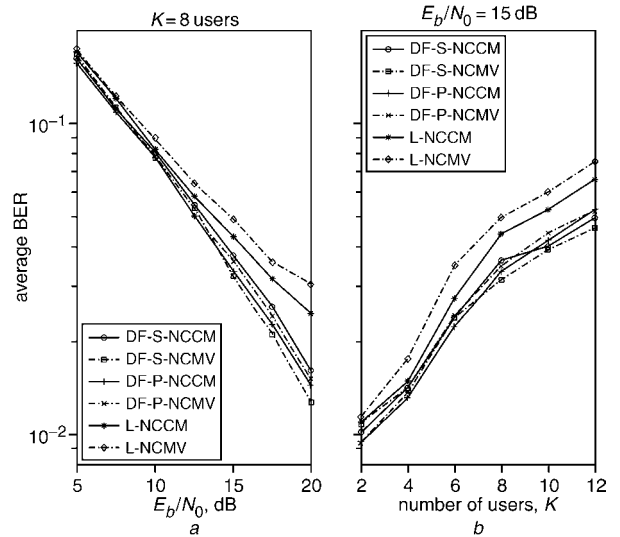
*Results:* The DS-CDMA system employs Gold sequences of length  $N = 31$  and has a bandwidth of 3.84 MHz, which corresponds to the data rate of 123.87 kbit/s. It is assumed here that the channels experienced by different users are statistically independent and identically distributed. The channel coefficients for each user  $k$  ( $k = 1, \dots, K$ ) are  $h_{k,l}(i) = p_{k,l} \alpha_{k,l}(i)$ , where  $\alpha_{k,l}(i)$ ,  $l = 0, 1, 2$ , is a

complex Gaussian random sequence obtained by passing complex white Gaussian noise through a filter with approximate transfer function  $\beta/\sqrt{1-(f/f_d)^2}$  where  $\beta$  is a normalisation constant,  $f_d = v/\lambda_c$  is the maximum Doppler shift,  $\lambda_c$  is the wavelength of the carrier frequency, and  $v$  is the speed of the mobile [8]. For each user this procedure corresponds to the generation of  $L_p$  independent sequences of correlated, unit power ( $E[|\alpha_{k,i}(i)|^2] = 1$ ), Rayleigh random variables. The channel parameters are  $p_{k,0} = 1$ ,  $p_{k,1} = 0.5$  and  $p_{k,2} = 0.3$ . In the absence of fading we have  $h_{k,i}(i) = p_{k,i}$ . We remark that the techniques examined here achieve the same hierarchy in terms of SINR [1] and BER for different scenarios, channel profiles, fading rates and power unbalances. We only show results for a simple channel profile and with perfect power control due to lack of space and for simplicity. The receivers process 2000 symbols, averaged over 100 experiments, the parameters of the algorithms are optimised for each scenario and the initial channel estimate is  $\mathbf{g}_k(0) = [0.5 \ 0 \ 0]^T$  for all approaches. The carrier frequency of the system is 1900 MHz. For linear receivers (L) and their algorithms we make  $\mathbf{f}(i) = \mathbf{0}$  and  $\mu_f = 0$ . Note that the L-NCMV and L-NCCM correspond to the algorithms in [3] and [4], respectively, with the normalised step size proposed here, and the curves shown are averaged over the user population.



**Fig. 1** SINR performance in power controlled situations  
a No fading  
b Fading scenario where mobiles move at  $v = 80$  km/h

Fig. 1 shows the SINR convergence performance of the receivers, which do not lock to an undesired user because of the joint channel estimation that allows the receiver to use the effective signature sequence, similarly to the technique reported in [3]. The performance of the DF receivers is improved by cancelling the interferers with the DF section, resulting in a higher SINR than linear ones. The BER performance with fading is shown in Fig. 2. We note that as the number of users is increased the S-DF versions outperform the P-DF ones, which suffer from error propagation at higher levels of BER. Note that a disadvantage of S-DF relative to P-DF is that it does not provide uniform performance over the user population. The blind DF techniques proposed here offer significant gains in performance at the expense of a small increase in complexity, saving transmission power and accommodating more users for the same BER performance.



**Fig. 2** BER performance against  $E_b/N_0$  and number of users ( $K$ )

a Against  $E_b/N_0$  b Against number of users ( $K$ )

Fading scenario with perfect power control, where mobiles move at  $v = 80$  km/h

**Conclusion:** Blind adaptive algorithms for DF receivers based on the CM and MV criteria are presented and examined with both successive and parallel DF versions. The proposed blind adaptive DF receivers for multipath channels showed satisfactory gains in performance over linear structures.

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