Reduced-Rank Interference Suppression for DS-CDMA Based on Interpolated FIR Filters

Rodrigo C. de Lamare and Raimundo Sampaio-Neto

***Abstract*— Reduced-rank receivers based on interpolated finite impulse response (FIR) filters for direct sequence code division multiple access (DS-CDMA) systems are proposed and a novel scheme where the interpolator is rendered time-varying is introduced. The interpolated minimum mean squared error (MMSE) and constrained minimum variance (CMV) solutions are derived for both receiver and interpolator to mitigate multiple access interference (MAI) and intersymbol interference (ISI) in a downlink scenario.**

***Index Terms*— Interference suppression, multiuser detection, DS-CDMA, interpolated filters, reduced-rank receivers.**

I. Introduction

REDUCED-RANK interference suppression for DS-CDMA is useful in situations where the processing gain N is large and/or it is desirable to reduce the number of parameters for estimation [1]-[4] for convergence and complexity issues. Interpolated FIR (IFIR) filters [5],[6] have been widely applied in the context of digital filtering, although their use for parameter estimation in communications remains unexplored. These structures retain the advantages of original FIR filters, show better convergence rate and can reduce the computational burden for parameter estimation, due to the reduced number of elements. In this letter, a novel IFIR scheme for the suppression of MAI and ISI with IFIR filters is proposed and supervised and blind solutions for both interpolator and receiver are presented. The new structure uses a more effective time-varying interpolator rather than the fixed interpolator approach of [6].

II. DS-CDMA system model

Consider the downlink of a synchronous DS-CDMA system with K users, N chips per symbol and L_p paths. Assuming that the receiver is synchronized and the channel is constant during each symbol interval, the received signal after filtering by a chip-pulse matched filter and sampled at chip rate yields the $(M = N + L_p - 1) \times 1$ received vector

$$\mathbf{r}(i) = \sum_{k=1}^{K} A_k b_k(i) \mathbf{h}(i) \star \mathbf{s}_k + \boldsymbol{\eta}(i) + \mathbf{n}(i) \quad (1)$$

where $\mathbf{n}(i) = [n_1(i) \ \ldots \ n_M(i)]^T$ is the complex Gaussian noise vector with $E[\mathbf{n}(k)\mathbf{n}^H(i)] = \sigma^2 \mathbf{I}$, where $(.)^T$ and $(.)^H$ denotes transpose and Hermitian transpose, respectively, $E[.]$ stands for ensemble average, $b_k(i) \in \{\pm 1 + j0\}$ is the symbol for user k with $j^2 = -1$, $\boldsymbol{\eta}(i)$ is the ISI, the amplitude of user k is A_k, the channel vector is $\mathbf{h}(i) = [h_0(i) \ \ldots \ h_{L_p-1}(i)]^T$, the symbol $\star$ denotes convolution and $\mathbf{s}_k = [a_k(1) \ldots a_k(N)]^T$ is the signature sequence for the k-th user.

Manuscript received July 5, 2004. The associate editor coordinating the review of this letter and approving it for publication was Prof. Paul Cotae. This work was supported by CNPq.

The authors are with CETUC/PUC-RIO, 22453-900, Rio de Janeiro, Brazil (e-mail: delamare@infolink.com.br).

Digital Object Identifier 10.1109/LCOMM.2005.03010.

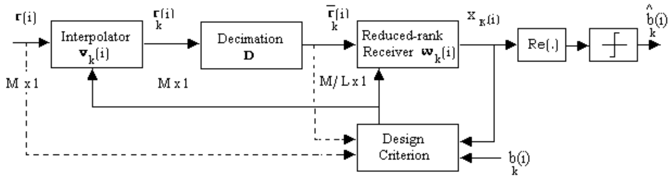

Fig. 1. Reduced-rank CDMA receiver structure.

III. Linearly Interpolated CDMA Receivers

The underlying principles of the proposed CDMA receiver structure are detailed here. Fig. 1 shows the structure of an IFIR receiver, where an interpolator and a reduced-rank receiver that are time-varying are employed. The $M \times 1$ received vector $\mathbf{r}(i) = [r_0^{(i)} \ \ldots \ r_{M-1}^{(i)}]^T$ is filtered by the interpolator filter $\mathbf{v}_k(i) = [v_{k,0}^{(i)} \ldots v_{k,N_I-1}^{(i)}]^T$, yielding the interpolated received vector $\mathbf{r}_k(i)$, which is projected onto an $M/L \times 1$-dimensional vector $\bar{\mathbf{r}}_k(i)$. This procedure corresponds to removing $L-1$ samples of $\mathbf{r}_k(i)$ of each set of L consecutive ones, and then computing the inner product of $\bar{\mathbf{r}}_k(i)$ with the M/L-dimensional vector of filter coefficients $\mathbf{w}_k(i) = [w_{k,0}^{(i)} \ \ldots \ w_{k,M/L-1}^{(i)}]^T$.

The projected interpolated observation vector $\bar{\mathbf{r}}_k(i) = \mathbf{D}\mathbf{r}_k(i)$ is obtained with the aid of the $M/L \times M$ projection matrix $\mathbf{D}$ that is mathematically equivalent to signal decimation on the $M \times 1$ vector $\mathbf{r}_k(i)$. An interpolated receiver with interpolation factor L can be designed by choosing $\mathbf{D}$ as:

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \underbrace{0 \ \ldots \ 0}_{(m-1)L \ zeros} & & & 1 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \multicolumn{7}{c}{\underbrace{0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ldots \ 0}_{(M/L-1)L \ zeros}} & 1 & \multicolumn{3}{c}{\underbrace{0 \ \ldots \ 0}_{(L-1) \ zeros}} \end{bmatrix} \quad (2)$$

where m ($m = 1, 2, \ldots, M/L$) denotes the m-th row. The strategy, that allows us to devise solutions for both interpolator and receiver, is to express the estimated symbol $x_k(i) = \mathbf{w}_k^H(i)\bar{\mathbf{r}}_k(i)$ as a function of $\mathbf{w}_k(i)$ and $\mathbf{v}_k(i)$:

$$x_k(i) = \mathbf{v}_k^H(i) \Big[\dot{\mathbf{r}}_0^{(i)} | \ \ldots \ | \dot{\mathbf{r}}_{M/L-1}^{(i)} \Big] \mathbf{w}_k^*(i) = \mathbf{v}_k^H(i) \boldsymbol{\Re}(i) \mathbf{w}_k^*(i) \quad (3)$$

where $\mathbf{u}_k(i) = \mathfrak{R}(i)\mathbf{w}_k^*(i)$ is an $N_I \times 1$ vector, $(.)^*$ denotes complex conjugate, the M/L coefficients of $\mathbf{w}_k(i)$ and the N_I elements of $\mathbf{v}_k(i)$ are assumed to be complex, the asterisk denotes complex conjugation and $\dot{\mathbf{r}}_s(i)$ is a length N_I segment of the received vector $\mathbf{r}(i)$ beginning at $r_{s\times L}(i)$ and

$$\mathfrak{R}(i) = \begin{bmatrix} r_0^{(i)} & r_L^{(i)} & \cdots & r_{(M/L-1)L}^{(i)} \\ r_1^{(i)} & r_{L+1}^{(i)} & \cdots & r_{(M/L-1)L+1}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ r_{N_I-1}^{(i)} & r_{L+N_I}^{(i)} & \cdots & r_{(M/L-1)L+N_I-1}^{(i)} \end{bmatrix} \quad (4)$$

IV. Interpolated MMSE Receivers

The MMSE solutions for $\mathbf{w}_k(i)$ and $\mathbf{v}_k(i)$ can be computed if we consider the optimization problem whose cost function is

$$J_{MSE}(\mathbf{w}_k(i), \mathbf{v}_k(i)) = E\Big[||b_k(i) - \mathbf{v}_k^H(i)\mathfrak{R}(i)\mathbf{w}_k^*(i)||^2\Big] \quad (5)$$

By minimizing (5) with respect to $\mathbf{w}_k(i)$ and $\mathbf{v}_k(i)$ the interpolated Wiener filter/receiver and interpolator weight vectors are

$$\mathbf{w}_k(i) = \bar{\mathbf{R}}_k^{-1}(i)\bar{\mathbf{p}}_k(i) \quad (6)$$

$$\mathbf{v}_k(i) = \bar{\mathbf{R}}_{u_k}^{-1}(i)\bar{\mathbf{p}}_{u_k}(i) \quad (7)$$

where $\bar{\mathbf{R}}_k(i) = E[\bar{\mathbf{r}}_k(i)\bar{\mathbf{r}}_k^H(i)]$, $\bar{\mathbf{p}}_k(i) = E[b_k^*(i)\bar{\mathbf{r}}_k(i)]$, $\bar{\mathbf{r}}_k(i) = \mathfrak{R}^T(i)\mathbf{v}_k^*(i)$, $\bar{\mathbf{R}}_{u_k}(i) = E[\mathbf{u}_k(i)\mathbf{u}_k^H(i)]$, $\bar{\mathbf{p}}_{u_k}(i) = E[b_k^*(i)\mathbf{u}_k(i)]$ and $\mathbf{u}_k(i) = \mathfrak{R}(i)\mathbf{w}_k^*(i)$. The novel structure trades off a full-rank matrix inversion against the inversion of two matrices with rank M/L and N_I. We remark that (6) and (7) are not closed-form solutions for $\mathbf{w}_k(i)$ and $\mathbf{v}_k(i)$ since (6) is a function of $\mathbf{v}_k(i)$ and (7) depends on $\mathbf{w}_k(i)$ and thus it is necessary to iterate (6) and (7) with an initial guess to obtain a solution.

V. Interpolated CMV Receivers

Consider the $M \times L_p$ constraint matrix $\mathbf{C}_k$ that contains one-chip shifted versions of the signature sequence of user k:

$$\mathbf{C}_k = \begin{bmatrix} a_k(1) & & \mathbf{0} \\ \vdots & \ddots & a_k(1) \\ a_k(N) & & \vdots \\ \mathbf{0} & \ddots & a_k(N) \end{bmatrix} \quad (8)$$

The interpolated CMV receiver parameter vector $\mathbf{w}_k$ and the interpolator parameter vector $\mathbf{v}_k$ are obtained by minimizing

$$J_{MV}(\mathbf{w}_k, \mathbf{v}_k) = E\Big[||x_k(i)||^2\Big] = E\Big[||\mathbf{v}_k^H(i)\mathfrak{R}(i)\mathbf{w}_k^*(i)||^2\Big] \quad (9)$$

subject to the proposed constraints $\mathbf{C}_k^H\mathbf{D}^H\mathbf{w}_k(i) = \mathbf{g}(i)$ and $||\mathbf{v}_k(i)|| = 1$, where $\mathbf{g}(i)$ is an $L_p \times 1$ channel vector to be determined. Note that the proposed constraint $||\mathbf{v}_k(i)|| = 1$ ensures adequate design values for the interpolator filter $\mathbf{v}_k$, whereas $\mathbf{C}_k^H\mathbf{D}^H\mathbf{w}_k(i) = \mathbf{g}(i)$ avoids the suppression of the desired signal. Using the method of Lagrange multipliers, the expressions for both receiver and interpolator are

$$\mathbf{w}_k(i) = \bar{\mathbf{R}}_k(i)^{-1}\mathbf{D}\mathbf{C}_k(\mathbf{C}_k^H\mathbf{D}^H\bar{\mathbf{R}}_k(i)^{-1}\mathbf{D}\mathbf{C}_k)^{-1}\mathbf{g}(i) \quad (10)$$

$$\mathbf{v}_k(i) = \arg\min_{\mathbf{v}_k} \ \mathbf{v}_k^H\bar{\mathbf{R}}_{u_k}(i)\mathbf{v}_k \quad (11)$$

The solution to $\mathbf{v}_k(i)$ is the eigenvector of $\bar{\mathbf{R}}_{u_k}(i)$, which corresponds to the minimum eigenvalue, via singular value decomposition (SVD). As occurs with the MMSE approach we iterate (10) and (11) with an initial guess to obtain a CMV solution. Note also that (10) assumes the knowledge of the channel. However, in applications where multipath is present these parameters are not known and thus channel estimation is required. To blindly estimate the channel, we adopt the method of [7], [8]:

$$\hat{\mathbf{g}}(i) = \arg\min_{\mathbf{g}} \ \mathbf{g}^H\mathbf{C}_k^H\mathbf{R}^{-1}(i)\mathbf{C}_k\mathbf{g} \quad (12)$$

subject to $||\hat{\mathbf{g}}|| = 1$, where $\mathbf{R}(i) = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ and whose solution is the eigenvector corresponding to the minimum eigenvalue of the $L_p \times L_p$ matrix $\mathbf{C}_k^H\mathbf{R}(i)^{-1}\mathbf{C}_k$ through SVD.

VI. Numerical Results

The performance of the proposed receivers in different situations and loads is assessed and compared with the MMSE [9] and CMV [7] full-rank, the eigen-decomposition (PC) [1], [2], the partial despreading (PD) [3] and the multi-stage Wiener filter (MWF) [4] reduced-rank techniques with rank D. The DS-CDMA system employs Gold sequences of length $N = 31$. Since the channel length is not known a priori, we will assume that $L_p = 6$ is an upper bound for all scenarios. The channel coefficients for the users are $h_l(i) = p_l\alpha_l(i)$, where $\alpha_l(i)$, $l = 0, 1, 2$, is obtained with Clarke's model [10]. We show the results in terms of the normalized Doppler frequency f_dT (cycles/symbol). We use three-path channels with relative powers given by 0, -3 and -6 dB, where in each run the second and third paths delays are uniformly distributed between 1 and 5 chips. The phase ambiguity derived from channel estimation is eliminated in our simulations by using the phase of $\mathbf{g}(0)$ as a reference to remove the ambiguity. The received powers of the interferers are log-normal random variables with associated standard deviation 6 dB, $\lambda = 0.998$ and $f_dT = 0.0025$ for all experiments. The matrix $\bar{\mathbf{R}}_k$ is estimated as $\hat{\bar{\mathbf{R}}}_k(i) = \frac{1}{i}\sum_{n=1}^{i}\lambda^{i-n}\bar{\mathbf{r}}_k(n)\bar{\mathbf{r}}_k^H(n)$, where λ is the forgetting factor. For the MMSE receivers [9], a pilot channel (with known transmitted symbols) is used for the estimation of $\hat{\bar{\mathbf{p}}}_k(i) = \frac{1}{i}\sum_{n=1}^{i}\lambda^{i-n}\bar{\mathbf{r}}_k(n)b_k^*(n)$ and $\hat{\bar{\mathbf{p}}}_{\mathbf{u}_k}(i) = \frac{1}{i}\sum_{n=1}^{i}\lambda^{i-n}\mathbf{u}_k(n)b_k^*(n)$ in (6) and (7), respectively, whereas for blind receivers, the signature sequence is assumed to be known. The remaining receiver techniques employ analogous recursions. For moderate fading processes, $\bar{\mathbf{p}}_k$ and $\bar{\mathbf{p}}_{\mathbf{u}_k}$ can be estimated as described above, whereas for very fast fading the receivers have to be modified as suggested in [9]. To obtain the most adequate dimension for the interpolator $\mathbf{v}_k(i)$, we have conducted experiments with values ranging from $N_I = 3$ to $N_I = 6$. The results indicated that the best performance is achieved with $N_I = 3$ for a wide range of scenarios and parameters such as N, L_p and K. Note that for the MWF

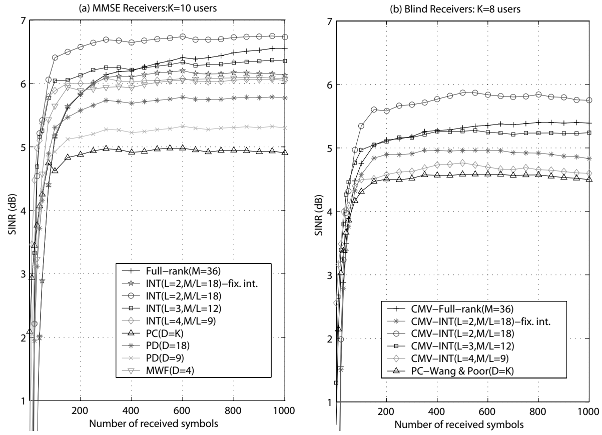

Fig. 2. SINR performance for both (a) MMSE and (b) CMV (or blind) receivers.

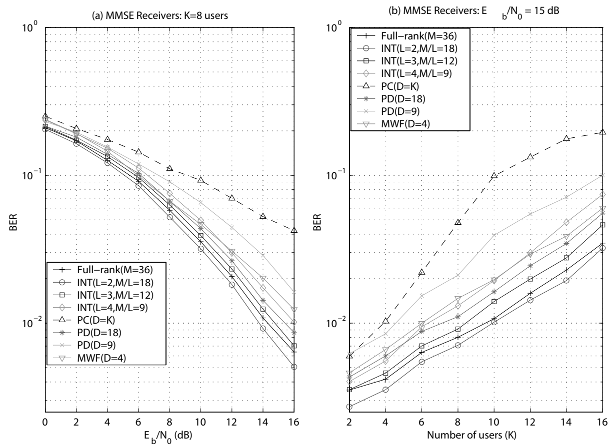

Fig. 3. BER versus (a) E_b/N_0 and (b) Number of users (K).

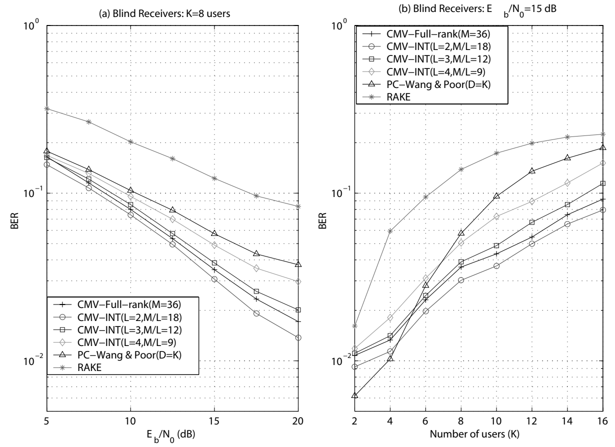

Fig. 4. BER versus (a) E_b/N_0 and (b) Number of users (K) for CMV receivers.

in the scenario considered, the use of $D = 4$ stages achieved the best performance, for the PC method in supervised [1] and blind (Wang & Poor) [2] modes we applied SVD, and the blind channel estimation of (12) [8] was used for all blind receivers.

In Fig. 2 we show the SINR [2] convergence performance of the analyzed receiver techniques. The curves show that the INT technique with L=2 and M/L=18 elements is superior to the full-rank receiver, whereas the INT(L=3,M/L=12) is close to the full-rank one, for both MMSE and CMV (blind) design approaches. Note that the INTs have a faster convergence performance than the full-rank and other reduced-rank techniques. Also, the use of a design criterion for the interpolator $\mathbf{v}_k$ can significantly increase the performance as compared to the fixed interpolator version of the INT(L=2,M/L=18)-fix. int. with $\mathbf{v}_k = [0.5\ 1\ 0.5]$ [6] (also used as an initial guess, i.e. $\mathbf{v}_k(0)$, for the new scheme).

The BER performance of the MMSE and blind receivers is illustrated in Figs. 3 and 4, where the curves are obtained after processing 2000 symbols averaged by 100 independent experiments. The results show that the INT(L=2,M/L=18) achieves the best BER performance, followed by the full-rank receiver, the INT(L=3,M/L=12), and the remaining techniques. For the blind receivers, the subspace receiver of Wang and Poor [2] has good performance for a small number of users (K). However, as K is increased, the INT receivers outperform it.

In terms of complexity, the INT requires the inversion of matrices with rank M/L and N_I (in blind mode the interpolator requires an SVD for an $N_I \times N_I$ matrix instead of an inversion), whereas the full-rank requires an $M \times M$ matrix inversion, PD requires an $D \times D$ matrix inversion, PC requires an $M \times M$ matrix SVD, and the MWF requires orthogonal decompositions.

VII. Conclusions

Reduced-rank receivers for DS-CDMA systems based on IFIR filters were presented and a new scheme where the interpolator is made time-varying was proposed. The novel structure was compared to previously reported methods and offers a very attractive trade-off between performance and complexity.

References

[1] A. M. Haimovich and Y. Bar-Ness, "An eigenanalysis interference canceler," *IEEE Trans. Signal Processing*, vol. 39, pp. 76-84, Jan. 1991.

[2] X. Wang and H. V. Poor, "Blind multiuser detection: a subspace approach," *IEEE Trans. Inform. Theory*, vol. 44, pp. 677-690, Mar., 1998.

[3] R. Singh and L. B. Milstein, "Interference suppression for DS/CDMA," *IEEE Trans. Commun.*, vol. 47, pp. 446-453, Mar. 1999.

[4] M. L. Honig and J. S. Goldstein, "Adaptive reduced-rank interference suppression based on the multistage Wiener filter," *IEEE Trans. Commun.*, vol. 50, pp. 986-994, June, 2002.

[5] Y. Neuvo, C. Y. Dong, and S. K. Mitra, "Interpolated finite impulse response filters," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol. 32, pp. 563-570, June 1984.

[6] L. S. Resende, C. A. F. Rocha, and M. G. Bellanger, "A linearly constrained approach to the interpolated FIR filtering problem", in *Proc. Int. Conf. Acoust. Speech and Sig. Proc.*, 2000, pp. 392-395.

[7] M. K. Tsatsanis and Z. Xu, "Performance analysis of minimum variance CDMA receivers," *IEEE Trans. Signal Processing*, vol. 46, pp. 3014-3022, Nov. 1998.

[8] X. G. Doukopoulos and G. V. Moustakides, "Blind channel estimation for downlink CDMA systems," in *Proc. IEEE Int. Conf. on Communications*, 2003, pp. 2416-2420.

[9] S. L. Miller, M. L. Honig, and L. B. Milstein, "Performance analysis of MMSE receivers for DS-CDMA in frequency-selective fading channels," *IEEE Trans. Commun.*, vol. 48, pp. 1919-1929, Nov. 2000.

[10] T. S. Rappaport, *Wireless Communications*. Englewood Cliffs, NJ: Prentice-Hall, 1996.